FACTORING AND ALGEBRAIC FRACTIONS PRETEST
MTH-4106-1

Name: Shanna

Date:

Duration: 2 hours 30 minutes

1. Factor the following polynomial. (5 marks)

$30a^5b^5 - 24a^4b^4 + 6a^2b^3 - 18a^3b^5 + 12a^7b^9 - 6a^3b^4$

$6a^2b^3(5a^3b^2 - 4a^2b + 1 - 3ab^2 + 2a^5b^6 - ab)$

2. Factor the following polynomial. (5 marks)

$a^2 - 13a - 30$

$p = -30$, $s = -13$

$(-15, +2)$

$(a-15)(a+2)$

3. Factor the following polynomial. (5 marks)

$3x^2 - 10xy + 8y^2$

$p = 24$

$s = -10$

$-4, -6$

$(3x^2 - 4xy)(-6xy + 8y^2)$

$x(3x - 4y) - 2y(3x - 4y)$

$\boxed{(x - 2y)(3x - 4y)}$

4. Factor the following polynomial. (5 marks)

$169 - 1.96a^2$

$\boxed{(13 - 1.4a)(13 + 1.4a)}$

5. Factor the following polynomial completely. Show all the steps in the solution. (5 marks)

$-5a^3b + 26a^2b^2 - 5ab^3$

$ab(-5a^2+26ab-5b^2)$

$p = 25$
$s = 26$ $\quad 25, 1$

$(-5a^2+25ab)+(1ab-5b^2)$

$-5a(a-5b)+b(a-5b)$

$(-5a+b)(a-5b)$

ANSWER: $\boxed{ab(-5a+b)(a-5b)}$

6. Factor the following polynomial completely. Show all the steps in the solution. (5 marks)

$4p^3x - 4x - y + 20q^4x + p^3y + 5q^4y$

$(4p^3x - 4x + 20q^4x) + (-y + p^3y + 5q^4y)$

$4x(p^3 - 1 + 5q^4) + y(-1 + p^3 + 5q^4)$

$\boxed{(4x+y)(-1+p^3+5q^4)}$

7. Factor the following polynomial completely.
Show all the steps in the solution. (5 marks)

$25x^2 - 225y^4$

$25(x^2 - 9y^4)$

$\boxed{25(x - 3y^2)(x + 3y^2)}$

8. Express the quotient of the following algebraic fractions in lowest terms.
Show all the steps in the solution. (10 marks)

$\frac{9 - y^2}{y^2 + 9} \div \frac{(x^2y^2 - 9x^2)}{1}$

$x^2y^2 - 9x^2 = x^2(y^2 - 9)$

$= x^2(y - 3)(y + 3)$

$\frac{(3-y)(3+y)}{y^2+9} \times \frac{1}{x^2(y-3)(y+3)}$

$\frac{-1(3-y)(3+y)}{y^2+9} \times \frac{1}{x^2(y-3)(y+3)} = \boxed{\frac{-1}{x^2(y^2+9)}}$

9. Reduce the following algebraic fraction to its lowest terms. Show all the steps in the solution. (5 marks)

$$\frac{36a^2-b^8}{6ab^8-36a^2b^4} = \frac{(6a-b^4)(6a+b^4)}{6ab^4(b^4-6a)}$$

$$= \frac{-1(\cancel{-6a+b^4})(6a+b^4)}{6ab^4(\cancel{b^4-6a})}$$

$$= \boxed{\frac{-(6a+b^4)}{6ab^4}}$$

10. Express the difference of the following two algebraic fractions in lowest terms. Show all the steps in the solution. (10 marks)

$$\frac{8}{m+4} - \frac{m+4}{m} \qquad \text{c.d.} = m(m+4)$$

$$\frac{8m}{\text{c.d.}} - \frac{(m+4)(m+4)}{\text{c.d.}}$$

$$\frac{8m}{\text{c.d.}} - \frac{(m^2+8m+16)}{\text{c.d.}}$$

$$\frac{8m-m^2-8m-16}{\text{c.d.}} = \boxed{\frac{-m^2-16}{m(m+4)}} \text{ OR } \boxed{\frac{-(m^2+16)}{m(m+4)}}$$

11. Express the product of the following two algebraic fractions in lowest terms. Show all the steps in the solution. (10 marks)

$$\frac{36-m^2}{m} \times \frac{6m-24}{-36m-6m^2}$$

$$\frac{(6-m)(6+m)}{m} \times \frac{6(m-4)}{-6m(6+m)}$$

$$\frac{(6-m)(6+m)}{m} \times \frac{6(m-4)}{-6m(6+m)} = \boxed{\frac{-(6-m)(m-4)}{m^2}}$$

bring that negative sign to the top.

12. Express the sum of the following two algebraic fractions in lowest terms. Show all the steps in the solution. (10 marks)

$$\frac{b^2}{3b^2+b} + \frac{2-b}{3b} = \frac{b^2}{b(3b+1)} + \frac{(2-b)}{3b}$$

c.d. $= 3b(3b+1)$ or $(3)(b)(3b+1)$

$$= \frac{3b^2}{c.d.} + \frac{(2-b)(3b+1)}{c.d.}$$

$(2-b)(3b+1)$

$6b+2-3b^2-b$

$-3b^2+5b+2$

$$= \frac{3b^2}{c.d.} + \frac{-3b^2+5b+2}{c.d.}$$

$$= \frac{3b^2-3b^2+5b+2}{c.d.} = \boxed{\frac{5b+2}{3b(3b+1)}}$$

13. The following two algebraic fractions are equivalent. Demonstrate their equivalence by transforming the expression on the left side. Show all the steps in the solution. (10 marks)

$$\frac{-t^2-t+20}{t^2+5t} + \frac{t}{t+4} = \frac{16}{t^2+4t}$$

$-t^2-t+20$

$p = -20$

$s = -1$

$-5, +4$

$(-t^2-5t)+(4t+20)$

$-t(t+5)+4(t+5)$

$(-t+4)(t+5)$

$$\frac{(-t+4)(t+5)}{t(t+5)} + \frac{t}{(t+4)}$$

$$\frac{-t+4}{t} + \frac{t}{(t+4)} \qquad \text{c.d.} = t(t+4)$$

$$\frac{(-t+4)(t+4)}{\text{c.d.}} + \frac{t^2}{\text{c.d.}}$$

$$\frac{-t^2+16}{\text{c.d.}} + \frac{t^2}{\text{c.d.}}$$

$$\frac{-t^2+16+t^2}{\text{c.d.}} = \boxed{\frac{16}{t(t+4)}} = \boxed{\frac{16}{t^2+4t}}$$

expression on right above!

14. The following two algebraic expressions are equivalent. This time, demonstrate their equivalence by transforming both expressions. Show all the steps in the solution. (10 marks)

$$\frac{(a^2-9)}{a^2+a-12} - \frac{(b^2-1)}{(b+1)^2} \equiv \frac{-1}{b+1} + \frac{-b+11+3a}{ab+4b+a+4}$$

$ab+4b+a+4$

$b(a+4)+1(a+4)$

$(b+1)(a+4)$

$$\frac{(a-3)(a+3)}{(a+4)(a-3)} - \frac{(b-1)(b+1)}{(b+1)(b+1)}$$

$$\frac{(a-3)(a+3)}{(a+4)(a-3)} - \frac{(b-1)(b+1)}{(b+1)(b+1)}$$

$$\frac{(a+3)}{(a+4)} - \frac{(b-1)}{(b+1)}$$

c.d. $= (a+4)(b+1)$

$$\frac{(a+3)(b+1)}{c.d.} - \frac{(b-1)(a+4)}{c.d.}$$

$$\frac{ab+a+3b+3}{c.d.} - \frac{(ab+4b-a-4)}{c.d.}$$

$$\frac{ab+a+3b+3-ab-4b+a+4}{c.d}$$

$$\frac{2a+7-b}{(a+4)(b+1)}$$

$$\frac{-1}{(b+1)} + \frac{-b+11+3a}{(b+1)(a+4)}$$

c.d. $= (b+1)(a+4)$

$$\frac{-1(a+4)}{c.d.} + \frac{-b+11+3a}{c.d.}$$

$$\frac{-a-4}{c.d.} + \frac{-b+11+3a}{c.d.}$$

$$\frac{-a-4-b+11+3a}{c.d.}$$

$$\frac{2a+7-b}{(b+1)(a+4)}$$

equivalent!