

MTH-4106-1 REVIEW: FACTORING AND ALGEBRAIC FRACTIONS

Question 1

Factor the following polynomials:

a) $4ab^2 - 16b^4$

b) $10x^6y^8 - 15x^7y^7 + 20x^5y^8 - 5x^5y^6 - 5x^4y^5 + 5x^5y^5$

c) $a^2 - 5a - 6$

d) $2x^2 - 5xy + 3y^2$

e) $256 - 1.69x^2$

f) $5x^4y - 15x - 16x^3y^3 + 48y^2$

g) $12a^4b^9c^3 + 6a^3b^7c^3 - 9a^6b^6c^2 - 3a^4b^5c + 3a^3b^6c + 3a^2b^4c$

h) $x^2 + x - 56$

i) $4x^2 - 17xy + 4y^2$

j) $\frac{4b^2}{49} - 25c^2$

k) $9a^4b - 45a - 4a^3b^3 + 20b^2$

l) $25yx^4 - 625x^6$

m) $a^2 - a - 72$

n) $5y^2 - 26xy + 5x^2$

o) $\frac{36y^2}{25} - 64z^2$

p) $8p^4r^2 - 48p - 25p^3r^4 + 150r^2$

q) $y^2 + y - 12$

r) $144 - 2.89b^2$

s) $5cy^2 - 5c - d + 20cz^5 + dy^2 + 4dz^5$

t) $b^2 - b - 6$

u) $6mx^2 - 6m - n + 42my^6 + nx^2 + 7ny^6$

v) $-4x^2 + 17xy - 4y^2$

w) $pq + 6 + 3p + 2q$

x) $4t^2 - 16u^4$

y) $-x^2 - xy + 6y^2$

z) $-2a^2 + 5ab - 2b^2$

Question 2

Factor the following polynomials completely.
Show all the steps in the solutions.

a) $18m^5n^4 - 78m^4n^5 + 24m^3n^6$

b) $-2a^3b + 5a^2b^2 - 2ab^3$

c) $4a^2 - 36y^4$

d) $32x^2y^5 - 18x^4y^3$

e) $20x^6y^3 - 64x^5y^4 + 12x^4y^5$

f) $-7y^3z + 50y^2z^2 - 7yz^3$

g) $16x^2 - 64a^4$

h) $15a^5b^5 - 63a^4b^6 + 12a^3b^7$

i) $-6x^3y + 13x^2y^2 - 2xy^3$

j) $150a^4b - 24a^2b^3$

k) $108a^5b^2 - 3a^3b^4$

Question 3

Reduce the following algebraic fractions to their lowest terms.
Show all the steps in the solutions.

a)
$$\frac{16m^4 - n^6}{3m^2n^4 - 12m^4n}$$

b)
$$\frac{x^2 - 8x + 7}{147 - 3x^2}$$

c)
$$\frac{25x^4 - 4y^2}{4xy^3 - 10x^3y^2}$$

$$d) \frac{t^2 - 5t + 6}{12 - 3t^2}$$

$$e) \frac{x^2 - 8x + 15}{54 - 6x^2}$$

Question 4

Perform the following operations. All answers must be expressed in lowest terms. Show all the steps in the solutions.

$$a) \frac{16 - a^2b^2}{x^2 + 16} \div (a^2b^2y^2 - 16y^2)$$

$$\text{b) } \frac{5x^2}{x^3} - \frac{4x-x^2}{(x-4)}$$

$$\text{c) } \frac{9-x^2}{x} \times \frac{3x-9}{-9x-3x^2}$$

$$\text{d) } \frac{1}{4-y} + \frac{y-3}{y(y-7)}$$

$$\text{e) } \frac{t^2 - 25}{t^2 + 25} \div (25c^2 - c^2t^2)$$

$$\text{f) } \frac{3a^3}{a^4} - \frac{2a - a^2}{a - 2}$$

$$\text{g) } \frac{4 - y^2}{y} \times \frac{2y - 4}{-4y - 2y^2}$$

$$\text{h) } \frac{1}{5-w} + \frac{w-4}{w(w-9)}$$

$$\text{i) } \frac{4-y^2}{y-2} \div \frac{y^2-2y-8}{y-4}$$

$$\text{j) } \frac{12m^4}{m^5} - \frac{6m-m^2}{m-6}$$

$$\text{k) } \frac{16x^2 - 8x + 1}{x - 3} \times \frac{x^2 - 8x + 15}{4x^2 - 21x + 5}$$

$$\text{l) } \frac{a^2}{4a^2 + a} + \frac{3 - a}{4a}$$

$$\text{m) } \frac{25 - a^2}{a - 5} \div \frac{a^2 - a - 30}{a - 6}$$

$$\text{n) } \frac{6}{x+3} - \frac{x+3}{x}$$

$$\text{o) } \frac{25y^2 - 10y + 1}{y-4} \times \frac{y^2 - 11y + 28}{5y^2 - 36y + 7}$$

$$\text{p) } \frac{x^2}{5x^2 + x} + \frac{4-x}{5x}$$

$$q) \frac{36a^2 - 12a + 1}{a - 7} \times \frac{a^2 - 9a + 14}{6a^2 - 13a + 2}$$

$$r) \frac{1}{6 - m} + \frac{m - 5}{m(m - 11)}$$

$$s) \frac{10}{y + 5} - \frac{y + 5}{y}$$

Question 5

In each of the following problems the two algebraic expressions are equivalent. In each problem, demonstrate the equivalence by transforming the expression on the left side. Show all the steps in the solutions.

a)
$$\frac{-b^2 - b + 30}{b^2 + 6b} + \frac{b}{b+5} = \frac{25}{b^2 + 5b}$$

$$\text{b) } \frac{5x+1}{x^2+5x+4} - \frac{5}{5x+20} = \frac{4x}{x^2+5x+4}$$

$$c) \quad \frac{-x^2 - x + 42}{x^2 + 7x} + \frac{x}{x+6} = \frac{36}{x^2 + 6x}$$

$$d) \frac{6y+1}{y^2+6y+5} - \frac{4}{4y+20} = \frac{5y}{y^2+6y+5}$$

Question 6

In each of the following problems the two algebraic expressions are again equivalent. This time, demonstrate that equivalence by transforming both algebraic expressions (i.e. on the left **and** right sides). Show all the steps in the solutions.

a)
$$\frac{3(x+2)}{x^2-2x-8} - \frac{2}{x-3} = \frac{1}{3-x} + \frac{2x-5}{x^2-7x+12}$$

$$\text{b) } \frac{(t^2 - 16)}{t^2 - t - 12} - \frac{(u^2 - 25)}{(u + 5)^2} = \frac{10}{u + 5} + \frac{5 + u}{tu + 15 + 5t + 3u}$$

$$c) \quad \frac{4(x+6)}{x^2+2x-24} - \frac{5}{x-5} = \frac{1}{5-x} - \frac{4}{x^2-9x+20}$$

$$d) \quad \frac{(m^2 - 4)}{m^2 - m - 2} - \frac{(n^2 - 1)}{(n+1)^2} = \frac{-2}{n+1} + \frac{4m+n+5}{mn+m+n+1}$$

Answers - Review Booklet

1. a) $4b^2(a-4b^2)$

b) $5x^4y^5(2x^2y^3-3x^3y^2+4xy^3-xy-1+x)$

c) $(a-6)(a+1)$

d) $(2x-3y)(x-y)$

e) $(16-1.3x)(16+1.3x)$

f) $(5x-16y^2)(x^3y-3)$

g) $3a^2b^4c(4a^2b^5c^2+2ab^3c^2-3a^4b^2c-a^2b+ab^2+1)$

h) $(x+8)(x-7)$

i) $(4x-y)(x-4y)$

j) $(\frac{2b}{7}-5c)(\frac{2b}{7}+5c)$

k) $(9a-4b^2)(a^3b-5)$

l) $25x^4(y-25x^2)$

m) $(a-9)(a+8)$

n) $(5y-x)(y-5x)$

o) $(\frac{6y}{5}-8z)(\frac{6y}{5}+8z)$

p) $(8p-25r^2)(p^3r^2-6)$

q) $(y+4)(y-3)$

r) $(12-1.7b)(12+1.7b)$

s) $(y^2+4z^5-1)(5c+d)$

t) $(b-3)(b+2)$

u) $(x^2-1+7y^6)(n+6m)$

v) $(-4x+y)(x-4y)$

w) $(g+3)(p+2)$

x) $4(t-2u^2)(t+2u^2)$

y) $(-x+2y)(x+3y)$

z) $(-2a+b)(a-2b)$

2. a) $6m^3n^4(3m-n)(m-4n)$

b) $ab(-2a+b)(a-2b)$

c) $4(a-3y^2)(a+3y^2)$

d) $2x^2y^3(4y-3x)(4y+3x)$

e) $4x^4y^3(5x-y)(x-3y)$

f) $yz(-7y+z)(y-7z)$

g) $16(x-2a^2)(x+2a^2)$

h) $3a^3b^5(5a-b)(a-4b)$

i) $xy(-6x+y)(x-2y)$

j) $6a^2b(5a-2b)(5a+2b)$

k) $3a^3b^2(6a-b)(6a+b)$

3. a) $\frac{-(4m^2+n^3)}{3m^2n}$

b) $\frac{-(x-1)}{3(7+x)}$

c) $\frac{-(5x^2+2y)}{2xy^2}$

d) $\frac{-(t-3)}{3(2+t)}$

e) $\frac{-(x-5)}{6(3+x)}$

Answers - Rev
Booklet
cont'

4. a) $\frac{-1}{y^2(x^2+16)}$

l) $\frac{11a+3}{4a(4a+1)}$

b) $\frac{5+x^2}{x}$

m) -1

c) $\frac{-(3-x)(x-3)}{x^2}$ OR $\frac{(x-3)^2}{x^2}$

n) $\frac{-x^2-9}{x(x+3)}$ OR $-\frac{(x^2+9)}{x(x+3)}$

d) $\frac{-12}{y(4-y)(y-7)}$

o) $5y-1$

p) $\frac{19x+4}{5x(5x+1)}$

e) $\frac{-1}{c^2(t^2+25)}$

q) $6a-1$

f) $\frac{3+a^2}{a}$

r) $\frac{-30}{m(m-11)(6-m)}$

g) $\frac{-(2-y)(y-2)}{y^2}$ OR $\frac{(y-2)^2}{y^2}$

s) $\frac{-y^2-25}{y(y+5)}$ OR $-\frac{(y^2+25)}{y(y+5)}$

h) $\frac{-20}{w(5-w)(w-9)}$

5. Make left side = value on right

6. Make both sides equal :

i) -1

a) $\frac{x-1}{(x-4)(x-3)}$

c) $\frac{-x}{(x-4)(x-5)}$

j) $\frac{12+m^2}{m}$

b) $\frac{10t+u+35}{(u+5)(t+3)}$

d) $\frac{2m+n+3}{(n+1)(m+1)}$

k) $4x-1$