MTH – 4105 – 1
EXPONENTS AND RADICALS

NAME: Shannon
DATE: _____

REVIEW BOOKLET

QUESTION 1

Perform the following multiplications by applying the laws of exponents. Make sure that your answers contain only positive exponents. Show all the steps in the solution.

a) $\left(32a^{-3}b^{2}\right)^{-2} \times \left(-4a^{4}b^{-2}\right)^{5}$

$\left(2^{5}a^{-3}b^{2}\right)^{-2} \times \left((-1)(2^{2})a^{4}b^{-2}\right)^{5}$

$2^{-10}a^{6}b^{-4} \times (-1)^{5}2^{10}a^{20}b^{-10}$

$2^{-10}a^{6}b^{-4} \times (-1)2^{10}a^{20}b^{-10} = \dfrac{a^{6}(-1)2^{10}a^{20}}{2^{10}b^{4}b^{10}}$

$= \boxed{\dfrac{-a^{26}}{b^{14}}}$

b) $\left(-3p^{-4}q^{2}\right)^{2} \times \left(9p^{3}q^{-5}\right)^{-1}$

$9 = 3^{2}$

$(-1)^{2}(3)^{2}p^{-8}q^{4} \times (3)^{-2}p^{-3}q^{5}$

$(1)\,3^{2}p^{-8}q^{4} \times 3^{-2}p^{-3}q^{5} = \dfrac{3^{2}q^{4}q^{5}}{p^{8}3^{2}p^{3}} = \boxed{\dfrac{q^{9}}{p^{11}}}$

c) $\left(-2a^2b^{-5}\right)^4 \times \left(4a^{-3}b^2\right)^{-2}$

$(-1)^4 2^4 a^8 b^{-20} \times 2^{-4} a^6 b^{-4}$

(1) $2^4 a^8 b^{-20} \times 2^{-4} a^6 b^{-4} = \dfrac{2^4 a^8 a^6}{b^{20} 2^4 b^4} = \boxed{\dfrac{a^{14}}{b^{24}}}$

d) $\left(xy^2z^2\right)^{-5} \times \left(x^6y^{-1}z\right)^{\frac{5}{6}}$

$x^{-5} y^{-10} z^{-10} \cdot x^5 y^{-\frac{5}{6}} z^{\frac{5}{6}} = \dfrac{x^5 z^{\frac{5}{6}}}{x^5 y^{10} z^{10} y^{\frac{5}{6}}}$ $\quad \rightarrow \frac{5}{6} - \frac{10}{1} = \frac{5}{6} - \frac{60}{6} = \frac{-55}{6}$

$= \dfrac{z^{\frac{5}{6}-10}}{y^{10+\frac{5}{6}}}$ $\quad \rightarrow 10\frac{5}{6} = \frac{65}{6}$

$= \dfrac{z^{-\frac{55}{6}}}{y^{\frac{65}{6}}} = \boxed{\dfrac{1}{y^{\frac{65}{6}} z^{\frac{55}{6}}}}$

e) $\left(\frac{9}{b}\right)^{-3} \times \left(\frac{b^3}{27}\right)^{-2}$

$\left(\frac{3^2}{b}\right)^{-3} \times \left(\frac{b^3}{3^3}\right)^{-2}$

$\frac{3^{-6}}{b^{-3}} \cdot \frac{b^{-6}}{3^{-6}} = \frac{b^{-6}}{b^{-3}} = \frac{b^3}{b^6} = \boxed{\frac{1}{b^3}}$

f) $\left(\frac{x^2}{125}\right)^{-3} \times \left(\frac{625}{x^3}\right)^{-2}$

$\left(\frac{x^2}{5^3}\right)^{-3} \times \left(\frac{5^4}{x^3}\right)^{-2}$

$\frac{x^{-6}}{5^{-9}} \times \frac{5^{-8}}{x^{-6}} = \frac{5^{-8}}{5^{-9}} = \frac{5^9}{5^8} = \boxed{5}$

QUESTION 2

Perform the following divisions by applying the laws of exponents. Make sure your answers contain only positive exponents. Show all the steps in the solutions.

a) $\left(-16x^{-3}yx^{4}\right)^{4} \div \left(32y^{-\frac{1}{4}}x^{4}\right)^{-4}$

$(-1)^4\, 2^{16} x^{-12} y^4 x^{16} \div 2^{-20} y^1 x^{-16}$

$(1)\, 2^{16} x^{-12} y^4 x^{16} \div \dfrac{y^1}{2^{20} x^{16}}$

$\dfrac{2^{16} y^4 x^{16}}{x^{12}} \times \dfrac{2^{20} x^{16}}{y}$

$2^{16} y^4 x^4 \cdot \dfrac{2^{20} x^{16}}{y} = \dfrac{2^{36} x^{20} y^4}{y} = \boxed{2^{36} x^{20} y^3}$

b) $\left(-27ab^{-3}a^{2}\right)^{7} \div \left(81b^{4}a^{-\frac{1}{5}}\right)^{5}$

$(-1)^7\, 3^{21} a^7 b^{-21} a^{14} \div \dfrac{3^{20} b^{20} a}{1}$ . . . . . $= \dfrac{3^{20} b^{20}}{a}$

$\dfrac{(-1)\, 3^{21} a^{21}}{b^{21}} \times \dfrac{a}{3^{20} b^{20}}$

$= \dfrac{(-1)\, 3^{21} a^{22}}{b^{21} 3^{20} b^{20}} = \dfrac{(-1)(3)\, a^{22}}{b^{41}} = \boxed{-\dfrac{3a^{22}}{b^{41}}}$

$= \frac{1}{-1} = -1$

c) $\left(-s^8 t^{-3}\right)^{-1} \div \left(5^{\frac{1}{3}} t^{\frac{1}{2}} u^{-1}\right)^6$

$(-1)^{-1} s^{-8} t^3 \div 5^2 t^3 u^{-6}$

$(-1)\, s^{-8} t^3 \div \frac{5^2 t^3}{u^6}$

$\frac{(-1)t^3}{s^8} \times \frac{u^6}{5^2 t^3} = \frac{(-1)\,u^6}{5^2 s^8} = \boxed{\frac{-u^6}{5^2 s^8} \text{ OR } \frac{-u^6}{25\, s^8}}$

d) $\left(\frac{9}{p^4}\right)^3 \div \left(\frac{p^5}{27}\right)^{-2}$

$\left[\frac{3^2}{p^4}\right]^3 \div \left[\frac{p^5}{3^3}\right]^{-2}$

$\frac{3^6}{p^{12}} \div \frac{p^{-10}}{3^{-6}}$

$\frac{3^6}{p^{12}} \div \frac{3^6}{p^{10}}$

$\frac{3^6}{p^{12}} \times \frac{p^{10}}{3^6} = \frac{3^6 p^{10}}{3^6 p^{12}} = \boxed{\frac{1}{p^2}}$

$16^4 = (2^4)^4 = 2^{16}$ (and then the outside -2 gets multiplied in!)

$(-1)^{-2} = \frac{1}{(-1)^2} = \frac{1}{1} = 1$

QUESTION 3

Simplify the following expressions. Make sure your answers contain only positive exponents. Show all the steps in the solutions.

a) $\left(\frac{-2x^3y^{-4}}{16^4x^{-2}y^3}\right)^{-2} = \frac{(-1)^{-2}2^{-2}x^{-6}y^8}{2^{-32}x^4y^{-6}}$

$= 2^4$

$= \frac{y^8 2^{32} y^6}{2^2 x^6 x^4} = \boxed{\frac{2^{30} y^{14}}{x^{10}}}$

↑ With these 2 problems I've multiplied the (a) (b) outside exponent $(-2, -4)$ through everything; on the next page I'll do it a different way... choose the method you like best! ☺

b) $\left(\frac{36^2 m^3 n^{-1}}{-6m^{-2}n^4}\right)^{-4}$

$= \left[\frac{(6^2)^2 m^3 n^{-1}}{-6m^{-2}n^4}\right]^{-4}$

$= \frac{6^{-16} m^{-12} n^4}{(-1)^{-4}(6)^{-4} m^8 n^{-16}} = \frac{n^4 6^4 n^{16}}{6^{16} m^{12} m^8} = \frac{6^4 n^{20}}{6^{16} m^{20}} = \boxed{\frac{n^{20}}{6^{12} m^{20}}}$

$\frac{1}{(-1)^4} = 1$ =

c) $\left(\frac{m^{\frac{4}{7}}n^{-3}o^{4}}{27m^{-1}n^{4}}\right)^{-2} = \left[\frac{27m^{-1}n^{4}}{m^{\frac{4}{7}}n^{-3}o^{4}}\right]^{2} = \left[\frac{3^{3}m^{-1}n^{4}}{m^{\frac{4}{7}}n^{-3}o^{4}}\right]^{2} = \frac{3^{6}m^{-2}n^{8}}{m^{\frac{8}{7}}n^{-6}o^{8}}$

$= \frac{3^{6}n^{8}n^{6}}{m^{2}m^{\frac{8}{7}}o^{8}} = \boxed{\frac{3^{6}n^{14}}{m^{\frac{22}{7}}o^{8}}}$

$\frac{2}{1} + \frac{8}{7}$

$= \frac{14}{7} + \frac{8}{7}$

$= \frac{22}{7}$

↑ With these two problems I've inversed the numerator & denominator to eliminate the negative outside exponent value.

d) $\left(\frac{-3a^{3}b^{-4}}{9^{2}a^{-5}b^{3}}\right)^{-3} = \left[\frac{9^{2}a^{-5}b^{3}}{-3a^{3}b^{-4}}\right]^{3} = \left[\frac{(3^{2})^{2}a^{-5}b^{3}}{-3a^{3}b^{-4}}\right]^{3} = \frac{3^{12}a^{-15}b^{9}}{(-1)^{3}(3)^{3}a^{9}b^{-12}}$

$\frac{3^{12}b^{9}b^{12}}{a^{15}(-1)3^{3}a^{9}} = \frac{3^{12}b^{21}}{(-1)3^{3}a^{24}} = \frac{3^{9}b^{21}}{(-1)a^{24}} = \boxed{\frac{-3^{9}b^{21}}{a^{24}}}$

QUESTION 4

It is imperative that you can do these problems manually (without use of a calculator). Show all your work – but I recommend that you do use a calculator to check your final answers!

Perform the following multiplications and divisions by using scientific notation and the laws of exponents. Express your answers in scientific notation. Show all the steps in the solutions.

a) $\dfrac{0.000\ 000\ 3}{4\times10^{3}} = \dfrac{3\times10^{-7}}{4\times10^{3}} = \dfrac{3}{4}\times10^{-7-3}$

$= 0.75\times10^{-10}$

$= 7.5\times10^{-11}$

b) $6\times10^{-3}\times0.000\ 002$

$6\times10^{-3}\times2\times10^{-6}$

$12\times10^{-3-6}$

12×10^{-9}

1.2×10^{-8}

c) $\dfrac{0.000\ 004}{5\times10^{3}} = \dfrac{4\times10^{-6}}{5\times10^{3}}$

$= \frac{4}{5}\times10^{-6-3}$

$= 0.8\times10^{-9}$

$= \boxed{8\times10^{-10}}$

d) $8\times10^{-9}\times0.000\ 08$

$8\times10^{-9}\times8\times10^{-5}$

$64\times10^{-9-5}$

64×10^{-14}

$\boxed{6.4\times10^{-13}}$

QUESTION 5

For each question, determine whether or not the two expressions are equivalent by applying the laws of exponents. Show all the steps in the solutions.

a) $\left(\frac{8}{9}\right)^{-7} \times \left(\frac{16}{9}\right)^{2} \times \left(\frac{32}{81}\right)^{3}$ and $\left(\frac{2}{3}\right)^{8} \times \left(\frac{4}{9}\right)^{5} \times \left(\frac{16}{81}\right)^{-4}$

$\left(\frac{2^3}{3^2}\right)^{-7} \cdot \left(\frac{2^4}{3^2}\right)^{2} \cdot \left(\frac{2^5}{3^4}\right)^{3}$

$\frac{2^{-21}}{3^{-14}} \cdot \frac{2^8}{3^4} \cdot \frac{2^{15}}{3^{12}}$

$\frac{3^{14}}{2^{21}} \cdot \frac{2^8}{3^4} \cdot \frac{2^{15}}{3^{12}}$

$\frac{3^{14}2^{23}}{3^{16}2^{21}}$

$\left(\frac{2}{3}\right)^{8} \cdot \left(\frac{2^2}{3^2}\right)^{5} \cdot \left(\frac{2^4}{3^4}\right)^{-4}$

$\frac{2^8}{3^8} \cdot \frac{2^{10}}{3^{10}} \cdot \frac{2^{-16}}{3^{-16}}$

$\frac{2^8}{3^8} \cdot \frac{2^{10}}{3^{10}} \cdot \frac{3^{16}}{2^{16}}$

$\frac{2^{18}3^{16}}{2^{16}3^{18}}$

$\frac{2^2}{3^2} = \frac{2^2}{3^2}$

b) $\left(\frac{25}{3}\right)^{-3} \times \left(\frac{125}{81}\right)^{3} \times \left(\frac{27}{25}\right)^{4}$ and $\left(\frac{625}{27}\right)^{-2} \times \left(\frac{25}{9}\right)^{6} \times \left(\frac{27}{125}\right)^{3}$

$\left(\frac{5^2}{3}\right)^{-3} \cdot \left(\frac{5^3}{3^4}\right)^{3} \cdot \left(\frac{3^3}{5^2}\right)^{4}$

$\frac{5^{-6}}{3^{-3}} \cdot \frac{5^9}{3^{12}} \cdot \frac{3^{12}}{5^8}$

$\frac{3^3}{5^6} \cdot \frac{5^9}{3^{12}} \cdot \frac{3^{12}}{5^8}$

$\frac{3^{15}5^9}{3^{12}5^{14}}$

$\left(\frac{5^4}{3^3}\right)^{-2} \cdot \left(\frac{5^2}{3^2}\right)^{6} \cdot \left(\frac{3^3}{5^3}\right)^{3}$

$\frac{5^{-8}}{3^{-6}} \cdot \frac{5^{12}}{3^{12}} \cdot \frac{3^9}{5^9}$

$\frac{3^6}{5^8} \cdot \frac{5^{12}}{3^{12}} \cdot \frac{3^9}{5^9}$

$\frac{3^{15}5^{12}}{3^{12}5^{17}}$

$\frac{3^3}{5^5} = \frac{3^3}{5^5}$

c) $\left(\frac{5}{4}\right)^{-2} \times \left(\frac{2}{25}\right)^{3} \times \left(\frac{125}{16}\right)^{-3}$ and $\left(\frac{8}{5}\right)^{3} \times \left(\frac{2}{125}\right)^{4} \times \left(\frac{625}{16}\right)^{-1}$

$\left(\frac{5}{2^2}\right)^{-2} \cdot \left(\frac{2}{5^2}\right)^{3} \cdot \left(\frac{5^3}{2^4}\right)^{-3}$ $\left(\frac{2^3}{5}\right)^{3} \cdot \left(\frac{2}{5^3}\right)^{4} \cdot \left(\frac{5^4}{2^4}\right)^{-1}$

$\frac{5^{-2}}{2^{-4}} \cdot \frac{2^3}{5^6} \cdot \frac{5^{-9}}{2^{-12}}$ $\frac{2^9}{5^3} \cdot \frac{2^4}{5^{12}} \cdot \frac{5^{-4}}{2^{-4}}$

$\frac{2^4}{5^2} \cdot \frac{2^3}{5^6} \cdot \frac{2^{12}}{5^9}$ $\frac{2^9}{5^3} \cdot \frac{2^4}{5^{12}} \cdot \frac{2^4}{5^4}$

$\frac{2^{19}}{5^{17}} \neq \frac{2^{17}}{5^{19}}$

d) $\left(\frac{9}{4}\right)^{-2} \times \left(\frac{3}{2}\right)^{-5} \times \left(\frac{2}{3}\right)^{3}$ and $\left(\frac{16}{81}\right)^{-1} \times \left(\frac{27}{8}\right) \times \left(\frac{3}{2}\right)^{5}$

$\left(\frac{3^2}{2^2}\right)^{-2} \cdot \left(\frac{3}{2}\right)^{-5} \cdot \left(\frac{2}{3}\right)^{3}$ $\left(\frac{2^4}{3^4}\right)^{-1} \cdot \left(\frac{3^3}{2^3}\right) \cdot \left(\frac{3}{2}\right)^{5}$

$\frac{3^{-4}}{2^{-4}} \cdot \frac{3^{-5}}{2^{-5}} \cdot \frac{2^3}{3^3}$ $\frac{2^{-4}}{3^{-4}} \cdot \frac{3^3}{2^3} \cdot \frac{3^5}{2^5}$

$\frac{2^4}{3^4} \cdot \frac{2^5}{3^5} \cdot \frac{2^3}{3^3}$ $\frac{3^4}{2^4} \cdot \frac{3^3}{2^3} \cdot \frac{3^5}{2^5}$

$\frac{2^{12}}{3^{12}} \neq \frac{3^{12}}{2^{12}}$

QUESTION 6 Let $m = -3$

a) If m is an odd negative integer, determine whether the following statements are true or false by replacing the variable with the number of your choice.

A) $\left(-\frac{1}{4}\right)^m \geq 4$ $\left(-\frac{1}{4}\right)^{-3} \geq 4$ $\left(-\frac{4}{1}\right)^{3} \geq 4$ $-64 \geq 4$ True ☐ or False ☑	B) $4m > 0$ $4(-3) > 0$ $-12 > 0$ True ☐ or False ☑
C) $\left(\frac{1}{4}\right)^{-m} \geq \frac{1}{4}$ $\left(\frac{1}{4}\right)^{+3} \geq \frac{1}{4}$ $\frac{1}{64} \geq \frac{1}{4}$ True ☐ or False ☑	D) $\frac{1}{4^m} > 0$ $\frac{1}{4^{-3}} > 0$ $4^3 > 0$ $64 > 0$ True ☑ or False ☐

* You can use your calculator (y^x button) to verify values for the above calculations - but work must be shown to prove that you understand the concepts!

Let $m = +2$

b) If m is an even positive integer, determine whether the following statements are true or false by replacing the variable with the number of your choice.

A) $5^{-m} > 0$ $5^{-2} > 0$ $\frac{1}{5^2} > 0$ $\frac{1}{25} > 0$ True ☑ or False ☐	B) $\left(-\frac{1}{5}\right)^m \geq \frac{1}{5}$ $\left(-\frac{1}{5}\right)^2 \geq \frac{1}{5}$ $\frac{1}{25} \geq \frac{1}{5}$ True ☐ or False ☑
C) $(-5)^m \geq 5$ $(-5)^2 \geq 5$ $25 \geq 5$ True ☑ or False ☐	D) $\left(\frac{1}{5}\right)^{-m} \leq \frac{1}{5}$ $\left(\frac{1}{5}\right)^{-2} \leq \frac{1}{5}$ $5^2 \leq \frac{1}{5}$ $25 \leq \frac{1}{5}$ True ☐ or False ☑

let $a = -\frac{1}{2}$

c) If $-1 < a < 0$, determine whether the following statements are true or false by replacing the variable with the number of your choice.

A) $\left(\frac{-1}{a^{-3}}\right)^2 > 0$

$\left(\frac{-1}{\left(-\frac{1}{2}\right)^{-3}}\right)^2 > 0$

$\left(\frac{-1}{(-2)^3}\right)^2 > 0$

$\left(\frac{-1}{-8}\right)^2 > 0$

$\left(\frac{1}{8}\right)^2 > 0$

$\frac{1}{64} > 0$

True ☑ or False ☐

B) $\left(\frac{1}{-a}\right)^3 < \frac{1}{a}$

$\left(\frac{1}{\frac{1}{2}}\right)^3 < \frac{1}{\left(-\frac{1}{2}\right)}$

$(2)^3 < -2$

$8 < -2$

True ☐ or False ☑

C) $a < a^2 < 1$

$-\frac{1}{2} < \left(-\frac{1}{2}\right)^2 < 1$

$-\frac{1}{2} < \frac{1}{4} < 1$

True ☑ or False ☐

D) $(-a)^{-3} < 0$

$\left(\frac{1}{2}\right)^{-3} < 0$

$2^3 < 0$

$8 < 0$

True ☐ or False ☑

Use calculator to verify answers!

Let $a = \frac{1}{2}$

d) If $0 < a < 1$, determine whether the following statements are true or false by replacing the variable with the number of your choice.

A) $\left(\frac{1}{a}\right)^2 > (-a)^{-1}$ $\left(\frac{1}{\frac{1}{2}}\right)^2 > \left(-\frac{1}{2}\right)^{-1}$ $2^2 > -2$ $4 > -2$ True ☑ or False ☐	B) $\left(\frac{-1}{a^2}\right)^3 < 0$ $\left(\frac{-1}{\left(\frac{1}{2}\right)^2}\right)^3 < 0$ $\left(\frac{-1}{\frac{1}{4}}\right)^3 < 0$ $(-4)^3 < 0$ $-64 < 0$ True ☑ or False ☐
C) $\frac{(-a)^{-2}}{-a^2} < 0$ $\frac{\left(-\frac{1}{2}\right)^{-2}}{-\left(\frac{1}{2}\right)^2} < 0$ $\frac{(-2)^2}{-\left(\frac{1}{4}\right)} < 0$ $\frac{4}{-\frac{1}{4}} < 0$ $-16 < 0$ True ☑ or False ☐	D) $a < 1 < a^{-3}$ $\frac{1}{2} < 1 < \left(\frac{1}{2}\right)^{-3}$ $\frac{1}{2} < 1 < 2^3$ $\frac{1}{2} < 1 < 8$ True ☑ or False ☐

QUESTION 7

For each problem given (i.e. on each of this and the next three pages), circle the algebraic expressions that are equivalent. In the spaces provided under each expression, show how you arrived at your conclusion.

a)

$216b^3$ $6^3 b^3$	$\frac{6^2}{6^{-1}}b^3$ $6^2 \cdot 6\ b^3$ $6^3 b^3$	$\frac{1}{(6^3 b^3)^{-1}}$ $6^3 b^3$
$\frac{6}{b^{-3}}$ $6b^3$	$(6b)^3$ $6^3 b^3$	$\frac{6}{-(36b^3)}$ $\frac{6}{-(6^2 b^3)}$ $\frac{-6}{6^2 b^3}$ $\frac{-1}{6b^3}$

b)

$-s^6t^8$	$-s^2t^4 \times s^4t^4$ $-s^6t^8$	$\frac{-s^6}{(t^{-2})^{-4}}$ $\frac{-s^6}{t^8}$
$\frac{-t^8}{s^{-6}}$ $-s^6t^8$	$(s^3t^{-4})^2$ $= s^6t^{-8}$ $= \frac{s^6}{t^8}$	$s^6t^5 \times t^3$ s^6t^8

c)

$-c^8d^{12}$	$\dfrac{-d^{12}}{c^{-8}}$ $-c^8d^{12}$	$-c^2d^6 \times c^4d^6$ $-c^6d^{12}$
$-\left(c^2d^{-3}\right)^4$ $-c^8d^{-12}$ $\dfrac{-c^8}{d^{12}}$	$\dfrac{-c^8}{\left(d^{-6}\right)^2}$ $\dfrac{-c^8}{d^{-12}}$ $-c^8d^{12}$	$-c^8d^3 \times d^4$ $-c^8d^7$

d)

$16x^4$ $2^4 x^4$	$\frac{2x}{-(2^3 x^3)}$ $\frac{2x}{-2^3 x^3}$ $= \frac{-1}{2^2 x^2}$	$(2x)^4$ $2^4 x^4$
$\frac{1}{(2^2 x^2)^{-2}}$ $(2^2 x^2)^2$ $2^4 x^4$	$\frac{2^2}{2^{-2}} x^4$ $2^2 \cdot 2^2 x^4$ $2^4 x^4$	$\frac{2}{x^{-4}}$ $2x^4$

QUESTION 8

Perform the operations indicated in the expressions below and simplify your answers. Show all the steps in the solutions.

a) $4\sqrt{2}+\sqrt{18}-\sqrt{72}$

$4\sqrt{2}+\sqrt{2\cdot 9}-\sqrt{8\cdot 9}$

$4\sqrt{2}+3\sqrt{2}-3\sqrt{8}$

$4\sqrt{2}+3\sqrt{2}-3\sqrt{2\cdot 4}$

$4\sqrt{2}+3\sqrt{2}-6\sqrt{2}$

$=\boxed{\sqrt{2}}$

* note that I simplified $\sqrt{72}$ in 2 steps (because 8·9 popped into my head first).
I could have done it in one step: $\sqrt{72}=\sqrt{2\cdot 36}=6\sqrt{2}$
* It doesn't matter – you'll get the same answer ever if you don't use the largest square right off!

b) $3\sqrt{3}+\sqrt{12}-\sqrt{108}$

$3\sqrt{3}+\sqrt{3\cdot 4}-\sqrt{3\cdot 36}$

$3\sqrt{3}+2\sqrt{3}-6\sqrt{3}$

$=\boxed{-\sqrt{3}}$

c) $\sqrt{12}-\sqrt{9}+\sqrt{27}$

$\sqrt{3\cdot4}-3+\sqrt{3\cdot9}$

$2\sqrt{3}-3+3\sqrt{3}$

$= \boxed{5\sqrt{3}-3}$

d) $\sqrt{54}+\sqrt{1}-\sqrt{96}$

$\sqrt{6\cdot9}+1-\sqrt{16\cdot6}$

$3\sqrt{6}+1-4\sqrt{6}$

$\boxed{1-\sqrt{6}}$

e) $3\sqrt{5}-\sqrt{180}+\sqrt{45}$

$3\sqrt{5}-\sqrt{36\cdot5}+\sqrt{5\cdot9}$

$3\sqrt{5}-6\sqrt{5}+3\sqrt{5}$

$= \boxed{0}$

QUESTION 9

Perform the operations indicated in the expressions below and simplify your answers. Show all the steps in the solutions.

a) $(3\sqrt{5}-3)\bullet(\sqrt{5}+1)$

$3\sqrt{25}+3\sqrt{5}-3\sqrt{5}-3$

$15-3$

$=\boxed{12}$

b) $(-3\sqrt{27}-2)\bullet(3\sqrt{3}+4)$

$-9\sqrt{81}-12\sqrt{27}-6\sqrt{3}-8$

$-81-12\sqrt{3\cdot 9}-6\sqrt{3}-8$

$-89-36\sqrt{3}-6\sqrt{3}$

$\boxed{-89-42\sqrt{3}}$

c) $(3\sqrt{3}+4)\bullet(\sqrt{3}-2)$

$3\sqrt{9} - 6\sqrt{3} + 4\sqrt{3} - 8$

$9 - 2\sqrt{3} - 8$

$= \boxed{1 - 2\sqrt{3}}$

d) $(5\sqrt{5}+4)\bullet(-2\sqrt{20}-3)$

$-10\sqrt{100} - 15\sqrt{5} - 8\sqrt{20} - 12$

$-100 - 15\sqrt{5} - 8\sqrt{4\cdot 5} - 12$

$-112 - 15\sqrt{5} - 16\sqrt{5}$

$\boxed{-112 - 31\sqrt{5}}$

e) $(5\sqrt{5}-3)\bullet(\sqrt{5}+5)$

$5\sqrt{25} + 25\sqrt{5} - 3\sqrt{5} - 15$

$25 + 25\sqrt{5} - 3\sqrt{5} - 15$

$\boxed{10 + 22\sqrt{5}}$

QUESTION 10

Perform the operations indicated in the following expressions. Simplify your answers and rationalize the denominators, if necessary. Show all the steps in the solutions.

a) $\dfrac{-4\sqrt{40}}{\sqrt{32}} = \dfrac{-4\sqrt{4\cdot 10}}{\sqrt{2\cdot 16}} = \dfrac{-8\sqrt{10}}{4\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{-2\sqrt{20}}{\sqrt{4}}$

$= \dfrac{-\cancel{2}\sqrt{4\cdot 5}}{\cancel{2}}$

$= \boxed{-2\sqrt{5}}$

b) $\dfrac{6\sqrt{108}}{-5\sqrt{98}} = \dfrac{6\sqrt{3\cdot 36}}{-5\sqrt{2\cdot 49}} = \dfrac{36\sqrt{3}}{-35\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{36\sqrt{6}}{-35\sqrt{4}}$

$= \dfrac{-36\sqrt{6}}{70}$

$= \boxed{\dfrac{-18\sqrt{6}}{35}}$

c) $\dfrac{-\sqrt{72}}{\sqrt{75}}$

$$\frac{-\sqrt{2\cdot 36}}{\sqrt{3\cdot 25}} = \frac{-6\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-6\sqrt{6}}{15}$$

$$= \boxed{\frac{-2\sqrt{6}}{5}}$$

d) $\dfrac{7\sqrt{20}}{-2\sqrt{27}} = \dfrac{7\sqrt{4\cdot 5}}{-2\sqrt{3\cdot 9}} = \dfrac{14\sqrt{5}}{-6\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{14\sqrt{15}}{-18}$

$$= \boxed{\frac{-7\sqrt{15}}{9}}$$

e) $\dfrac{-2\sqrt{24}}{\sqrt{18}} = \dfrac{-2\sqrt{4\cdot 6}}{\sqrt{2\cdot 9}} = \dfrac{-4\sqrt{6}}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{-4\sqrt{12}}{6}$

$$= \frac{-2\sqrt{3\cdot 4}}{3}$$

$$= \boxed{\frac{-4\sqrt{3}}{3}}$$

QUESTION 11

Perform the operations indicated in the following expressions. Simplify your answers and rationalize the denominators, if necessary. Show all the steps in the solutions.

a) $\dfrac{2\sqrt{5}}{3\sqrt{5}-1}$

$\dfrac{(3\sqrt{5}+1)}{(3\sqrt{5}+1)} = \dfrac{6\sqrt{25}+2\sqrt{5}}{9\sqrt{25}-1}$

$= \dfrac{30+2\sqrt{5}}{45-1}$

$= \dfrac{2\sqrt{5}+30}{44}$

$= \boxed{\dfrac{\sqrt{5}+15}{22}}$

b) $\dfrac{-4\sqrt{7}}{4\sqrt{2}+8}$

$= \dfrac{-\sqrt{7}}{\sqrt{2}+2} \dfrac{(\sqrt{2}-2)}{(\sqrt{2}-2)} = \dfrac{-\sqrt{14}+2\sqrt{7}}{2-4}$

$= \dfrac{-\sqrt{14}+2\sqrt{7}}{-2}$

$= \boxed{\dfrac{\sqrt{14}-2\sqrt{7}}{2}}$

c) $\frac{-5\sqrt{2}}{2\sqrt{6}+6} \quad \frac{(2\sqrt{6}-6)}{(2\sqrt{6}-6)} = \frac{-10\sqrt{12}+30\sqrt{2}}{24-36}$

$= \frac{-10\sqrt{3\cdot 4}+30\sqrt{2}}{-12}$

$= \frac{-20\sqrt{3}+30\sqrt{2}}{-12}$

$= \frac{-10\sqrt{3}+15\sqrt{2}}{-6} = \boxed{\frac{10\sqrt{3}-15\sqrt{2}}{6}}$

d) $\frac{6\sqrt{2}}{3\sqrt{2}-6} \quad \frac{(3\sqrt{2}+6)}{(3\sqrt{2}+6)}$

$= \frac{36+36\sqrt{2}}{18-36} = \frac{36\sqrt{2}+36}{-18}$

$= \frac{-36\sqrt{2}-36}{18}$

$= \frac{-2\sqrt{2}-2}{1} = \boxed{-2\sqrt{2}-2}$

e) $\frac{-2\sqrt{5}}{3\sqrt{3}+7} \quad \frac{(3\sqrt{3}-7)}{(3\sqrt{3}-7)} = \frac{-6\sqrt{15}+14\sqrt{5}}{27-49}$

$= \frac{-6\sqrt{15}+14\sqrt{5}}{-22}$

$= \frac{-3\sqrt{15}+7\sqrt{5}}{-11} = \boxed{\frac{3\sqrt{15}-7\sqrt{5}}{11}}$

QUESTION 12

For each question, determine whether or not the two expressions are equivalent. Show all the steps in the solutions.

a) $(4\sqrt{3}+2)(3\sqrt{15}-4)$ and $2\sqrt{5}(\sqrt{3}+18)+2\sqrt{3}(2\sqrt{5}-8)-8$

$12\sqrt{45}-16\sqrt{3}+6\sqrt{15}-8$

$12\sqrt{5\cdot 9}-16\sqrt{3}+6\sqrt{15}-8$

$36\sqrt{5}-16\sqrt{3}+6\sqrt{15}-8$

$2\sqrt{15}+36\sqrt{5}+4\sqrt{15}-16\sqrt{3}-8$

$6\sqrt{15}+36\sqrt{5}-16\sqrt{3}-8$

Yes, they're equivalent.

b) $(5\sqrt{2}+3)(4\sqrt{6}-2)$ and $2\sqrt{2}(\sqrt{3}-5)+5\sqrt{3}(2\sqrt{2}+8)-6$

$20\sqrt{12}-10\sqrt{2}+12\sqrt{6}-6$

$20\sqrt{3\cdot 4}-10\sqrt{2}+12\sqrt{6}-6$

$40\sqrt{3}-10\sqrt{2}+12\sqrt{6}-6$

$2\sqrt{6}-10\sqrt{2}+10\sqrt{6}+40\sqrt{3}-6$

$12\sqrt{6}-10\sqrt{2}+40\sqrt{3}-6$

Yes, they're equivalent.

c) $(4-2\sqrt{3})\bullet(3\sqrt{3}+6)$ and $2\sqrt{5}+18-\sqrt{20}-12$

$12\sqrt{3}+24-6\sqrt{9}-12\sqrt{3}$ $\qquad$ $2\sqrt{5}+18-\sqrt{4\cdot5}-12$

$12\sqrt{3}+24-18-12\sqrt{3}$ $\qquad$ $2\sqrt{5}+18-2\sqrt{5}-12$

(6) $=$ (6)

yes, they're equivalent

d) $(6-\sqrt{5})\bullet(2\sqrt{5}+3)$ and $\sqrt{45}+20+7\sqrt{5}-2$

$12\sqrt{5}+18-2\sqrt{25}-3\sqrt{5}$ $\qquad$ $\sqrt{5\cdot9}+20+7\sqrt{5}-2$

$12\sqrt{5}+18-10-3\sqrt{5}$ $\qquad$ $3\sqrt{5}+20+7\sqrt{5}-2$

$9\sqrt{5}+8$ $\qquad$ $10\sqrt{5}+18$

No, they're not equivalent

QUESTION 13

For each question, determine whether or not the two expressions are equivalent by converting them to exponential form. Show all the steps in the solutions.

a) $x^{\frac{5}{3}}\sqrt{x^4}$ and $\left(\frac{1}{x}\right)^{\frac{-7}{3}} \cdot \sqrt[3]{x^4}$

$x^{\frac{5}{3}} \cdot x^{\frac{4}{2}}$

$x^{\frac{5}{3}} \cdot x^{2}$

$x^{\frac{5}{3}} \cdot x^{\frac{6}{3}}$

$x^{\frac{11}{3}}$

$x^{\frac{7}{3}} \cdot x^{\frac{4}{3}}$

$x^{\frac{11}{3}}$

$=$

Equivalent

b) $x^2\sqrt{\left(\frac{1}{x}\right)^{-7}}$ and $\left(x^4\right)^{\frac{1}{2}} \cdot \sqrt[4]{x^{10}}$

$x^2 \cdot \left(\frac{1}{x}\right)^{\frac{-7}{2}}$

$x^2 \cdot x^{\frac{7}{2}}$

$x^{\frac{4}{2}} \cdot x^{\frac{7}{2}}$

$x^{\frac{11}{2}}$

$x^{\frac{4}{2}} \cdot x^{\frac{10}{4}}$

$x^{\frac{4}{2}} \cdot x^{\frac{5}{2}}$

$x^{\frac{9}{2}}$

$\neq$

Not equivalent

c) $b^{\frac{5}{4}}\sqrt{b^2}$ and $\left(\frac{1}{b}\right)^{\frac{-3}{4}} \bullet \sqrt[6]{b^4}$

$b^{\frac{5}{4}} \cdot b^{\frac{2}{2}}$

$b^{\frac{5}{4}} \cdot b^{\frac{4}{4}}$

$b^{\frac{9}{4}}$

$b^{\frac{3}{4}} \cdot b^{\frac{4}{6}}$

$b^{\frac{18}{24}} \cdot b^{\frac{16}{24}}$

$b^{\frac{34}{24}}$

$b^{\frac{17}{12}}$

Not equivalent

d) $a^2\sqrt{\left(\frac{1}{a}\right)^{-5}}$ and $\left(a^2\right)^{\frac{3}{2}} \bullet \sqrt{a^3}$

$a^2 \cdot \left(\frac{1}{a}\right)^{-\frac{5}{2}}$

$a^2 \cdot a^{\frac{5}{2}}$

$a^{\frac{4}{2}} \cdot a^{\frac{5}{2}}$

$a^{\frac{9}{2}}$

$a^3 \cdot a^{\frac{3}{2}}$

$a^{\frac{6}{2}} \cdot a^{\frac{3}{2}}$

$a^{\frac{9}{2}}$

Equivalent

e) $y^{\frac{9}{2}}\sqrt{y}$ and $\left(\frac{1}{y}\right)^{\frac{-5}{3}} \bullet \sqrt[3]{y^{10}}$

$y^{\frac{9}{2}} \cdot y^{\frac{1}{2}}$

$y^{\frac{10}{2}}$

y^5 $=$ $y^{\frac{15}{3}}$

y^5

Equivalent

QUESTION 14

Simplify the following expressions. For this question, please convert exponent answers to radical form. Rationalize the denominators, if necessary.

a) $\dfrac{3^{\frac{5}{2}}}{3^{1}}$ $\quad 3^{\frac{5}{2}-\frac{2}{2}} = 3^{\frac{3}{2}} =$ $\sqrt{3^3}$ OR $\sqrt{27}$

b) $\dfrac{7}{4}\sqrt{80} = \dfrac{7}{4}\sqrt{5 \cdot 16} = \dfrac{7}{4} \cdot 4\sqrt{5} =$ $7\sqrt{5}$

c) $2\sqrt{7} \times 1\sqrt{7} = 1\sqrt{7}$ OR $\sqrt{7}$ $\quad -2\sqrt{49} = -2(7)$ $= -14$

d) $\dfrac{1}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} =$ $\dfrac{\sqrt{5}}{10}$

e) $\dfrac{\sqrt{50}}{5\sqrt{3}} = \dfrac{\sqrt{2 \cdot 25}}{5\sqrt{3}} = \dfrac{5\sqrt{2}}{5\sqrt{3}} = \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} =$ $\dfrac{\sqrt{6}}{3}$

f) $5\sqrt{7} - 1\sqrt{7} =$ $4\sqrt{7}$

g) $\dfrac{1}{7^{\frac{-5}{2}}} = 7^{\frac{5}{2}} = \boxed{\sqrt{7^5}}$

h) $\dfrac{7^{\frac{3}{2}}}{7^{1}} \quad 7^{\frac{3}{2}-\frac{2}{2}} = 7^{\frac{1}{2}} = \boxed{\sqrt{7}}$

i) $\dfrac{5}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \boxed{\dfrac{5\sqrt{7}}{7}}$

j) $\dfrac{1}{7^{\frac{-3}{2}}} = 7^{\frac{3}{2}} = \boxed{\sqrt{7^3}}$

k) $4\sqrt{3} \times -2\sqrt{3} = -8\sqrt{9} = \boxed{-24}$

l) $\dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \boxed{\dfrac{3\sqrt{5}}{5}}$