MTH-4101 Word Problems

Type 1 : Age Problems

1. Jack and Mac are talking one day and Mac says, "In 9 years I will be three times your age, but today, I am four times your age. How old are Jack and Mac?

	Present	In 9 years
Jack	x	$x+9$
Mac	y	$y+9$

① $y + 9 = 3(x+9)$

② $y = 4x$

① $y + 9 = 3x + 27$

$y = 3x + 27 - 9$

$y = 3x + 18$

$y = y$

$4x = 3x + 18$

$4x - 3x = 18$

$x = 18$ = Jack's age

$y = 4x$

$= 4(18)$

$= 72$ = Mac's age

Jack is 18 yrs old; Mac is 72 yrs old

2. The difference in age between Blythe and Roger is 15 years. In 7 years, Blythe will be twice as old as Roger. Find the age of each.

Let x = Blythe's age

Let y = Roger's age

Note: Blythe is older than Roger (imp. for first equation...

older person − younger person = 15

① $x - y = 15$

② $x + 7 = 2(y + 7)$

① $\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{15}{-1}$

$y = x - 15$

② $x + 7 = 2y + 14$

$-2y = -x + 14 - 7$

$\dfrac{-2y}{-2} = \dfrac{-x}{-2} + \dfrac{7}{-2}$

$y = \frac{1}{2}x - \frac{7}{2}$

	Present	In 7 yrs
Blythe	x	$x+7$
Roger	y	$y+7$

Let $y_1 = y_2$

$x - 15 = \frac{1}{2}x - \frac{7}{2}$

$1x - \frac{1}{2}x = 15 - \frac{7}{2}$

$\frac{1}{2}x = \frac{30}{2} - \frac{7}{2}$

$\cancel{2}^{2}\left(\frac{1}{\cancel{2}}x\right) = \left(\frac{23}{2}\right)\frac{\cancel{2}}{1}$

$x = 23$

$y = x - 15$

$= 23 - 15 = 8 = y$

Blythe is 23 yrs

Roger is 8 yrs

3. A grandfather says to his grandson, "Today I am four times as old as you are, but in seven years I will be three times as old as your age." Find the age of each person.

	Present	In 7 yrs
Grandfather	x	$x+7$
Grandson	y	$y+7$

① $x = 4y$

② $x + 7 = 3(y+7)$

① $\dfrac{4y}{4} = \dfrac{x}{4}$

$y_1 = \dfrac{1}{4}x$

② $x + 7 = 3y + 21$

$-3y = -x + 21 - 7$

$\dfrac{-3y}{-3} = \dfrac{-x}{-3} + \dfrac{14}{-3}$ $\Big\}$ $y_2 = \dfrac{1}{3}x - \dfrac{14}{3}$

Let $y_1 = y_2$

$\dfrac{1}{4}x = \dfrac{1}{3}x - \dfrac{14}{3}$

$\dfrac{1 \cdot 3}{4 \cdot 3}x - \dfrac{1 \cdot 4}{3 \cdot 4}x = -\dfrac{14}{3}$

$\dfrac{3}{12}x - \dfrac{4}{12}x = \dfrac{-14}{3}$

$-\dfrac{12}{1}\left(-\dfrac{1}{12}x\right) = \left(\dfrac{-14}{3}\right)\dfrac{-12}{1}$

$x = 56$

$y = \dfrac{1}{4}x$

$= \dfrac{1}{4}(56) = 14$

The grandfather is 56, the grandson is 14.

4. A father said to his son, "Today I am four times as old as you are, but in 14 years I will be twice your age". Find the age of each person.

	Present	In 14 yrs
Father	x	$x+14$
Son	y	$y+14$

① $x = 4y$

② $x + 14 = 2(y+14)$

① $\dfrac{4y}{4} = \dfrac{x}{4}$

$y_1 = \dfrac{1}{4}x$

② $2(y+14) = x + 14$

$2y + 28 = x + 14$

$2y = x + 14 - 28$

$\dfrac{2y}{2} = \dfrac{x}{2} - \dfrac{14}{2}$ $\Big\}$ $y_2 = \dfrac{1}{2}x - 7$

$y_1 = y_2$

$\dfrac{1}{4}x = \dfrac{1}{2}x - 7$

$\dfrac{1}{4}x - \dfrac{1 \cdot 2}{2 \cdot 2}x = -7$

$\dfrac{1}{4}x - \dfrac{2}{4}x = -7$

$-\dfrac{4}{1}\left(-\dfrac{1}{4}x\right) = (-7)-4$

$x = 28$

$y = \dfrac{1}{4}x$

$= \dfrac{1}{4}(28)$

$y = 7$

The father is 28, the son is 7 yrs.

-2-

5. Eight years ago, Pam's age was double that of Natalie's. In 12 years, Natalie's age will be ¾ that of Pam's age. How old are they now?

	Present	8 yrs ago	In 12 yrs
Pam	x	$x-8$	$x+12$
Natalie	y	$y-8$	$y+12$

① $x - 8 = 2(y-8)$

$x - 8 = 2y - 16$

$-2y = -x - 16 + 8$

$\dfrac{-2y}{-2} = \dfrac{-x}{-2} - \dfrac{8}{-2}$

$y_1 = \dfrac{1}{2}x + 4$

② $y + 12 = \dfrac{3}{4}(x+12)$

$y + 12 = \dfrac{3}{4}x + 9$

$y = \dfrac{3}{4}x + 9 - 12$

$y_2 = \dfrac{3}{4}x - 3$

$y_1 = y_2$

$\dfrac{1}{2}x + 4 = \dfrac{3}{4}x - 3$

$\dfrac{1 \cdot 2}{2 \cdot 2}x - \dfrac{3}{4}x = -3 - 4$

$\dfrac{2}{4}x - \dfrac{3}{4}x = -7$

$-\dfrac{4}{1}\left(-\dfrac{1}{4}x\right) = \left(\dfrac{-7}{1}\right)\dfrac{-4}{1}$

$x = 28$

$y = \dfrac{1}{2}x + 4$

$= \dfrac{1}{2}(28) + 4$

$= 14 + 4 = 18$

Pam is 28 yrs, Natalie is 18 yrs.

6. 19 years ago Pat's mom was 24 times his age. In 3 years Pat's mom will be double his age. Determine Pat's age and his mom's age.

	Present	19 yrs ago	In 3yrs
Pat	x	$x-19$	$x+3$
Mom	y	$y-19$	$y+3$

① $y - 19 = 24(x-19)$

② $y + 3 = 2(x+3)$

① $y - 19 = 24(x-19)$

$y - 19 = 24x - 456$

$y = 24x - 456 + 19$

$y_1 = 24x - 437$

② $y + 3 = 2(x+3)$

$y + 3 = 2x + 6$

$y = 2x + 6 - 3$

$y_2 = 2x + 3$

$y_1 = y_2$

$24x - 437 = 2x + 3$

$24x - 2x = 3 + 437$

$\dfrac{22x}{22} = \dfrac{440}{22}$

$x = 20$

$y = 2x + 3$

$= 2(20) + 3$

$= 40 + 3$

$y = 43$

Pat is 20yrs; Mom is 43yrs.

7. Stephen is twice Matthew's age. The difference between their ages is 30 years. What are their ages?

	Present
Stephen	x
Matthew	y

① $x = 2y$

② $x - y = 30$

① $x = 2y$

$\dfrac{2y}{2} = \dfrac{x}{2}$

$y_1 = \dfrac{1}{2}x$

② $x - y = 30$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{30}{-1}$

$y_2 = x - 30$

$y_1 = y_2$

$\dfrac{1}{2}x = x - 30$

$\dfrac{1}{2}x - 1x = -30$

$-\dfrac{2}{1}\left(-\dfrac{1}{2}x\right) = (-30) - 2$

$x = 60$

$y = \dfrac{1}{2}x = \dfrac{1}{2}(60)$

$= 30$

Stephen is 60 yrs old;
Matthew is 30 yrs old.

8. Today Max is three times as old as Mary. In thirteen years Max will be twice Mary's age. How old is each person?

	Present	In 13 yrs
Max	x	$x + 13$
Mary	y	$y + 13$

① $x = 3y$ $\Big\}$ $y_1 = \dfrac{1}{3}x$

② $x + 13 = 2(y + 13)$

$x + 13 = 2y + 26$

$-2y = -x + 26 - 13$

$\dfrac{-2y}{-2} = \dfrac{-x}{-2} + \dfrac{13}{-2}$

$y_2 = \dfrac{1}{2}x - \dfrac{13}{2}$

$y_1 = y_2$

$\dfrac{1}{3}x = \dfrac{1}{2}x - \dfrac{13}{2}$

$\dfrac{1 \cdot 2}{3 \cdot 2}x - \dfrac{1 \cdot 3}{2 \cdot 3}x = \dfrac{-13}{2}$

$\dfrac{2}{6}x - \dfrac{3}{6}x = \dfrac{-13}{2}$

$-\dfrac{6}{1}\left(-\dfrac{1}{6}x\right) = \left(-\dfrac{13}{2}\right)\dfrac{-6}{1}$

$x = 39$

$y = \dfrac{1}{3}x = \dfrac{1}{3}(39) = 13$

Max is 39 yrs;
Mary is 13 yrs.

9. Seven years ago Sara was six times as old as her sister Julie. In sixteen years, Julie will be 5/7 Sara's age. How old is each sister?

	Present	7 yrs ago	In 16 yrs
Sara	x	$x-7$	$x+16$
Julie	y	$y-7$	$y+16$

① $x-7=6(y-7)$

$x-7=6y-42$

$-6y=-x-42+7$

$\dfrac{-6y}{-6}=\dfrac{-x}{-6}-\dfrac{35}{-6}$

$y_1=\dfrac{1}{6}x+\dfrac{35}{6}$

② $y+16=\dfrac{5}{7}(x+16)$

$y+16=\dfrac{5}{7}x+\dfrac{80}{7}$

$y=\dfrac{5}{7}x+\dfrac{80}{7}-16\cdot\dfrac{7}{7}$

$y=\dfrac{5}{7}x+\dfrac{80}{7}-\dfrac{112}{7}$

$y_2=\dfrac{5}{7}x-\dfrac{32}{7}$

$y_1=y_2$

$\dfrac{1}{6}x+\dfrac{35}{6}=\dfrac{5}{7}x-\dfrac{32}{7}$

$\dfrac{1\cdot7}{6\cdot7}x-\dfrac{5\cdot6}{7\cdot6}x=-\dfrac{32\cdot6}{7\cdot6}-\dfrac{35\cdot7}{6\cdot7}$

$\dfrac{7}{42}x-\dfrac{30}{42}x=\dfrac{-192}{42}-\dfrac{245}{42}$

$\dfrac{42}{23}\left(-\dfrac{23}{42}x\right)=\left(-\dfrac{437}{42}\right)\left(\dfrac{-42}{23}\right)$

$x=19$

$y=\dfrac{1}{6}x+\dfrac{35}{6}$

$y=\dfrac{1}{6}\left(\dfrac{19}{1}\right)+\dfrac{35}{6}$

$=\dfrac{19}{6}+\dfrac{35}{6}=\dfrac{54}{6}$

$y=9$

Sara is 19 yrs;
Julie is 9 yrs.

10. Five years ago Mark was three times as old as Gabrielle. In five years, he will be twice her age. How old is each person?

	Present	5 yrs ago	In 5 yrs
Mark	x	$x-5$	$x+5$
Gabrielle	y	$y-5$	$y+5$

① $x-5=3(y-5)$

$x-5=3y-15$

$-3y=-x-15+5$

$\dfrac{-3y}{-3}=\dfrac{-x}{-3}-\dfrac{10}{-3}$

$y_1=\dfrac{1}{3}x+\dfrac{10}{3}$

② $x+5=2(y+5)$

$x+5=2y+10$

$-2y=-x+10-5$

$\dfrac{-2y}{-2}=\dfrac{-x}{-2}+\dfrac{5}{-2}$

$y_2=\dfrac{1}{2}x-\dfrac{5}{2}$

$y_1=y_2$

$\dfrac{1}{3}x+\dfrac{10}{3}=\dfrac{1}{2}x-\dfrac{5}{2}$

$\dfrac{1\cdot2}{3\cdot2}x-\dfrac{1\cdot3}{2\cdot3}x=-\dfrac{5\cdot3}{2\cdot3}-\dfrac{10\cdot2}{3\cdot2}$

$\dfrac{2}{6}x-\dfrac{3}{6}x=-\dfrac{15}{6}-\dfrac{20}{6}$

$\dfrac{-6}{1}\left(-\dfrac{1}{6}x\right)=\left(-\dfrac{35}{6}\right)\dfrac{-6}{1}$

$x=35$

$y=\dfrac{1}{2}x-\dfrac{5}{2}$

$=\dfrac{1}{2}\left(\dfrac{35}{1}\right)-\dfrac{5}{2}$

$=\dfrac{35}{2}-\dfrac{5}{2}$

$y=\dfrac{30}{2}=15$

Mark is 35 yrs;
Gabrielle is 15 yrs.

Type 2: Money Problems

1. You buy two sweaters and a pair of jeans to complete your work uniform. The total bill comes to $125. Your friend buys one sweater and three pairs of jeans of the same type. Her bill comes to $175. What is the cost of one sweater? What is the cost of one pair of jeans?

Let x = cost 1 sweater
Let y = cost 1 pr jeans

① $2x + y = 125$ ② $x + 3y = 175$

① $y = -2x + 125$ ② $\dfrac{3y}{3} = \dfrac{-x}{3} + \dfrac{175}{3}$

② $y = -\dfrac{1}{3}x + \dfrac{175}{3}$

$y_1 = y_2$

$-2x + 125 = -\dfrac{1}{3}x + \dfrac{175}{3}$

$-2\dfrac{x \cdot 3}{1 \cdot 3} + \dfrac{1}{3}x = \dfrac{175}{3} - \dfrac{125 \cdot 3}{1 \cdot 3}$

$-\dfrac{6}{3}x + \dfrac{1}{3}x = \dfrac{175}{3} - \dfrac{375}{3}$

$-\dfrac{3}{5}\left(-\dfrac{5}{3}x\right) = \left(\dfrac{-200}{3}\right)\dfrac{-3}{5}$

$x = 40$

$y = -2x + 125$
$= -2(40) + 125$
$= -80 + 125 = 45$

The cost of a sweater is $40; the cost of a pair of jeans is $45.

2. The attendance at a circus was 40 000 persons. If a child's ticket cost $3.00 and an adult's ticket cost $5.00, and if the total receipts amounted to $170 000, how many children and how many adults saw the show?

Let x = # children
y = # adults

① $x + y = 40\,000$

$y_1 = -x + 40\,000$

② $3x + 5y = 170\,000$

$\dfrac{5y}{5} = \dfrac{-3x}{5} + \dfrac{170\,000}{5}$

$y_2 = -\dfrac{3}{5}x + 34\,000$

$y_1 = y_2$

$-x + 40\,000 = -\dfrac{3}{5}x + 34\,000$

$-\dfrac{1}{1}x + \dfrac{3}{5}x = 34\,000 - 40\,000$

$-\dfrac{5}{5}x + \dfrac{3}{5}x = -6000$

$-\dfrac{5}{2}\left(-\dfrac{2}{5}x\right) = (-6000)\dfrac{-5}{2}$

$x = 15000 = $ # children who attended

$y = -x + 40\,000$
$= -15\,000 + 40\,000$
$y = 25\,000 = $ # adults who attended

3. Jaime has two different jobs. When she works 8 hours as a math tutor and 12 hours as a cashier at a local store she earns a total of \$361.00. If she works 12 hours doing the tutoring and 5 hours at the store then she earns \$453.75. What is her hourly wage at each job?

x = hourly wage as tutor
y = hourly wage as cashier

① $8x + 12y = 361$

$$\frac{12y}{12} = \frac{-8x}{12} + \frac{361}{12}$$

$$y_1 = -\frac{2}{3}x + \frac{361}{12}$$

② $12x + 5y = 453.75$

$$\frac{5y}{5} = \frac{-12x}{5} + \frac{453.75}{5}$$

$$y_2 = -\frac{12}{5}x + 90.75$$

$y_1 = y_2$

$$-\frac{2}{3}x + \frac{361}{12} = -\frac{12}{5}x + 90.75$$

$$-\frac{2 \cdot 5}{3 \cdot 5}x + \frac{12 \cdot 3}{5 \cdot 3}x = 90.75 - \frac{361}{12}$$

$$-\frac{10}{15}x + \frac{36}{15}x = 90.75 - 30.08$$

$$\frac{15}{26}\left(\frac{26}{15}x\right) = \frac{(60.67)}{1}\frac{15}{26}$$

$x = \$35$ = hourly wage tutoring

$$y = -\frac{2}{3}x + \frac{361}{12}$$
$$= -\frac{2}{3}(35) + 30.08$$
$$= -23.33 + 30.08 = \$6.75 = \text{hourly wage as cashier}$$

4. A customer at a bar purchased 3 beer and 5 single rum-and-cokes. His total came to \$28.50. At the next table, Mike bought a beer for each of the four guys and a rum-and-coke for each of the three girls. It cost him \$25.35. What does the bar charge for a beer and what does it charge for a rum-and-coke?

x = cost of beer
y = cost of rum-and-coke

① $3x + 5y = 28.50$

$$\frac{5y}{5} = \frac{-3x}{5} + \frac{28.50}{5}$$

$$y_1 = -\frac{3}{5}x + 5.7$$

② $4x + 3y = 25.35$

$$\frac{3y}{3} = \frac{-4x}{3} + \frac{25.35}{3}$$

$$y_2 = -\frac{4}{3}x + 8.45$$

$y_1 = y_2$

$$-\frac{3}{5}x + 5.7 = -\frac{4}{3}x + 8.45$$

$$-\frac{3 \cdot 3}{5 \cdot 3}x + \frac{4 \cdot 5}{3 \cdot 5}x = 8.45 - 5.7$$

$$-\frac{9}{15}x + \frac{20}{15}x = 2.75$$

$$\frac{15}{11}\left(\frac{11}{15}x\right) = (2.75)\frac{15}{11}$$

$x = \$3.75$ = cost of beer

$$y = -\frac{4}{3}x + 8.45$$
$$= -\frac{4}{3}(3.75) + 8.45$$
$$= -5 + 8.45 = \$3.45 = \text{cost of rum + coke}$$

5. A patron in a bar can purchase 6 shots and 2 beers for $22.50. For the same price he can buy 4 beers and 2 shots. What is the respective cost of a beer and a shot?

Let x = cost of beer
$\quad y$ = cost of shot

① $6y + 2x = 22.50$

$\dfrac{6y}{6} = \dfrac{-2x}{6} + \dfrac{22.50}{6}$

$y_1 = -\frac{1}{3}x + 3.75$

② $4x + 2y = 22.50$

$\dfrac{2y}{2} = \dfrac{-4x}{2} + \dfrac{22.50}{2}$

$y_2 = -2x + 11.25$

$y_1 = y_2$

$-\frac{1}{3}x + 3.75 = -2x + 11.25$

$-\frac{1}{3}x + \dfrac{2 \cdot 3}{1 \cdot 3}x = 11.25 - 3.75$

$-\frac{1}{3}x + \frac{6}{3}x = 7.5$

$\frac{3}{5}\left(\frac{5}{3}x\right) = (7.5)\frac{3}{5}$

$\boxed{x = 4.50 = \text{cost of beer}}$

$y = -2x + 11.25$

$\quad = -2(4.50) + 11.25$

$\quad = -9 + 11.25 = \boxed{2.25 = \text{cost of shot}}$

6. A school office sells pens and pencils. Anastasia buys 5 pencils and 2 pens and it costs her $5.25. Michael buys 4 pencils and 8 pens for a total cost of $15.40. What does the school charge for a pencil? What does the school charge for a pen?

x = cost of pencil
y = cost of pen

① $5x + 2y = 5.25$

② $4x + 8y = 15.40$

① $5x + 2y = 5.25$

$\dfrac{2y}{2} = \dfrac{-5x}{2} + \dfrac{5.25}{2}$

$y_1 = -\frac{5}{2}x + 2.63$

② $4x + 8y = 15.40$

$\dfrac{8y}{8} = \dfrac{-4x}{8} + \dfrac{15.40}{8}$

$y_2 = -\frac{1}{2}x + 1.925$

$y_1 = y_2$

$-\frac{5}{2}x + 2.63 = -\frac{1}{2}x + 1.925$

$-\frac{5}{2}x + \frac{1}{2}x = 1.925 - 2.63$

$-\frac{4}{2}x = -0.705$

$\dfrac{-2x}{-2} = \dfrac{-0.705}{-2}$

$\boxed{x = 0.35 \quad (35¢) = \text{cost of pencil}}$

$y = -\frac{1}{2}x + 1.925$

$\quad = -\frac{1}{2}(0.35) + 1.925$

$\quad = -0.175 + 1.925$

$\boxed{= 1.75 = \text{cost of pen}}$

7. Tanya has some quarters and dimes in her pocket. There are 50 coins in all for a total of $7.25. How many dimes and how many quarters does she have?

Let x = # dimes
Let y = # quarters

① $x + y = 50$

$y_1 = -x + 50$

② $0.10x + 0.25y = 7.25$

$$\frac{0.25y}{0.25} = \frac{-0.10x}{0.25} + \frac{7.25}{0.25}$$

$y_2 = -0.4x + 29$

$y_1 = y_2$

$-x + 50 = -0.4x + 29$

$-1x + 0.4x = 29 - 50$

$$\frac{-0.6x}{-0.6} = \frac{-21}{-0.6}$$

$\boxed{x = 35 = \text{# dimes}}$

$y = -x + 50$
$= -35 + 50$
$= \boxed{15 = \text{# quarters}}$

8. Benny has some dimes and quarters. She has 30 coins in all for a total of $5.70. How many coins of each type does she have?

Let x = # dimes

Let y = # quarters

① $x + y = 30$

$y_1 = -x + 30$

② $0.10x + 0.25y = 5.70$

$$\frac{0.25y}{0.25} = \frac{-0.10x}{0.25} + \frac{5.70}{0.25}$$

$y_2 = -0.4x + 22.8$

$y_1 = y_2$

$-x + 30 = -0.4x + 22.8$

$-1x + 0.4x = 22.8 - 30$

$$\frac{-0.6x}{-0.6} = \frac{-7.2}{-0.6}$$

$\boxed{x = 12 = \text{# dimes}}$

$y = -x + 30$
$= -12 + 30$
$= \boxed{18 = \text{# quarters}}$

9. Zara has $5.00 in her coat pocket. This is made up of 38 coins in the form of dimes and quarters. How many dimes and how many quarters does Zara have?

Let x = # dimes

Let y = # quarters

① $x + y = 38$

$y_1 = -x + 38$

② $0.10x + 0.25y = 5$

$$\frac{0.25y}{0.25} = \frac{-0.10x}{0.25} + \frac{5}{0.25}$$

$$y_2 = -0.4x + 20$$

$y_1 = y_2$

$-x + 38 = -0.4x + 20$

$-1x + 0.4x = 20 - 38$

$$\frac{-0.6x}{-0.6} = \frac{-18}{-0.6}$$

$\boxed{x = 30 = \text{# dimes}}$

$y = -x + 38$

$= -30 + 38$

$= \boxed{8 = \text{# quarters}}$

10. Stef's piggy bank has 50 coins totalling $4.15. The coins are all dimes and nickels. How many dimes and how many nickels are there?

Let x = # dimes

Let y = # nickels

① $x + y = 50$

$y_1 = -x + 50$

② $0.10x + 0.05y = 4.15$

$$\frac{0.05y}{0.05} = \frac{-0.10x}{0.05} + \frac{4.15}{0.05}$$

$$y_2 = -2x + 83$$

$y_1 = y_2$

$-x + 50 = -2x + 83$

$-1x + 2x = 83 - 50$

$\boxed{x = 33 = \text{# dimes}}$

$y = -x + 50$

$= -33 + 50 \left(= 17 = \text{# nickels}\right)$

11. Michelle cashes her paycheck, which is for \$530.00. The bank teller gives her 38 bills, some of which are twenty-dollar bills and some of which are ten-dollar bills. How many of each kind of bill does Michelle receive?

Let x = #\$20 bills

Let y = #\$10 bills

① $x + y = 38$

$y_1 = -x + 38$

② $20x + 10y = 530$

$\dfrac{10y}{10} = -\dfrac{20x}{10} + \dfrac{530}{10}$

$y_2 = -2x + 53$

$y_1 = y_2$

$-x + 38 = -2x + 53$

$-1x + 2x = 53 - 38$

$\boxed{x = 15 = \#\$20 \text{ bills}}$

$y = -x + 38$

$= -15 + 38$

$= \boxed{23 = \#\$10 \text{ bills}}$

12. Jessie has \$8.45 made up of quarters and nickels. He has 77 coins in all. How many quarters and how many nickels does he have?

Let x = # quarters

Let y = # nickels

① $x + y = 77$

$y_1 = -x + 77$

② $0.25x + 0.05y = 8.45$

$\dfrac{0.05y}{0.05} = \dfrac{-0.25x}{0.05} + \dfrac{8.45}{0.05}$

$y_2 = -5x + 169$

$y_1 = y_2$

$-x + 77 = -5x + 169$

$-1x + 5x = 169 - 77$

$\dfrac{4x}{4} = \dfrac{92}{4}$

$\boxed{x = 23 = \# \text{quarters}}$

$y = -x + 77$

$= -23 + 77$

$= \boxed{54 = \# \text{nickels}}$

Type 3: Number Problems

1. The sum of two numbers is 63, and one of the numbers is double the other. What are the numbers?

Let x = larger #

Let y = smaller #

① $x + y = 63$

② $x = 2y$

① $x + y = 63$

$y_1 = -x + 63$

② $\dfrac{2y}{2} = \dfrac{x}{2}$

$y_2 = \dfrac{1}{2}x$

$y_1 = y_2$

$-x + 63 = \dfrac{1}{2}x$

$\dfrac{-1x^2 - \dfrac{1}{2}x}{1.2} = -63$

$-\dfrac{2}{2}x - \dfrac{1}{2}x = -63$

$-\dfrac{2}{3}\left(-\dfrac{3}{2}x\right) = (-63)\dfrac{-2}{3}$

$x = 42$

$y = \dfrac{1}{2}x$

$= \dfrac{1}{2}(42) = 21$

The numbers are 42 and 21

2. Find two numbers whose difference is 20 if a third of their sum is 40.

Let x = larger #

Let y = smaller #

① $x - y = 20$

② $\dfrac{1}{3}(x+y) = 40$

① $x - y = 20$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{20}{-1}$

$y_1 = x - 20$

② $\dfrac{1}{3}x + \dfrac{1}{3}y = 40$

$\dfrac{3}{1}\left(\dfrac{1}{3}y\right) = \dfrac{3}{1}\left(-\dfrac{1}{3}x + 40\right)$

$y_2 = -x + 120$

$y_1 = y_2$

$x - 20 = -x + 120$

$1x + 1x = 120 + 20$

$\dfrac{2x}{2} = \dfrac{140}{2}$

$x = 70$

$y = x - 20$

$= 70 - 20$

$y = 50$

The numbers are 70 and 50

-12-

3. The difference between two numbers is 7. One-third of the larger number added to one-quarter of the smaller number is 7. Find the numbers.

Let x = larger number
y = smaller number

① $x - y = 7$

② $\frac{1}{3}x + \frac{1}{4}y = 7$

① $x - y = 7$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{7}{-1}$

① $y = x - 7$

② $\frac{1}{3}x + \frac{1}{4}y = 7$

$\frac{4}{1}\left(\frac{1}{4}y\right) = \left(-\frac{1}{3}x + 7\right)\frac{4}{1}$

② $y = -\frac{4}{3}x + 28$

$y_1 = y_2$

$x - 7 = -\frac{4}{3}x + 28$

$1x + \frac{4}{3}x = 28 + 7$

$\frac{3}{3}x + \frac{4}{3} = 35$

$\frac{3}{7}\left(\frac{7}{3}x\right) = \left(\frac{35}{1}\right)\frac{3}{7}$

$\boxed{x = 15}$

$y = x - 7$
$= 15 - 7 \boxed{= 8}$

$\boxed{\text{The numbers are 8 and 15.}}$

4. The difference between two numbers is 13. One-half of the smaller number added to one-third of the larger number is 16. Find the numbers.

Let x = larger number
y = smaller number

① $x - y = 13$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{13}{-1}$

① $y = x - 13$

② $\frac{1}{2}y + \frac{1}{3}x = 16$

$\frac{2}{1}\left(\frac{1}{2}y\right) = \frac{2}{1}\left(-\frac{1}{3}x + 16\right)$

② $y = -\frac{2}{3}x + 32$

$y_1 = y_2$

$x - 13 = -\frac{2}{3}x + 32$

$1x + \frac{2}{3}x = 32 + 13$

$\frac{3}{3}x + \frac{2}{3}x = 45$

$\frac{3}{5}\left(\frac{5}{3}x\right) = \left(\frac{45}{1}\right)\frac{3}{5}$

$\boxed{x = 27}$

$y = x - 13$
$= 27 - 13$
$\boxed{= 14}$

$\boxed{\text{The numbers are 14 and 27.}}$

-13-

5. The difference between two numbers is 18. One-half of the larger number added to one-quarter of the smaller number is 21. Find the two numbers.

Let $x =$ larger #
$\quad y =$ smaller #

① $x - y = 18$

② $\frac{1}{2}x + \frac{1}{4}y = 21$

$\frac{4}{1}\left(\frac{1}{4}y\right) = \frac{4}{1}\left(-\frac{1}{2}x + 21\right)$

② $y = -2x + 84$

① $\frac{-y}{-1} = \frac{-x}{-1} + \frac{18}{-1}$

① $y = x - 18$

$y_1 = y_2$

$x - 18 = -2x + 84$

$1x + 2x = 84 + 18$

$\frac{3x}{3} = \frac{102}{3}$

$x = 34$

$y = x - 18$
$\quad = 34 - 18$
$\quad = 16$

The numbers are 16 and 34

6. Two numbers differ by 52. One is triple the other. What are the two numbers?

Let $x =$ larger number
$\quad y =$ smaller number

① $x - y = 52$

$\frac{-y}{-1} = \frac{-x}{-1} + \frac{52}{-1}$

① $y = x - 52$

② $x = 3y$

$\frac{3y}{3} = \frac{x}{3}$

② $y = \frac{1}{3}x$

$y_1 = y_2$

$x - 52 = \frac{1}{3}x$

$1x - \frac{1}{3}x = 52$

$\frac{3}{3}x - \frac{1}{3}x = 52$

$\frac{3}{2}\left(\frac{2}{3}x\right) = (52)\frac{3}{2}$

$x = 78$

$y = x - 52$
$\quad = 78 - 52$
$\quad = 26$

The numbers are 26 and 78.

-14-

7. Two number differ by 11. One-sixth of the larger number added to one-fifth of the smaller number also results in 11. Find the two numbers.

Let x = larger number

y = smaller number

$y_1 = y_2$

$x - 11 = -\frac{5}{6}x + 55$

① $x - y = 11$

$1x + \frac{5}{6}x = 55 + 11$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{11}{-1}$

$\frac{6}{6}x + \frac{5}{6}x = 66$

The numbers are 25 and 36

① $y = x - 11$

$\frac{6}{11}\left(\frac{11}{6}x\right) = \left(\frac{66}{1}\right)\frac{6}{11}$

② $\frac{1}{6}x + \frac{1}{5}y = 11$

$\boxed{x = 36}$

$\frac{5}{1}\left(\frac{1}{5}y\right) = \frac{5}{1}\left(-\frac{1}{6}x + 11\right)$

$y = x - 11$

② $y = -\frac{5}{6}x + 55$

$= 36 - 11$

$\boxed{= 25}$

8. One number is four times another number. The difference between the two numbers is 51. Find the numbers.

Let x = larger number.

y = smaller number

$y_1 = y_2$

$\frac{1}{4}x = x - 51$

① $x = 4y$

$\frac{1}{4}x - 1x = -51$

$\dfrac{4y}{4} = \dfrac{x}{4}$

$\frac{1}{4}x - \frac{4}{4}x = -51$

① $y = \frac{1}{4}x$

$-\frac{4}{3}\left(-\frac{3}{4}x\right) = \left(\frac{-51}{1}\right)\frac{-4}{3}$

② $x - y = 51$

$\boxed{x = 68}$

$\dfrac{-y}{-1} = \dfrac{-x}{-1} + \dfrac{51}{-1}$

$y = \frac{1}{4}x$

The numbers are 17 and 68

② $y = x - 51$

$= \frac{1}{4}(68)$

$= \boxed{17}$