MTH-4108-1 Quadratic Functions Quiz #1

Name: Shannon

Date:

Problem #1

A car dealer is trying to maximize his sales. A survey has shown that 90 people would buy the Intrepid model if it were to sell for $19 320; however, for each $345 price reduction, three more people would buy this vehicle.

Given that x represents the number of $345 price reductions, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total revenue from the sale of these vehicles. (10 marks)

Number of $345 reductions	Price of vehicle $	Total number of buyers	Total revenue $
0	19 320	90	1 738 800
1	19 320 – (345 × 1) = 18 975	90 + (1·3) 93	1 764 675
2	19 320 – (345×2) = 18 630	90 + (2·3) = 96	1788 480
x	19 320 – (345·x)	90 + (x·3)	(19 320 – 345x)(90 + 3x)

Determine the equation for calculating the total revenue.

Equation: $y = -1035x^2 + 26\,910x + 1\,738\,800$

$(19\,320 - 345x)(90 + 3x)$

$-1035x^2 - 31\,050x + 57\,960x + 1738800$

Problem #2

The Brick is selling a particular leather sofa for $3 300. At this price, they sell an average of 12 such sofas per week. Accountants for The Brick have ascertained that for each $60 decrease in the price of the sofa, there would be three more sofas sold per week.

Given that x represents the number of $60 price reductions, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the weekly revenue from the sale of these sofas. (10 marks)

Number of $ 60 reductions	Price of the sofa	Number of buyers per week	Revenue per week ($)
0	3 300	12	39 600
1	3 300 − 1 x 60	12 + 1 x 3	48 600
2	3 300 − 2 x 60	$12 + 2 \cdot 3$	(3180)(18) 57 240
x	$3300 - x \cdot 60$	$12 + x \cdot 3$	$(3300 - 60x)(12 + 3x)$

Equation: $y = -180x^2 + 9180x + 39\,600$

$$y = (3300 - 60x)(12 + 3x)$$

$$-180x^2 - 720x + 9900x + 39600$$

MTH-4108-1 Quadratic Functions Quiz #2

Name: Shannon

Date:

1. Solve the following equation using the appropriate factoring method. Show all the steps in the solution. (5 marks)

$2x^2 + 9 = 9x$

$p = 18$

$s = -9$

$-3, -6$

$2x^2 - 9x + 9 = 0$

$(2x^2 - 3x)(-6x + 9) = 0$

$x(2x-3) - 3(2x-3) = 0$

$(x-3)(2x-3) = 0$

① $x - 3 = 0$

$x = 3$

② $2x - 3 = 0$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$ or 1.5

$\boxed{\text{Solution: } x = 3 \text{ and } x = \frac{3}{2}}$

2. Solve the following equations using the quadratic formula. Show all the steps in the solutions. (5 marks each)

a) $-x^2 + 6x - 9 = 0$ $a = -1$ $b = 6$ $c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(-1)(-9)}}{2(-1)}$$

$$= \frac{-6 \pm \sqrt{36 - 36}}{-2} = \frac{-6 \pm \sqrt{0}}{-2}$$

$$= \frac{-6}{-2} = 3$$

$\boxed{\text{Solution: } x = 3}$

b) $-9x^2 + 6x - 1 = 0$ $a = -9$ $b = 6$ $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(-9)(-1)}}{2(-9)}$$

$$= \frac{-6 \pm \sqrt{36 - 36}}{-18}$$

$$= \frac{-6}{-18} = \frac{1}{3}$$

$\boxed{\text{Solution: } x = \frac{1}{3}}$

c) $3x - x^2 = -4$

$-x^2 + 3x + 4 = 0$

$a = -1 \quad b = 3 \quad c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(-1)(4)}}{2(-1)}$$

$$= \frac{-3 \pm \sqrt{9 + 16}}{-2}$$

$$= \frac{-3 \pm 5}{-2}$$

$\nearrow \frac{-3+5}{-2} = \frac{2}{-2} = -1$

$\searrow \frac{-3-5}{-2} = \frac{-8}{-2} = 4$

$\boxed{\text{Solution: } x = -1 \text{ and } x = 4}$

d) $3x^2 = 12$

$3x^2 - 12 = 0$

$a = 3 \quad b = 0 \quad c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{0^2 - 4(3)(-12)}}{2(3)}$$

$$= \frac{0 \pm \sqrt{144}}{6}$$

$$= \frac{0 \pm 12}{6}$$

$\nearrow \frac{0+12}{6} = \frac{12}{6} = 2$

$\searrow \frac{0-12}{6} = \frac{-12}{6} = -2$

$\boxed{\text{Solution: } x = 2 \text{ and } x = -2}$

MTH – 4108-1 Quadratic Functions Quiz #3

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Date:

1. Determine whether the following statements are true or false. (8 marks)

a) If a quadratic equation has one zero, then the discriminant (Δ) of this equation is less than zero. F

b) The discriminant (Δ) of a quadratic equation is greater than zero. The zeros of this equation could be 6 and −3. + T

c) The zeros of a quadratic equation whose discriminant (Δ) is −5 could be 4 and −2 F

d) If a quadratic equation has no zero, its discriminant (Δ) is equal to zero. F

e) The zero of a quadratic equation is 1. Its discriminant (Δ) could be less than 0. F

f) The discriminant (Δ) of a quadratic equation is −4. This equation has no zeros. T

g) The zeros of a quadratic equation are 6 and 2. The discriminant (Δ) of this equation is zero. F

h) If the discriminant (Δ) of a quadratic equation is greater than 0, then the equation has two distinct zeros. T

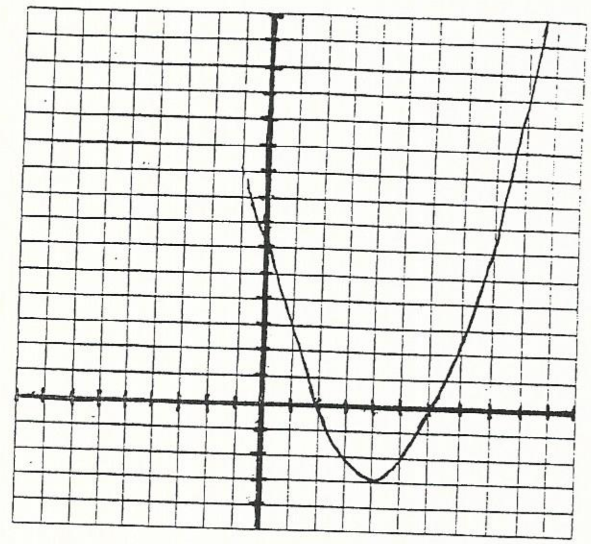
Δ	# zeros
+	2
0	1
−	0 (none)

2. By referring to the following graph, determine the characteristics listed below.

Scale

x-axis:
0.5 cm ≙ 1 unit

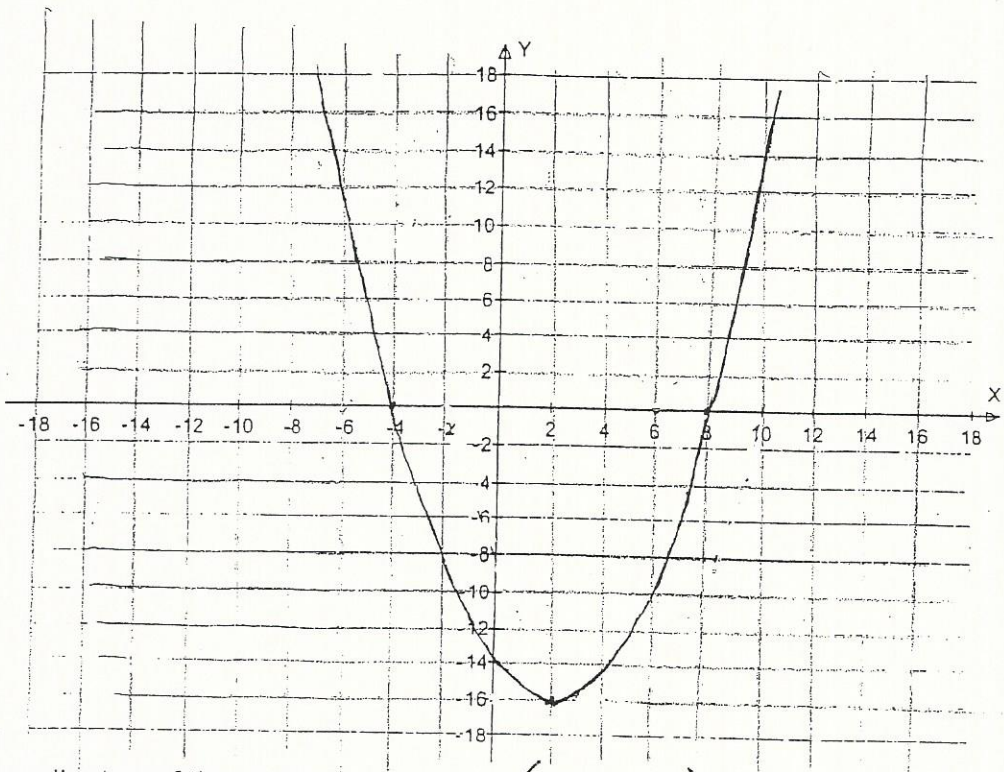
y-axis:
0.5 cm ≙ 1 unit



Coordinates of the vertex:	(4,-3)
Zero(s):	(2,0), (6,0) or 2 and 6
Equation of the axis of symmetry:	x = 4
y-intercept:	6 or (0,6)
Minimum:	-3 or (4,-3)

5 marks

3. By referring to the following graph, determine the characteristics listed below.



Coordinates of the vertex: (2, −16)

y-intercept: −14 or (0, −14)

Zero(s): −4 and 8 or (−4,0), (8,0)

Minimum: −16 or (2, −16)

Equation of the axis of symmetry: $x = 2$

5 marks

$-x^2+4x-4$ $p=4$
$s=4$
$2,2$

$a=-1$
$b=4$
$c=-4$

4. Graph the equation below:

$y=-x^2+4x-4$

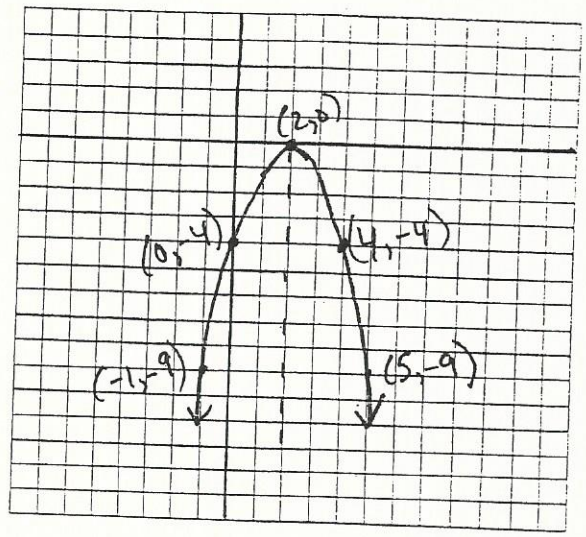
Opens down

$(-x^2+2x)+(2x-4)$
$-x(x-2)+2(x-2)$
$(-x+2)(x-2)$

$x=2$

Then determine the characteristics listed below and draw the axis of symmetry.

x	y
-1	-9
5	-9
2	0
1	-1



$\Delta = b^2-4ac$
$= 16-4(-1)(-4)$
$= 16-16=0$ (1 zero)

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)=\left(\frac{-4}{-2}, \frac{-0}{4a}\right)=(2,0)$

Coordinates of the y-intercept: $(0,-4)$

Coordinates of the point symmetric with the y-intercept: $(4,-4)$

Coordinates of the zeros: $(2,0)$

Equation of the axis of symmetry: $x=2$

10 marks

5. Graph the equation below:

$y = 3x^2 - 6x + 0$

$a = 3$

$b = -6$

$c = 0$

Opens up.

$3x^2 - 6x$

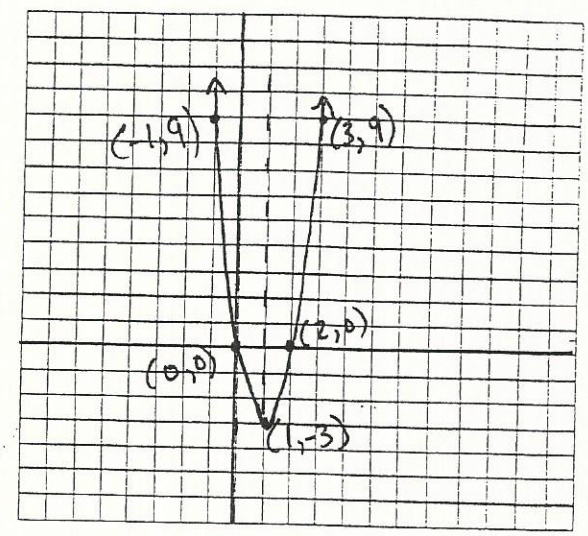
$3x(x-2) =$

$x = 2$

$x = 0$

Then determine the characteristics listed below and draw the axis of symmetry.

x	y
-1	9
3	9



$\Delta = b^2 - 4ac$

$= 36 - 4(3)(0)$

$= 36$

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right), \left(\frac{6}{6}, \frac{-36}{12}\right), (1,-3)$

Coordinates of the y-intercept: $(0,0)$

Coordinates of the point symmetric with the y-intercept: $(2,0)$

Coordinates of the zeros: $(0,0), (2,0)$

Equation of the axis of symmetry: $x = 1$

10 marks

$a = \frac{1}{2}$

6. Graph the equation below: $b = 1$

$$y = \frac{1}{2}x^2 + x + \frac{7}{2}$$

$c = \frac{7}{2}$

opens up

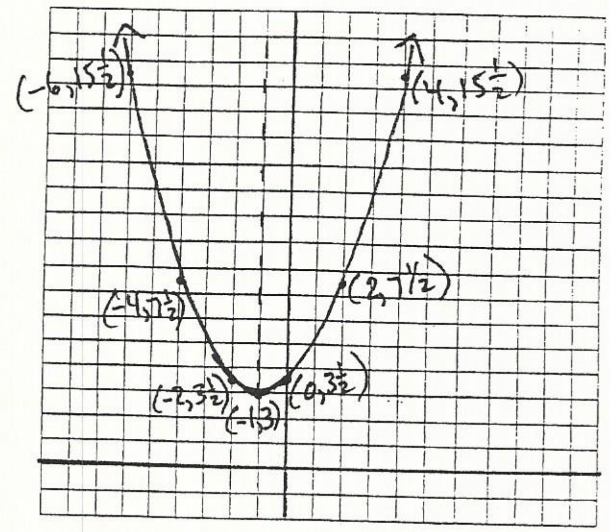
Then determine the characteristics listed below and draw the axis of symmetry.

-3 $\quad$ 5

-1 $\quad$ 3

x	y
-2	$3\frac{1}{2}$
-4	$7\frac{1}{2}$
2	$7\frac{1}{2}$
-6	$15\frac{1}{2}$
4	$15\frac{1}{2}$

1 $\quad$ 5



$\Delta = b^2 - 4ac$ $\quad 3,\ 11$

$= 1 - 4\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)$

$= 1 - 7 = -6$ (no zeros)

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right), \left(\frac{-1}{1}, \frac{6}{2}\right), (-1, 3)$

Coordinates of the *y*-intercept: $(0, 3\frac{1}{2})$ or $(0, \frac{7}{2})$

Coordinates of the point symmetric with the *y*-intercept: $(-2, 3\frac{1}{2})$

Coordinates of the zeros: none

Equation of the axis of symmetry: $x = -1$

10 marks

7. Graph the equation below: $a = \frac{1}{2}$

$$y = \frac{1}{2}x^2 - 2x + 4$$

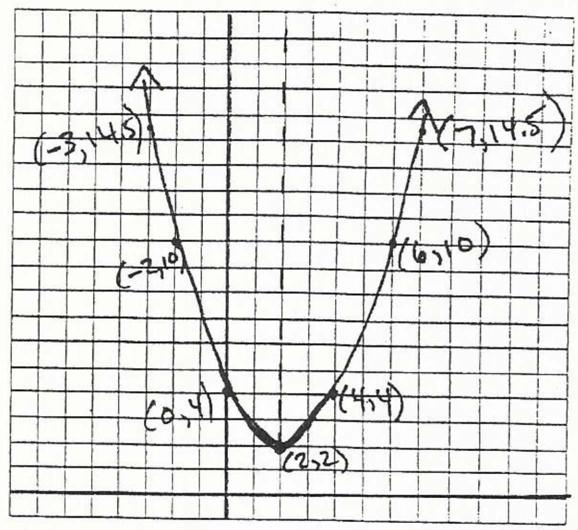
$b = -2$

$c = 4$

Opens up.

Then determine the characteristics listed below and draw the axis of symmetry.

x	y
-2	10
6	10
-3	14.5
7	14.5



$\Delta = b^2 - 4ac$

$= 4 - 4(\frac{1}{2})(4)$

$= 4 - 8 = -4$ (no zeros)

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right), \left(\frac{2}{1}, \frac{4}{2}\right), (2,2)$

Coordinates of the y-intercept: $(0, 4)$

Coordinates of the point symmetric with the y-intercept: $(4, 4)$

Coordinates of the zeros: no zeros

Equation of the axis of symmetry: $x = 2$

10 marks

$-x^2 + 10x - 16$ $\quad p = +16$

$a = -1 \quad (-x^2 + 2x) + (8x - 16) \quad s = +10$

8. Graph the equation below: $b = 10 \quad -x(x-2) + 8(x-2) \quad 2, 8$

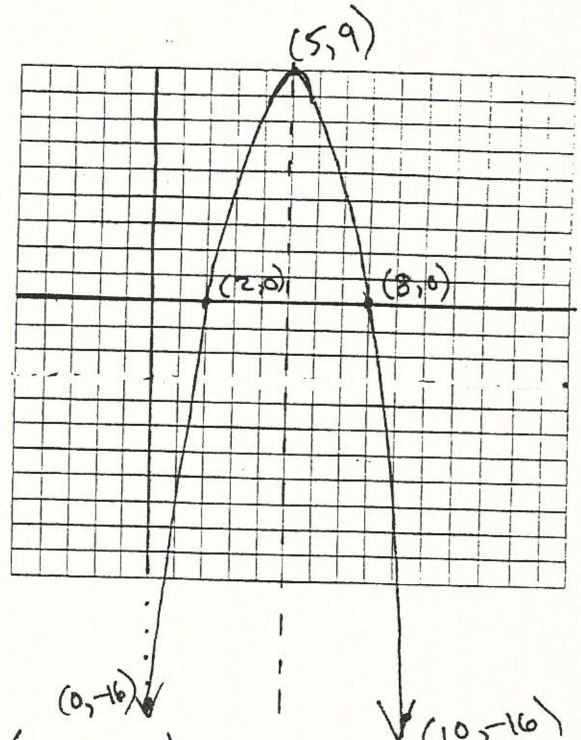
$y = -x^2 + 10x - 16 \quad c = -16 \quad (-x+8)(x-2) = 0$

$x = 2$

or $x = 8$

Then determine the characteristics listed below and draw the axis of symmetry.

opens down



x	y

$\Delta = b^2 - 4ac$

$= 100 - 4(-1)(-16)$

$= 100 - 64 = 36$

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right), \left(\frac{-10}{-2}, \frac{-36}{-4}\right) = (5, 9)$

Coordinates of the y-intercept: $(0, -16)$

Coordinates of the point symmetric with the y-intercept: $(10, -16)$

Coordinates of the zeros: $(2, 0)$ and $(8, 0)$

Equation of the axis of symmetry: $x = 5$

10 marks

9. Graph the equation below:

$y = 2x^2 + 2x - \frac{9}{2}$

$a = 2$

$b = 2$

$c = -\frac{9}{2}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{40}}{4}$

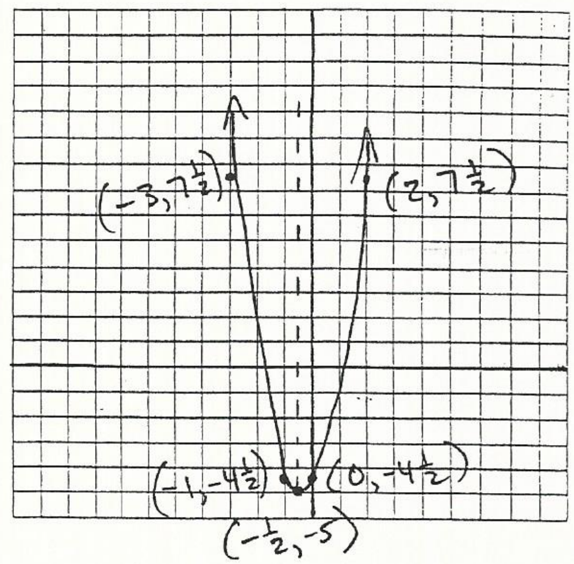
$= \frac{-2 + \sqrt{40}}{4} = 1.08$

or $\frac{-2 - \sqrt{40}}{4} = -2.08$

Then determine the characteristics listed below and draw the axis of symmetry.

opens up

x	y
2	$7\frac{1}{2}$
-3	$7\frac{1}{2}$



$\Delta = b^2 - 4ac$

$= 4 - 4(2)(-\frac{9}{2})$

$= 4 + 36$

$= 40$ (2 zeros)

Coordinates of the vertex: $\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right) = \left(\frac{-2}{4}, \frac{-40}{8}\right) = \left(-\frac{1}{2}, -5\right)$

Coordinates of the *y*-intercept: $(0, -\frac{9}{2})$ or $(0, -4\frac{1}{2})$

Coordinates of the point symmetric with the *y*-intercept: $(-1, -4\frac{1}{2})$

Coordinates of the zeros: $(1.08, 0)$, $(-2.08, 0)$

Equation of the axis of symmetry: $x = -\frac{1}{2}$

10 marks

MTH – 4108-1 Quadratic Functions Quiz #4 Name:

Date:

1. The bases are loaded, and Jackson steps up to the plate. He *swaks* the ball, whose trajectory is defined by the equation: $y = \frac{-2}{49}(x-21)^2 + 18$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work. (10 marks)

$$y = -\frac{2}{49}(x-21)(x-21) + 18$$

$$= -\frac{2}{49}(x^2 - 42x + 441) + 18$$

$$= -\frac{2}{49}x^2 + 1\tfrac{5}{7}x - 18 \quad + 18$$

$$y = -\frac{2}{49}x^2 + 1\tfrac{5}{7}x$$

$$\left(0 = -\frac{2}{49}x^2 + 1\tfrac{5}{7}x\right)49$$

$$0 = -2x^2 + 84x$$

$$0 = -2x(x - 42)$$

↑

42m = answer

2. An object is thrown from the top of a building 18 m high. The graph below represents the height in metres reached by the object with respect to the time in seconds. The equation that represents this situation is:

$$y = -\frac{15}{7}x^2 + \frac{180}{7}x + 18$$

Determine how long it will take for the object to reach its maximum height. Clearly show all your work. (10 marks)

$$x = \frac{-b}{2a} = \frac{-\frac{180}{7}}{2\left(-\frac{15}{7}\right)} = \frac{-\frac{180}{7}}{-4\frac{2}{7}} = 6$$

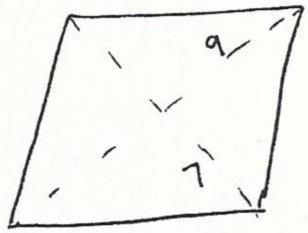
$\boxed{\text{Answer: } \underline{6 \text{ seconds}}}$

3. Marcela tracked the price of her company's shares over a one-month period. She noticed that the share price fluctuated according to the equation
$y = \frac{x^2}{10} - 2x + 6.5$, where x represents a specific day during the observation period and y, the share price in \$.
On which day did the share price reach its lowest point? Clearly show all your work. (10 marks)

$$x = \frac{-b}{2a} = \frac{2}{2\left(\frac{1}{10}\right)} = \frac{2}{\frac{1}{5}} = 10$$

$\boxed{\text{Answer: Day } 10}$

4. A rhombus has an area of $126cm^2$. Given that the long diagonal measures 4 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work. (10 marks)



Let x = short diagonal

$x+4$ = long diagonal

$$\frac{D \cdot d}{2} = A$$

$$\frac{D \cdot d}{2} = 126$$

$$\frac{(x+4)x}{2} = 126$$

$$\frac{1}{2}x^2 + 2x - 126 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4\left(\frac{1}{2}\right)(-126)}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 + 252}}{1}$$

$$= -2 \pm 16$$

$$= 14$$

$x = 14$

$x + 4 = 18$

$c^2 = 7^2 + 9^2$

$c^2 = 49 + 81$

$= 130$

$c = 11.4$

$4c = 45.6\,cm$

MTH – 4108-1 Quadratic Functions Quiz #5

Name: Shannon

Date:

1. A freight train must travel a distance of 534 km on its run. If an 18-wheeler were to take the same route, but travel 70 km/h faster than the freight train, the travel time could be reduced by 9.2 hours. What is the speed of the freight train? Round off your answer to the nearest unit.

Clearly show all your work.

Let x = speed of freight train

x + 70 = speed of truck

$$\frac{534}{x} - \frac{534}{x+70} = 9.2$$

$$\frac{534(x+70)}{c.d.} - \frac{534x}{c.d.} = 9.2$$

$$\frac{534x + 37380}{c.d.} - \frac{534x}{c.d.} = 9.2$$

$$\frac{37380}{x(x+70)} = \frac{9.2}{1}$$

$$\frac{37380}{x^2+70x} = \frac{9.2}{1}$$

$$9.2x^2 + 644x = 37380$$

$$9.2x^2 + 644x - 37380 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-644 \pm \sqrt{(644)^2 - 4(9.2)(-37380}}{2(9.2)}$$

$$= \frac{-644 \pm \sqrt{414736 + 1375584}}{18.4}$$

$$= \frac{-644 \pm 1338.0284}{18.4}$$

$$= \boxed{37.7 \text{ km/h} = \text{speed of freight train}}$$

2. In a factory, 150-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 10 litres per minute, it would take 30 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x + 10$ = hypothetical new rate of flow

$$\frac{150}{x} - \frac{150}{x+10} = 0.5$$

$$\frac{150(x+10)}{c.d.} - \frac{150x}{c.d.} = 0.5$$

$$\frac{150x + 1500}{c.d.} - \frac{150x}{c.d.} = 0.5$$

$$\frac{1500}{x(x+10)} = 0.5$$

$$\frac{1500}{x^2 + 10x} = \frac{0.5}{1}$$

$$0.5x^2 + 5x = 1500$$

$$0.5x^2 + 5x - 1500 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(0.5)(-1500)}}{2(0.5)}$$

$$= \frac{-5 \pm \sqrt{25 + 3000}}{1}$$

$$= -5 \pm 55$$

$$= \boxed{50 \text{ L/min} = \text{flow from tap}}$$

3. Sue measured the area of the front of a picture frame. She found that it measured 514 cm^2. Given that the frame is 13 cm longer than it is wide, determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation. (5 marks)

let x = width

$x + 13$ = length

$x(x+13) = 514$

$\boxed{x^2 + 13x - 514 = 0}$

4. The square of a number increased by twice that number is equal to 168. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation. (5 marks)

let x = #

$x^2 + 2x = 168$

$\boxed{x^2 + 2x - 168 = 0}$