MTH – 4107 – 1
Straight Lines II

NAME: Shannon
DATE: _____

Duration: 2 hours 30 minutes

PRETEST

QUESTION 1 (10 marks)

Determine the equation of the line that passes through point $\left(5,-\frac{1}{3}\right)$ and is perpendicular to the line whose equation is $3x-2y+5=0$. Clearly show all your work.

l_1

$3x-2y+5=0$

$\frac{-2y}{-2} = \frac{-3x}{-2} \frac{-5}{-2}$

$y = \frac{3}{2}x + \frac{5}{2}$

$m = \frac{3}{2}$

l_2

$m = -\frac{2}{3}$

$y = mx+b$

$-\frac{1}{3} = \left(-\frac{2}{3}\right)(5) + b$

$-\frac{1}{3} = -\frac{10}{3} + b$

$-\frac{1}{3} + \frac{10}{3} = b$

$\frac{9}{3} = b$

$3 = b$

$y = -\frac{2}{3}x + 3$

QUESTION 2 (10 marks)

Determine the equation of the line that passes through point $(3,1)$ and is parallel to the line whose equation is $x - 3y = 6$. Clearly show all your work.

l_1

$x - 3y = 6$

$\frac{-3y}{-3} = \frac{-x}{-3} + \frac{6}{-3}$

$y = \frac{1}{3}x - 2$

$m = \frac{1}{3}$

l_2

$m = \frac{1}{3}$

$y = mx + b$

$1 = \left(\frac{1}{3}\right)(3) + b$

$1 = 1 + b$

$1 - 1 = b$

$0 = b$

$y = \frac{1}{3}x$

QUESTION 3 (10 marks)

Determine the equation of the line that passes through point $\left(-\frac{2}{5},5\right)$ and is parallel to the line whose equation is $-4x-9=0$. Clearly show all your work.

l_1

$-4x-9=0$

$-4x=9$

$\frac{-4x}{-4}=\frac{9}{-4}$

$x=-\frac{9}{4}$

Vertical line.

l_2

$x=-\frac{2}{5}$

Do these over for students (no room here)

QUESTION 4

Given the following five equations:

l_1: $2y = -5$ $\quad y = \frac{-5}{2}$

l_2: $-4y + 8x = 10$ $\longrightarrow$ $\frac{-4y}{-4} = \frac{-8x}{-4} + \frac{10}{-4}$ $\quad y = 2x - \frac{10}{4}$

l_3: $x = -2y$ $\longrightarrow$ $\frac{2y}{2} = \frac{-x}{2}$ $\quad y = -\frac{1}{2}x$

l_4: $-x - 2y = 12$

l_5: $\left(\frac{-4}{3}\right) - \frac{3}{4}x = 3\left(\frac{-4}{3}\right)$ $\quad x = -\frac{12}{3} = -4$

$\frac{-2y}{-2} = \frac{x}{-2} + \frac{12}{-2}$ $\quad y = -\frac{1}{2}x - 6$

a) Determine whether l_2 is perpendicular to l_4. Clearly show all your work and justify it. (2 marks)

yes $\quad l_2: m = 2$ $\quad l_4: m = -\frac{1}{2}$

b) Find a line that is parallel to l_3. Clearly show all your work and justify it. (2 marks)

l_4

c) Find a line that is parallel to l_2. Clearly show all your work and justify it. (2 marks)

None

d) Determine what line is concurrent with l_5 in point (-4,-4). Clearly show all your work and justify it. (2 marks)

l_4: $-x - 2y = 12$

$-(-4) - 2(-4) = 12$

$4 + 8 = 12$

e) Determine what line is concurrent with l_2 in its y-intercept. Clearly show all your work and justify it. (2 marks)

l_2: y int $= -\frac{10}{4} = \frac{-5}{2}$

answer: Q_1

QUESTION 5 (5 marks)

$x_1\ y_1 \qquad x_2\ y_2$

Calculate the distance between points E (-4,9) and F (3,-7). Round off your answer to the nearest hundredth, if necessary. Show all the steps in the solution.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3+4)^2 + (-7-9)^2}$$

$$= \sqrt{7^2 + (-16)^2}$$

$$= \sqrt{49 + 256}$$

$$= 17.46 \text{ u}$$

QUESTION 6 (5 marks)

The following expressions represent the distance between two points.

1) $\sqrt{(7+1)^2+(-2-5)^2}$

2) $|-2-2|$

3) $\sqrt{(-3+1)^2+(-2-5)^2}$

4) $|-5-5|$

5) $\sqrt{(7+3)^2+(-2+2)^2}$

Points A (-3,-2), B (7,-2) and C (-1,5) were used to define the segments below.

Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AC}$ 3

b) $\overline{BA}$ 4, 5

c) $\overline{BC}$ 1

AC : $\sqrt{(5+2)^2+(-1+3)^2}$

$= \sqrt{7^2+2^2}$

$= \sqrt{49+4} = \sqrt{53}$

BA : $\sqrt{(-2+2)^2+(7+3)^2}$

$= \sqrt{0^2+(10)^2}$

$= \sqrt{100} = 10$

BC : $\sqrt{(5+2)^2+(-1-7)^2}$

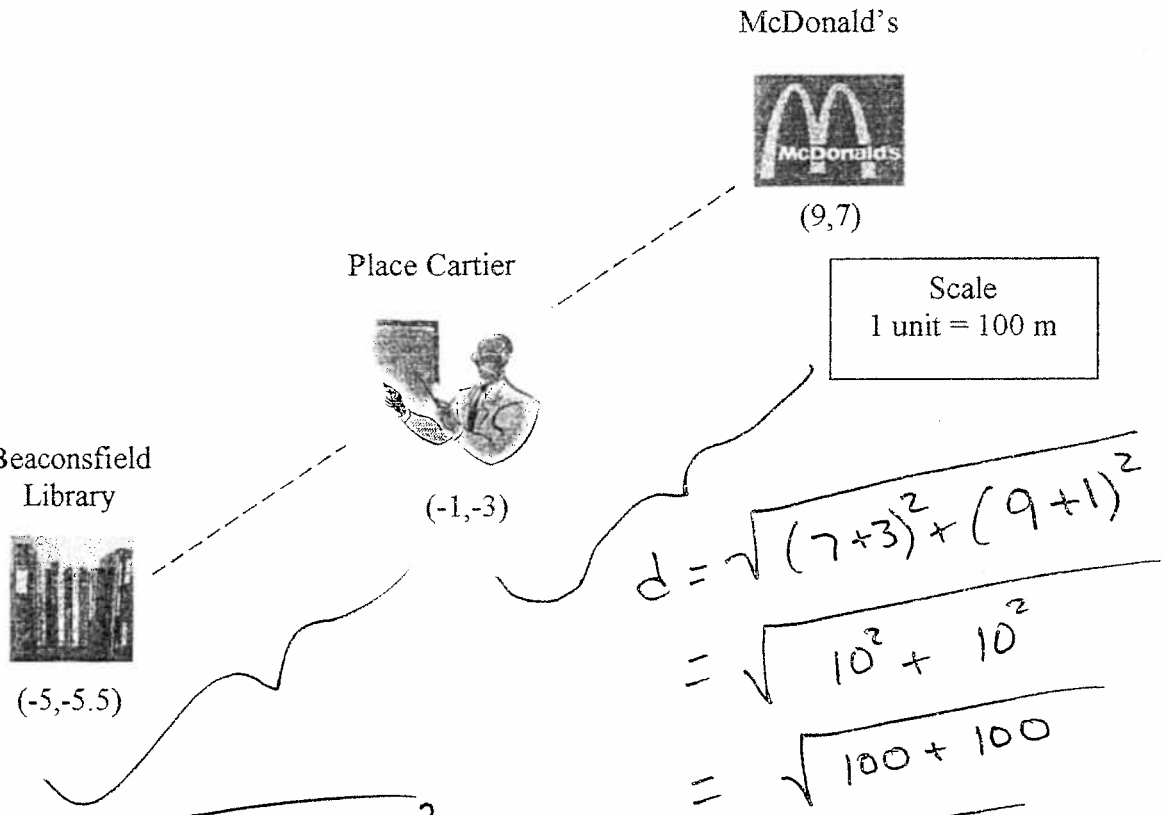
$= \sqrt{7^2+(-8)^2}$

QUESTION 7 (10 marks)

Two friends meet in the hall at Cartier. They decide to walk to McDonald's for lunch, and then to walk back out to the Beaconsfield Library to study (which of course will take them back past Cartier).

What distance will the friends cover by the time they reach the library?

Clearly show all your work.



$d = \sqrt{(-3+5.5)^2+(-1+5)^2}$

$= \sqrt{6.25+16}$

$= \sqrt{22.25}$

$= 4.72u$

$d = \sqrt{(7+3)^2+(9+1)^2}$

$= \sqrt{10^2+10^2}$

$= \sqrt{100+100}$

$= \sqrt{200}$

$= 14.142$

$\times 2 = 28.284u.$

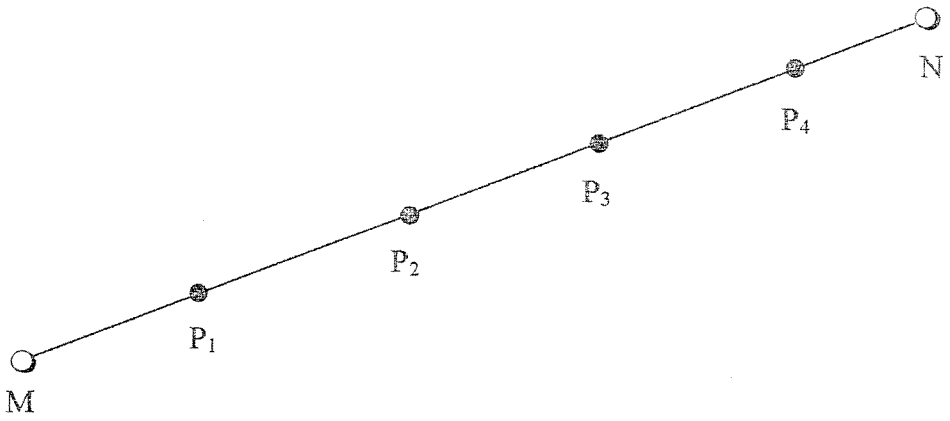
$4.72u + 28.284u = 33u$

$33 \times 100m$

$= 3300\,m$

QUESTION 8 (5 marks)

Points P_1, P_2, P_3, and P_4 divide segment $\overline{MN}$ into five equal parts.



Determine the point that corresponds to each statement below.

a) Divides MN in a ratio of $\frac{3}{2}$ P_3

b) Is located two-thirds of the way along $\overline{P_4P_1}$ P_2

c) Is located at the midpoint of $\overline{MP_4}$ P_2

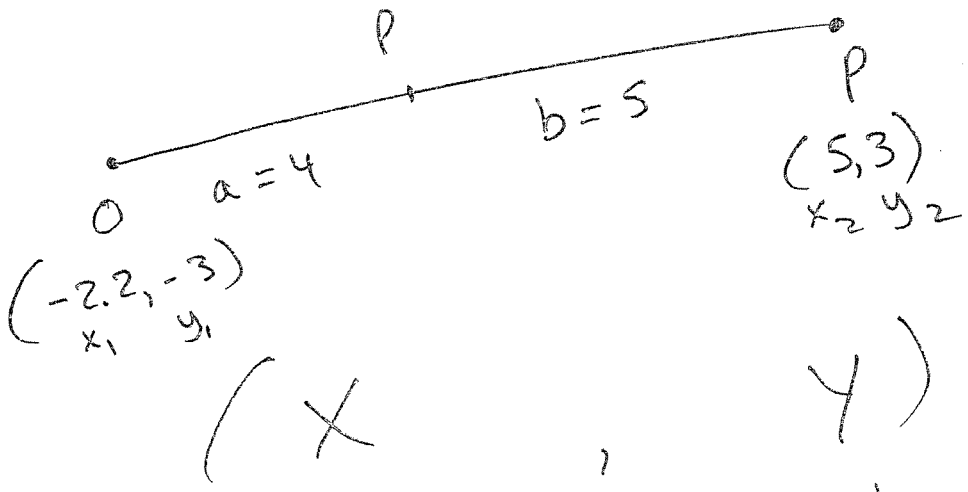
d) Divides $\overline{NP_1}$ in a ratio of $\frac{3}{1}$ P_2

e) Divides P_4P_1 in a ration of $\frac{1}{2}$ P_3

QUESTION 9 (5 marks)

Calculate the coordinates of the point that divides segment $\overline{OP}$ in a ratio of $\frac{4}{5}$.

The coordinates of point O are (-2.2,-3) and those of point P are (5,3). Show all the steps in the solution.



$$(X \quad , \quad Y)$$

$$\frac{ax_2 + bx_1}{a+b} \qquad \frac{ay_2 + by_1}{a+b}$$

$$\frac{(4)(5)+(5)(-2.2)}{9} \qquad \frac{(4)(3)+(5)(-3)}{9}$$

$$\frac{20+(-11)}{9} \qquad \frac{12-15}{9}$$

$$\frac{9}{9} \quad , \quad \frac{-3}{9}$$

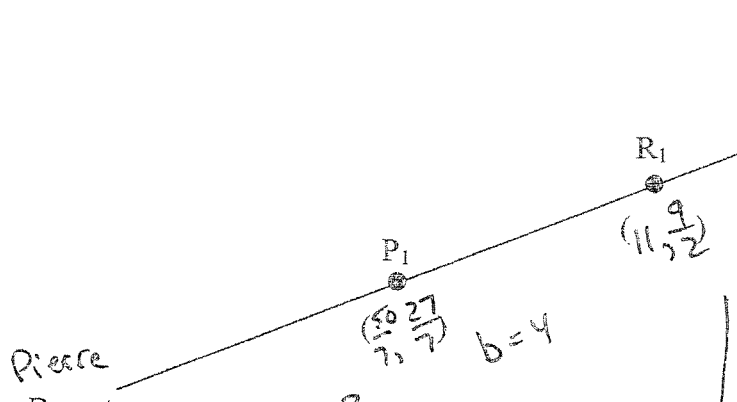
ANS: $\left(1, -\frac{1}{3}\right)$

QUESTION 10 (10 marks)

Pierre lives in downtown Montréal, while Ray lives on the West Island. The two friends decide to cycle toward one another along Highway 20 (just for fun – to see where they will meet!). The men leave at the same time. After 30 minutes, Pierre covers $\frac{3}{7}$ of the total distance. He is now at point P_1. Ray, travelling east from the West Island, has now reached point R_1. This point R_1 divides the distance from his starting point on the West Island to Pierre's starting point in a ratio of $\frac{1}{3}$.

One unit corresponds to 200m. Determine the distance between the cyclists after 30 minutes of cycling by identifying the coordinates of points P_1 and R_1.

Clearly show all your work.



P_1

x_1 y_1

$\frac{ax_2+bx_1}{a+b}$ $\frac{ay_2+by_1}{a+b}$

$\frac{(3)(14)+(4)(2)}{7}$, $\frac{(3)(5)+(4)(3)}{7}$

$\frac{42+8}{7}$, $\frac{15+12}{7}$

$\frac{50}{7}$, $\frac{27}{7}$

$a=3$ $b=4$

$d = \sqrt{\left(\frac{9}{2}-\frac{27}{7}\right)^2+\left(11-\frac{50}{7}\right)^2}$

$= \sqrt{0.4133+14.878}$

$= 3.91u$

$\times 200m$

782m

ANS

Ray R_1

$a=1$ x_1, y_1 Ray

$b=3$

$\frac{ax_2+bx_1}{a+b}$, $\frac{ay_2+by_1}{a+b}$

$\frac{(1)(2)+(3)(14)}{4}$, $\frac{(1)(3)+(3)(5)}{4}$

$\frac{2+42}{4}$, $\frac{3+15}{4}$

$\frac{44}{4}$, $\frac{18}{4}$

$(11, \frac{9}{2})$

QUESTION 11 (10 marks)

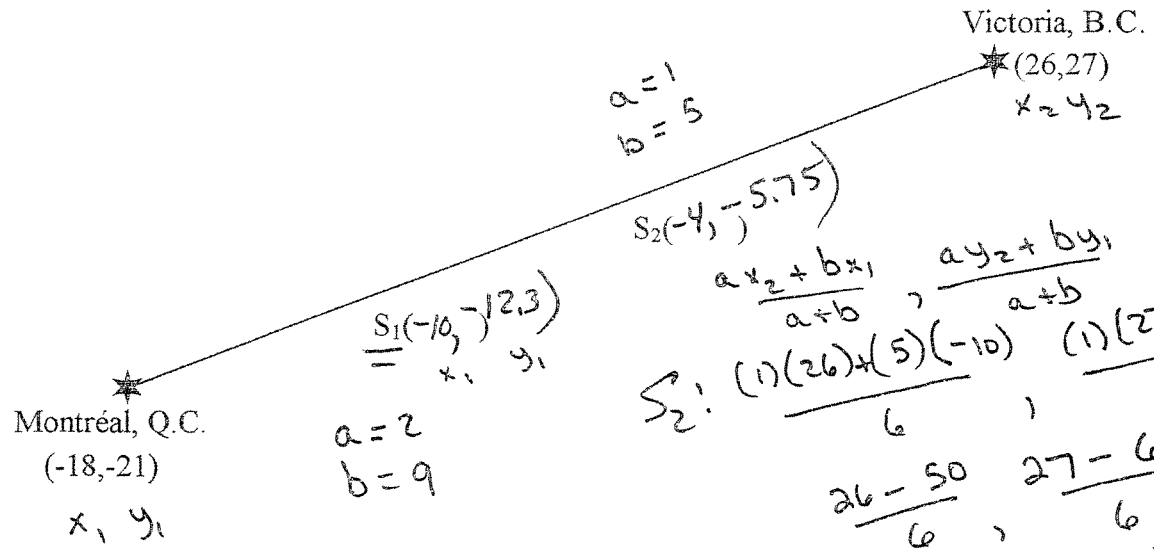
A group of friends decide to drive from Montréal to Victoria, B.C. for their summer vacation. They expect the drive to take close to a week.

On the first day, they reach a point (point S_1) that divides the total distance in a ratio of $\frac{2}{9}$.

On the second day, they cover, they cover one-sixth of the remaining distance (to reach point S_2).

One unit corresponds to 75 Km. Determine the distance they have left to cover to reach Victoria (S_2 – Victoria) by identifying the coordinates of points S_1 and S_2.

Clearly show all your work.



S_1: $\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}$

$\frac{(2)(26)+(9)(-18)}{11}, \frac{(2)(27)+(9)(-21)}{2+9}$

$\frac{52-162}{11}, \frac{54-189}{11}$

$(-10, -12.3)$

S_2: $\frac{(1)(26)+(5)(-10)}{6}, \frac{(1)(27)+(5)(-12.3)}{6}$

$\frac{26-50}{6}, \frac{27-61.5}{6}$

$(-4, -5.75)$

$d: \sqrt{(27+5.75)^2+(26+4)^2}$

$= \sqrt{1072.5625 + 900}$

$= 44.41$ u

$\times 75$ Km $=$

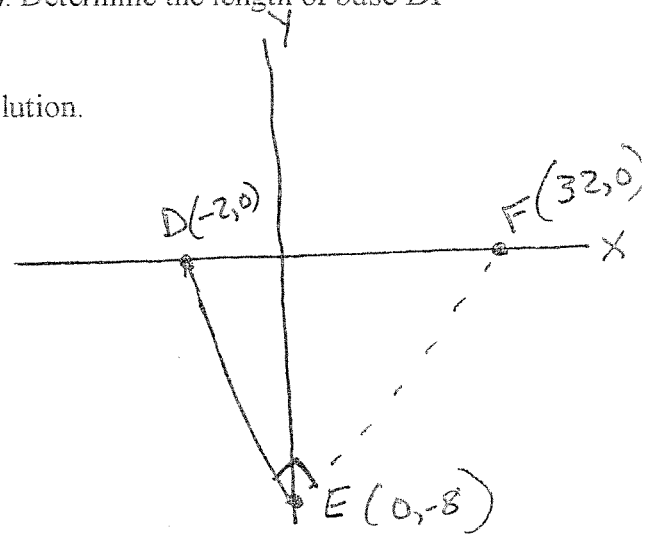
3331 Km left.

QUESTION 12 (10 marks)

In triangle DEF, angle E is a right angle.

The coordinates of D are (-2,0) and those of E are (0,-8). Determine the length of base DF given that point F is located on the x-axis.

Clearly show all your work. Show all the steps in the solution.



DE

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0+8}{-2-0} = \frac{8}{-2}$

$= -4$

F

$m = +\frac{1}{4}$

$y = mx + b$

$-8 = \left(\frac{1}{4}\right)(0) + b$

$-8 = 0 + b$

$-8 = b$

Eqn = $y = \frac{1}{4}x - 8$

F: $(x, 0)$ $\quad 0 = \frac{1}{4}x - 8$

$\left(\frac{4}{1}\right)8 = \frac{1}{4}x\left(\frac{4}{1}\right)$

$32 = x$

DF: $32 + 2 = 34$ u