MTH-4106-1
Factoring and Algebraic Fractions

Name: Shannon

Date:

Quiz # 1

Factor the following polynomials. (10 marks)

1. $9cd^4 - 81d^2$

$9d^2(cd^2 - 9)$

2. $24x^5y^9 + 20x^3y^4 - 16x^{11}y^3 - 32x^2y^7 + 8x^3y^4 - 4x^2y^3$

$4x^2y^3(6x^3y^6 + 5xy - 4x^9 - 8y^4 + 2xy - 1)$

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Quiz # 2

Factor the following polynomials. (15 marks)

1. $(5x^2z^2 - 20z - 4x^4z + 16x^2)$

$5z(x^2z - 4) - 4x^2(x^2z - 4)$

$\boxed{(5z - 4x^2)(x^2z - 4)}$

2. $2ax^2 - 2a - b + 6ay^3 + bx^2 + 3by^3$

$(6ay^3 + 3by^3) + (2ax^2 + bx^2) + (-2a - b)$

$3y^3(2a+b) + x^2(2a+b) - 1(2a+b)$

$\boxed{(3y^3 + x^2 - 1)(2a+b)}$

3. $xy + 3x + 2y + 6$

$(xy + 3x) + (2y + 6)$

$x(y+3) + 2(y+3)$

$\boxed{(x+2)(y+3)}$

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Quiz # 3

Factor the following polynomials. (35 marks)

1. $x^2 - 10x - 24 = (x-12)(x+2)$

2. $b^2 + b - 30 = (b+6)(b-5)$

3. $t^2 + t - 6 = (t+3)(t-2)$

4. $x^2 - 3x + 2 = (x-2)(x-1)$

5. $a^2 + 2a - 3 = (a+3)(a-1)$

6. $m^2 + 3m + 2 = (m+2)(m+1)$

7. $p^2 - 6p + 5 = (p-5)(p-1)$

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Quiz # 4

Factor the following polynomials. (30 marks)

1. $4a^2 - 17ab + 4b^2$

$p = +16$

$s = -17$

$-16, -1$

$(4a^2 - 16ab)(-ab + 4b^2)$

$4a(a - 4b) - b(a - 4b)$

1. $(4a - b)(a - 4b)$

2. $2x^2 - 5xy + 3y^2$

$p = +6$

$s = -5$

$-2, -3$

$(2x^2 - 2xy)(-3xy + 3y^2)$

$2x(x - y) - 3y(x - y)$

2. $(2x - 3y)(x - y)$

3. $-b^2 - b + 6$

$p = -6$

$s = -1$

$-3, +2$

$(-b^2 - 3b)(2b + 6)$

$-b(b + 3) + 2(b + 3)$

3. $(-b + 2)(b + 3)$

4. $-3x^2 + 10xy - 3y^2$

$p = +9$

$s = +10$

$+9, +1$

$(-3x^2 + 9xy) + (1xy - 3y^2)$

$3x(-x + 3y) - y(-x + 3y)$

$(3x - y)(-x + 3y)$

5. $15a^2 - 13ab + 2b^2$

$p = 30$

$s = -13$

$-3, -10$

$(15a^2 - 3ab)(-10ab + 2b^2)$

$3a(5a - b) - 2b(5a - b)$

$(3a - 2b)(5a - b)$

6. $3a^2 - 10ab + 3b^2$

$p = 9$

$s = -10$

$-9, -1$

$(3a^2 - 9ab)(-1ab + 3b^2)$

$3a(a - 3b) - b(a - 3b)$

$(3a - b)(a - 3b)$

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Quiz # 5

Factor the following polynomials. (15 marks)

1. $\frac{16x^2}{9} - 36y^2 = \left(\frac{4x}{3} - 6y\right)\left(\frac{4x}{3} + 6y\right)$

2. $196 - 1.21x^2 = (14 - 1.1x)(14 + 1.1x)$

3. $9cd^4 - 81d^2 = 9d^2(cd^2 - 9)$

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Quiz # 6

Factor the following polynomials completely. Show all the steps in the solutions. (20 marks)

1. $9x^5y^2 - 57x^4y^3 + 18x^3y^4$

$3x^3y^2(3x^2 - 19xy + 6y^2)$

$p = 18$

$s = -19$

$-18, -1$

$(3x^2 - 18xy)(1xy + 6y^2)$

$3x(x-6y) - y(x-6y)$

$(3x-y)(x-6y)$

ANS: $3x^3y^2(3x-y)(x-6y)$

2. $-4x^3y + 17x^2y^2 - 4xy^3$

$p = 16$

$s = 17$

$16, 1$

$xy(-4x^2 + 17xy - 4y^2)$

$(-4x^2 + 16xy)(1xy - 4y^2)$

$-4x(x-4y) + y(x-4y)$

$(-4x+y)(x-4y)$

ANS: $xy(-4x+y)(x-4y)$

3. $9c^2 - 225m^4$

$9(c^2 - 25m^4)$

ANS: $9(c-5m^2)(c+5m^2)$

4. $20y^4 - 45x^2y^2$

$5y^2(4y^2 - 9x^2)$

ANS: $5y^2(2y-3x)(2y+3x)$

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Quiz # 7

Reduce the following algebraic fractions to their lowest terms. Show all the steps in the solutions. (10 marks)

1. $\dfrac{9a^2-b^4}{4a^3b^3-12a^4b}$

$$= \frac{(3a-b^2)(3a+b^2)}{4a^3b(b^2-3a)}$$

$$= \frac{-1(-3a+b^2)(3a+b^2)}{4a^3b(b^2-3a)}$$

$$= \boxed{\frac{-(3a+b^2)}{4a^3b}}$$

2. $\dfrac{x^2-5x+4}{48-3x^2}$

$$= \frac{(x-4)(x-1)}{3(16-x^2)}$$

$$= \frac{(x-4)(x-1)}{3(4-x)(4+x)}$$

$$= \frac{-1(-x+4)(x-1)}{3(4-x)(4+x)}$$

$$= \frac{-(x-1)}{3(4+x)}$$

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Quiz # 8

Express the products of the following algebraic fractions in lowest terms. Show all the steps in the solutions. (20 marks)

1. $\frac{16-q^2}{q} \times \frac{4q-12}{-16q-4q^2}$

$= \frac{(4-q)(4+q)}{q} \times \frac{4(q-3)}{-4q(4+q)}$

$= \frac{(4-q)(\cancel{4+q})}{q} \times \frac{4(q-3)}{-4q(\cancel{4+q})}$

$= \boxed{\frac{-(4-q)(q-3)}{q^2}}$

2. $\frac{4b^2-4b+1}{b-6} \times \frac{b^2-10b+24}{2b^2-9b+4}$

$= \frac{(2b-1)(2b-1)}{(b-6)} \times \frac{(b-4)(b-6)}{(2b-1)(b-4)}$

① $p=4$, $s=-4$, $-2,-2$

$(4b^2-2b)(-2b+1)$

$2b(2b-1)-1(2b-1)$

$(2b-1)(2b-1)$

② $2b^2-9b+4$

$p=8$, $s=-9$, $-8,-1$

$(2b^2-8b)(-1b+4)$

$2b(b-4)-1(b-4)$

$(2b-1)(b-4)$

$\frac{(2b-1)(\cancel{2b-1})}{(\cancel{b-6})} \times \frac{(\cancel{b-4})(\cancel{b-6})}{(\cancel{2b-1})(\cancel{b-4})}$

$= \boxed{2b-1}$

Express the quotients of the following algebraic fractions in lowest terms. Show all the steps in the solutions. (20 marks)

3. $\dfrac{4-x^2y^2}{x^2y^2+4} \div \dfrac{(x^2y^2z^2-4z^2)}{1}$

$z^2(x^2y^2-4)$

$z^2(xy-2)(xy+2)$

$$\frac{(2-xy)(2+xy)}{x^2y^2+4} \cdot \frac{1}{z^2(xy-2)(xy+2)}$$

$$\frac{-1(-2+xy)(2+xy)}{x^2y^2+4} \cdot \frac{1}{z^2(xy-2)(xy+2)} = \boxed{\frac{-1}{z^2(x^2y^2+4)}}$$

4. $\dfrac{16-x^2}{x-4} \div \dfrac{x^2-x-20}{x-5}$

$$\frac{(4-x)(4+x)}{x-4} \cdot \frac{(x-5)}{(x-5)(x+4)}$$

$$\frac{(4-x)(4+x)}{(x-4)} \cdot \frac{(x-5)}{(x-5)(x+4)}$$

$$\frac{-1(-4+x)}{(x-4)} = \boxed{-1}$$

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Quiz # 9

Express the sums of the following algebraic fractions in lowest terms. Show all the steps in the solutions. (~~20~~ 30 marks)

1. $\frac{1}{3-x} + \frac{x-2}{x(x-5)}$

c.d $= (3-x)(x-5)x$

$\frac{(x)(x-5)}{c.d.} + \frac{(x-2)(3-x)}{c.d.}$

$(x-2)(3-x)$
$= 3x - x^2 - 6 + 2x$

$\frac{x^2 - 5x}{c.d.} + \frac{-x^2 + 5x - 6}{c.d.}$

$\frac{x^2 - 5x - x^2 + 5x - 6}{c.d.} = \boxed{\frac{-6}{x(3-x)(x-5)}}$

2. $\frac{m^2}{2m^2+m} + \frac{1-m}{2m} = \frac{m^2}{(m)(2m+1)} + \frac{(1-m)}{2(m)}$

You could cancel m here, but you'd have to multiply it back in, anyway since it must be in the c.d. (it's in the 2nd denominator)

c.d. $= (2)(m)(2m+1)$

$(1-m)(2m+1)$
$2m + 1 - 2m^2 - m$
$-2m^2 + m + 1$

$\frac{2m^2}{c.d.} + \frac{(1-m)(2m+1)}{c.d.}$

$\frac{2m^2}{c.d.} + \frac{-2m^2 + m + 1}{c.d.}$

$\frac{2m^2 - 2m^2 + m + 1}{c.d.} = \boxed{\frac{m+1}{2m(2m+1)}}$

3. $\frac{-b^2-b+6}{b^2+3b} + \frac{b}{b+2} =$

$\frac{(-b+2)(b+3)}{b(b+3)} + \frac{b}{(b+2)}$

$-b^2-b+6$

$p = -6$

$s = -1$

$-3, +2$

$\frac{(-b+2)(b+3)}{b(b+3)} + \frac{b}{(b+2)}$

$(-b^2-3b)+(2b+6)$

$-b(b+3)+2(b+3)$

$(-b+2)(b+3)$

$\frac{(-b+2)}{b} + \frac{b}{(b+2)}$

c.d. $= b(b+2)$

$\frac{(-b+2)(b+2)}{c.d.} + \frac{b^2}{c.d.}$

$\frac{-b^2+4}{c.d.} + \frac{b^2}{c.d.}$

$\frac{-b^2+4+b^2}{c.d.}$

$\frac{4}{b(b+2)}$

Express the differences of the following algebraic fractions in lowest terms. Show all the steps in the solutions. (20 marks)

4. A. $\frac{4n^2}{n^3} - \frac{2n-n^2}{n-2}$ $= \frac{4n^2}{n^3} - \frac{n(2-n)}{(n-2)}$

$\frac{4n^2}{n^3} - \frac{n(-1)(-2+n)}{(n-2)}$

$\frac{4}{n} - \frac{-n}{1} = \frac{4}{n} + \frac{n}{1}$ c.d. = n

$\frac{4}{n} + \frac{n^2}{n}$

$\boxed{\frac{4+n^2}{n}}$

5. $\frac{4}{(x+2)} - \frac{x+2}{x}$ c.d. = $x(x+2)$

$\frac{4x}{c.d.} - \frac{(x+2)(x+2)}{c.d.}$

$\frac{4x}{c.d.} - \frac{(x^2+4x+4)}{c.d.}$

$\frac{4x - x^2 - 4x - 4}{c.d.}$

$\boxed{\frac{-x^2-4}{x(x+2)}}$ OR $\boxed{\frac{-(x^2+4)}{x(x+2)}}$

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Quiz # 10

The following two algebraic expressions are equivalent. Demonstrate their equivalence by transforming the expression on the left side.
Show all the steps in the solution. (10 marks)

$$\frac{-a^2-a+12}{a^2+4a}+\frac{a}{a+3}=\frac{9}{a^2+3a}$$

$\frac{(-a+3)(a+4)}{a(a+4)}+\frac{a}{a+3}$

$\frac{-a+3}{a}+\frac{a}{(a+3)}$ c.d. $= a(a+3)$

$\frac{(-a+3)(a+3)}{c.d.}+\frac{a^2}{c.d.}$

$\frac{-a^2+9}{c.d.}+\frac{a^2}{c.d.}$

$\frac{-a^2+9+a^2}{c.d.} = \boxed{\frac{9}{a(a+3)}} = \boxed{\frac{9}{a^2+3a}}$

expression on right!

$-a^2-a+12$

$p=-12$
$s=-1$
$-4, +3$

$(-a^2-4a)+(3a+12)$
$-a(a+4)+3(a+4)$
$(-a+3)(a+4)$

$(-a+3)(a+3)$
$-a^2-3a+3a+9$
$-a^2+9$

2. The following two algebraic expressions are equivalent. Demonstrate their equivalence by transforming the expression on the left side. Show all the steps in the solution. (10 marks)

$$\frac{4m+1}{m^2+4m+3} - \frac{4}{4m+12} = \frac{3m}{m^2+4m+3}$$

$$\frac{4m+1}{(m+3)(m+1)} - \frac{4}{4(m+3)}$$

$$\frac{4m+1}{(m+3)(m+1)} - \frac{\cancel{4}\,1}{\cancel{4}(m+3)}$$

$$\frac{4m+1}{(m+3)(m+1)} - \frac{1}{(m+3)} \qquad \text{c.d.} = (m+3)(m+1)$$

$$\frac{4m+1}{\text{c.d.}} - \frac{(m+1)}{\text{c.d.}}$$

$$\frac{4m+1-m-1}{\text{c.d.}}$$

$$= \frac{3m}{\text{c.d.}}$$

expression on right!

$$\frac{3m}{(m+3)(m+1)} = \frac{3m}{m^2+4m+3}$$

3. The following two algebraic expressions are equivalent. This time, demonstrate their equivalence by transforming both expressions. Show all the steps in the solution. (10 marks)

$$\frac{2(x+4)}{x^2+x-12} - \frac{4}{x-2} = \frac{2}{2-x} + \frac{2}{x^2-5x+6}$$

$$\frac{2(x+4)}{(x+4)(x-3)} - \frac{4}{(x-2)} \qquad \frac{2}{(2-x)} + \frac{2}{(x-2)(x-3)}$$

$$\frac{2(x+4)}{(x+4)(x-3)} - \frac{4}{(x-2)} \qquad \frac{-2}{(x-2)} + \frac{2}{(x-2)(x-3)}$$

$$\text{c.d.} = (x-2)(x-3)$$

$$\frac{2}{(x-3)} - \frac{4}{(x-2)} \qquad \frac{-2(x-3)}{\text{c.d.}} + \frac{2}{\text{c.d.}}$$

$$\text{c.d.} = (x-3)(x-2)$$

$$\frac{2(x-2)}{\text{c.d.}} - \frac{4(x-3)}{\text{c.d.}} \qquad \frac{-2x+6}{\text{c.d.}} + \frac{2}{\text{c.d.}}$$

$$\frac{2x-4}{\text{c.d.}} - \frac{(4x-12)}{\text{c.d.}} \qquad \frac{-2x+8}{\text{c.d.}}$$

$$\frac{2x-4-4x+12}{\text{c.d.}}$$

$$\frac{-2x+8}{\text{c.d.}}$$

$$\frac{2(-x+4)}{(x-3)(x-2)} = \frac{2(-x+4)}{(x-3)(x-2)}$$

4. The following two algebraic expressions are equivalent. Again, demonstrate that they are equivalent by transforming both expressions. Show all the steps in the solution. (10 marks)

$$\frac{(x^2-1)}{x^2+x-2} - \frac{(y^2-9)}{(y+3)^2} = \frac{3}{y+3} + \frac{-y+3x+3}{xy+6+2y+3x}$$

$\frac{(x-1)(x+1)}{(x+2)(x-1)} - \frac{(y-3)(y+3)}{(y+3)(y+3)}$; $\frac{3}{y+3} + \frac{-y+3x+3}{xy+6+2y+3x}$

$\frac{\cancel{(x-1)}(x+1)}{(x+2)\cancel{(x-1)}} - \frac{(y-3)\cancel{(y+3)}}{(y+3)\cancel{(y+3)}}$; grouping $\rightarrow (xy+2y)+(3x+6)$

$\frac{(x+1)}{(x+2)} - \frac{(y-3)}{(y+3)}$; $y(x+2)+3(x+2)$

$(y+3)(x+2)$

c.d. $= (x+2)(y+3)$; $\frac{3}{(y+3)} + \frac{-y+3x+3}{(y+3)(x+2)}$

c.d. $= (y+3)(x+2)$

$\frac{(x+1)(y+3)}{c.d.} - \frac{(y-3)(x+2)}{c.d.}$; $\frac{3(x+2)}{c.d.} + \frac{-y+3x+3}{c.d.}$

$\frac{xy+3x+y+3}{c.d.} - \frac{(xy+2y-3x-6)}{c.d.}$; $\frac{3x+6}{c.d.} + \frac{-y+3x+3}{c.d.}$

$\frac{xy+3x+y+3-xy-2y+3x+6}{c.d.}$; $\frac{3x+6-y+3x+3}{c.d.}$

$\cancel{xy}+3x+y+3-\cancel{xy}-2y+3x+6$

$\boxed{\frac{6x-y+9}{(x+2)(y+3)}}$ equal! $\boxed{\frac{6x-y+9}{(x+2)(y+3)}}$