

Duration: 2 hours 30 minutes

PRETEST

QUESTION 1 (5 marks)

Perform the following multiplication by applying the laws of exponents. Make sure that your answer contains only positive exponents. Show all the steps in the solution.

$$\begin{aligned}
 & (a^5 b^{-1} c)^{\frac{4}{5}} \times (a^2 b c^4)^{-2} \\
 & a^4 b^{-\frac{4}{5}} c^{\frac{4}{5}} \times a^{-4} b^{-2} c^{-8} \\
 & a^{4-4} b^{-\frac{4}{5}-2} c^{\frac{4}{5}-8} \\
 & a^0 b^{-2\frac{4}{5}} c^{-\frac{36}{5}} \\
 & b^{-\frac{14}{5}} c^{-\frac{36}{5}} = \boxed{\frac{1}{b^{\frac{14}{5}} c^{\frac{36}{5}}}}
 \end{aligned}$$

QUESTION 2 (5 marks)

Perform the following division by applying the laws of exponents. Make sure that your answer contains only positive exponents. Show all the steps in the solution.

$$\begin{aligned}
 & (-m^2 n^{-2})^{-1} \div \left(m^{\frac{1}{2}} n^{\frac{1}{2}} o\right)^4 \\
 & \frac{1}{-m^2 n^{-2}} \div \frac{-2 \quad 2 \quad 4}{m \quad n \quad o} \\
 & \frac{1}{-m^2 n^{-2}} \times \frac{1}{m^2 n^2 o^4} \\
 & \frac{n^2}{-m^2} \times \frac{m^2}{n^2 o^4} \\
 & \rightarrow = \frac{n^2 m^2}{-m^2 n^2 o^4} \\
 & = \frac{1}{-o^4} \\
 & = \boxed{\frac{-1}{o^4}}
 \end{aligned}$$

NB: a negative sign should never be left on the bottom!

QUESTION 3 (5 marks)

Simplify the following expression. Make sure that your answer contains only positive exponents. Show all the steps in the solution.

$$5^2 \rightarrow \left[\frac{x^{-3}y^2z^{\frac{3}{5}}}{(25)^{-3}z^{-1}} \right]^{-3} = \frac{x^9 y^{-6} z^{-\frac{9}{5}}}{5^{-6} y^9 z^3} = \frac{5^6 x^9}{y^6 y^9 z^3 z^{\frac{9}{5}}} = \frac{5^6 x^9}{y^{15} z^{3+\frac{9}{5}}}$$

$$= z^{\frac{15}{5} + \frac{9}{5}} = z^{\frac{24}{5}}$$

You are allowed to give the numerical value of 5⁶, but it's okay to just leave it as 5⁶.

$$= \frac{5^6 x^9}{y^{15} z^{\frac{24}{5}}}$$

QUESTION 4 (5 marks)

Perform the following division. Make sure that your answer contains only positive exponents. Show all the steps in the solution.

$$\left(\frac{25}{b^2} \right)^3 \div \left(\frac{b^2}{25} \right)^{-2}$$

$$\left(\frac{5^2}{b^2} \right)^3 \div \left(\frac{b^2}{5^2} \right)^{-2}$$

$$\frac{5^6}{b^6} \div \frac{b^{-4}}{5^{-4}}$$

$$\frac{5^6}{b^6} \times \frac{5^{-4}}{b^{-4}}$$

$$\frac{5^6}{b^6} \times \frac{b^4}{5^4}$$

$$= \frac{5^6 b^4}{5^4 b^6}$$

$$= \boxed{\frac{5^2}{b^2} \text{ OR } \frac{25}{b^2}}$$

QUESTION 5 (5 marks)

Perform the following division by using scientific notation and the laws of exponents. Express your answer in scientific notation. **Show all the steps in the solution.**

$$\begin{aligned} \frac{0.000\ 003}{5 \times 10^7} &= \frac{3 \times 10^{-6}}{5 \times 10^7} = \frac{3}{5} \times 10^{-6-7} \\ &= \frac{3}{5} \times 10^{-13} \\ &= 0.6 \times 10^{-13} \\ &= 6 \times 10^{-14} \end{aligned}$$

* No marks will be given for just the correct answer - which is easily obtained with a scientific calculator.
 → Your steps show that you understand ① scientific notation and ② the laws of exponents

QUESTION 6 (10 marks)

Determine whether or not the following two expressions are equivalent by applying the laws of exponents. Show all the steps in the solution.

$$\begin{aligned} \left(\frac{27}{2}\right)^{-3} \times \left(\frac{8}{9}\right)^4 \times \left(\frac{3}{8}\right)^{-2} &\quad \text{and} \quad \left(\frac{3^3}{32}\right)^{-1} \times \left(\frac{4}{27}\right)^4 \times \left(\frac{8}{9}\right)^2 \\ \left(\frac{3^3}{2}\right)^{-3} \times \left(\frac{2^3}{3^2}\right)^4 \times \left(\frac{3}{2^3}\right)^{-2} &\quad \left(\frac{3^3}{2^5}\right)^{-1} \cdot \left(\frac{2^2}{3^3}\right)^4 \cdot \left(\frac{2^3}{3^2}\right)^2 \\ \frac{3^{-9}}{2^{-3}} \cdot \frac{2^{12}}{3^8} \cdot \frac{3^{-2}}{2^{-6}} &\quad \frac{3^{-3}}{2^{-5}} \cdot \frac{2^8}{3^{12}} \cdot \frac{2^6}{3^4} \\ \frac{2^3}{3^9} \cdot \frac{2^{12}}{3^8} \cdot \frac{2^6}{3^2} &\quad \frac{2^5}{3^3} \cdot \frac{2^8}{3^{12}} \cdot \frac{2^6}{3^4} \\ \frac{2^{21}}{3^{19}} &\neq \frac{2^{19}}{3^{19}} \end{aligned}$$

Answer: The two expressions are thus equivalent.

Yes
 No

QUESTION 7 (10 marks)

Among the following algebraic expressions, circle those that are equivalent. In the space provided under each expression, show how you arrived at your conclusion.

$\frac{2^4}{2^{-2}} y^6$ $2^{4-(-2)} y^6$ $2^{4+2} y^6$ $2^6 y^6$	$\frac{4}{-(16y^6)}$ $\frac{2^2}{-(2^4 y^6)}$ $-\frac{2^2}{2^4 y^6}$	$64y^6$ $2^6 y^6$
$(4y^2)^3$ $(2^2 y^2)^3$ $2^6 y^6$	$\frac{1}{(2^6 y^6)^{-1}}$ $\frac{1}{2^{-6} y^{-6}}$ $2^6 y^6$	$\frac{4y}{y^{-5}}$ $\frac{2^2 y^1}{y^{-5}}$ $2^2 y^6$

QUESTION 8 (10 marks)

Let $a = \frac{1}{2}$ (or any other fraction between 0 + 1)

If $0 < a < 1$, determine whether the following statements are true or false by replacing the variable with the number of your choice.

<p>A) $\left(\frac{-1}{a}\right)^{-3} < 0$</p> <p>$\left(\frac{-1}{\frac{1}{2}}\right)^{-3} < 0$</p> <p>$(-2)^{-3} < 0$</p> <p>$\frac{1}{(-2)^3} < 0$</p> <p>$\frac{1}{-8} < 0$</p> <p>$-\frac{1}{8} < 0$</p> <p>True <input checked="" type="checkbox"/> or False <input type="checkbox"/></p>	<p>B) $\left(\frac{1}{a}\right)^2 > \frac{1}{a^{-1}}$</p> <p>$\left(\frac{1}{\frac{1}{2}}\right)^2 > \frac{1}{\left(\frac{1}{2}\right)^{-1}}$</p> <p>$2^2 > \frac{1}{2}$</p> <p>$4 > \frac{1}{2}$</p> <p>True <input checked="" type="checkbox"/> or False <input type="checkbox"/></p>
<p>C) $(-a)^{-3} > 0$</p> <p>$\left(-\frac{1}{2}\right)^{-3} > 0$</p> <p>$(-2)^3 > 0$</p> <p>$-8 > 0$</p> <p>True <input type="checkbox"/> or False <input checked="" type="checkbox"/></p>	<p>D) $a^2 < a < 1$</p> <p>$\left(\frac{1}{2}\right)^2 < \frac{1}{2} < 1$</p> <p>$\frac{1}{4} < \frac{1}{2} < 1$</p> <p>True <input checked="" type="checkbox"/> or False <input type="checkbox"/></p>

QUESTION 9 (5 marks)

Perform the operations indicated in the expression below and simplify your answer. Show all the steps in the solution.

$$\sqrt{50} - \sqrt{4} - \sqrt{98}$$

$$\sqrt{2 \cdot 25} - 2 - \sqrt{2 \cdot 49}$$

$$5\sqrt{2} - 2 - 7\sqrt{2}$$

$$\boxed{-2 - 2\sqrt{2}}$$

OR

$$\boxed{-2\sqrt{2} - 2}$$

QUESTION 10 (5 marks)

Perform the operations indicated in the expression below and simplify your answer. Show all the steps in the solution.

$$(-4\sqrt{2} + 3)(2\sqrt{32} - 6)$$

$$-8\sqrt{64} + 24\sqrt{2} + 6\sqrt{32} - 18$$

$$-8(8) + 24\sqrt{2} + 6\sqrt{2 \cdot 16} - 18$$

$$-64 + 24\sqrt{2} + 24\sqrt{2} - 18$$

$$\boxed{48\sqrt{2} - 82}$$

QUESTION 11 (5 marks)

Perform the operations indicated in the following expression. Simplify your answer and rationalize the denominator, if necessary. Show all the steps in the solution.

$$\begin{aligned} \frac{2\sqrt{54}}{-3\sqrt{125}} &= \frac{2\sqrt{6 \cdot 9}}{-3\sqrt{5 \cdot 25}} = \frac{6\sqrt{6} \cdot \sqrt{5}}{-15\sqrt{5} \cdot \sqrt{5}} = \frac{6\sqrt{30}}{-15\sqrt{25}} \\ &= \frac{6\sqrt{30}}{-75} \\ &= \frac{2\sqrt{30}}{-25} \\ &= \boxed{\frac{-2\sqrt{30}}{25}} \end{aligned}$$

QUESTION 12 (5 marks)

Perform the operations indicated in the following expression. Simplify your answer and rationalize the denominator, if necessary. Show all the steps in the solution.

$$\begin{aligned} \frac{4\sqrt{6}}{2\sqrt{6}-3} (2\sqrt{6}+3) &= \frac{8\sqrt{36} + 12\sqrt{6}}{4\sqrt{36} - 9} \\ &= \frac{48 + 12\sqrt{6}}{24 - 9} \\ &= \boxed{\frac{12\sqrt{6} + 48}{15}} \end{aligned}$$

QUESTION 13 (10 marks)

Determine whether or not the following two expressions are equivalent. Show all the steps in the solution.

$$(2\sqrt{2} + 4)(3\sqrt{12} - 5) \quad \text{and} \quad 2\sqrt{2}(3\sqrt{3} - 5) + 6\sqrt{3}(\sqrt{2} + 4) - 20$$

$$6\sqrt{24} - 10\sqrt{2} + 12\sqrt{12} - 20$$

$$6\sqrt{4 \cdot 6} - 10\sqrt{2} + 12\sqrt{3 \cdot 4} - 20$$

$$12\sqrt{6} - 10\sqrt{2} + 24\sqrt{3} - 20$$

$$\underline{6\sqrt{6}} - 10\sqrt{2} + \underline{6\sqrt{6}} + 24\sqrt{3} - 20$$

$$12\sqrt{6} - 10\sqrt{2} + 24\sqrt{3} - 20$$

Answer: The two expressions are thus equivalent.

Yes

No

QUESTION 14 (5 marks)

Determine whether or not the following two expressions are equivalent by converting them to exponential form. Show all the steps in the solution.

$$y^3 \sqrt[3]{\left(\frac{1}{y}\right)^{-4}} \quad \text{and} \quad (y^4)^{\frac{1}{6}} \cdot \sqrt[3]{y^5}$$

$$\begin{aligned} y \cdot \left(\frac{1}{y}\right)^{-\frac{4}{3}} & \quad y^{\frac{4}{6}} \cdot y^{\frac{5}{3}} \\ y \cdot y^{\frac{4}{3}} & \quad y^{\frac{4}{6}} \cdot y^{\frac{10}{6}} \\ y^{\frac{3}{3}} \cdot y^{\frac{4}{3}} & \quad y^{\frac{14}{6}} \\ y^{\frac{7}{3}} & = y^{\frac{7}{3}} \end{aligned}$$

Answer: The two expressions are thus equivalent.

Yes
 No

QUESTION 15 (10 marks)

Match each expression in the left-hand column with the equivalent expression in the right-hand column. In the space under each expression on the left, show how you arrived at your answers.

<p>A) $5\sqrt{5} - 1\sqrt{5}$</p> <p>$4\sqrt{5}$</p>
<p>B) $\frac{\sqrt{45}}{3\sqrt{2}} = \frac{\sqrt{5 \cdot 9}}{3\sqrt{2}} = \frac{3\sqrt{5} \cdot \sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}}$</p> <p>$= \frac{3\sqrt{10}}{6}$</p> <p>$= \frac{\sqrt{10}}{2}$</p>
<p>C) $\frac{1}{5^{\frac{3}{2}}} = 5^{\frac{3}{2}}$</p> <p>$= \sqrt{5^3}$</p> <p>$= \sqrt{125} = \sqrt{25 \cdot 5}$</p> <p>$= 5\sqrt{5}$</p>
<p>D) $\frac{5^{\frac{3}{2}}}{5} = \frac{5^{\frac{3}{2}}}{5^1} = 5^{\frac{3}{2}-1}$</p> <p>$= 5^{\frac{3}{2}-\frac{2}{2}}$</p> <p>$= 5^{\frac{1}{2}}$</p> <p>$= \sqrt{5}$</p>

1) $\frac{\sqrt{10}}{2}$
2) $\sqrt{5}$
3) 5
4) $4\sqrt{5}$
5) $\frac{\sqrt{5}}{2}$
6) $5\sqrt{5}$
Answers:
A) <u>4</u>
B) <u>1</u>
C) <u>6</u>
D) <u>2</u>