ANSWER KEY

SETS, RELATIONS & FUNCTIONS

Review Package

MATH 4109-1

Sets

1. Let $A = \{1, 3, 7, 9, 10, 13, 14, 15\}$
$B = \{3, 9\}$
$C = \{9, 10, 14, 15\}$

Find $B \setminus (A \cap C)$

$A \cap C \rightarrow \{9, 10, 14, 15\}$
$B \rightarrow \{3, 9\}$
$B \setminus (A \cap C) \rightarrow \{3\}$

2. Let $A = \{6, 7, 10, 12, 16, 21\}$
$B = \{0, 1, 2, 3\}$
$C = \{3, 5, 6, 7, 10, 15\}$

Find $(A \cap C) \setminus B$

$A \cap C \rightarrow \{6, 7, 10\}$
$B \rightarrow \{0, 1, 2, 3\}$
$(A \cap C) \setminus B \rightarrow \{6, 7, 10\}$

3. Write the following in interval notation:

a) $\{x \in \mathbb{R} \mid x > 5\}$

$]5, \infty[$

b) $\{x \in \mathbb{R} \mid x \leq -2\}$

$]-\infty, -2]$

c) $\{x \in \mathbb{R} \mid -1 < x \leq 4\}$

$]-1, 4]$

4. Write the following in set-builder notation:

d) $-\infty, 6[$

$\{x \in \mathbb{R} \mid x < 6\}$

e) $]2, 7[$

$\{x \in \mathbb{R} \mid 2 < x < 7\}$

f) $]-4, 3]$

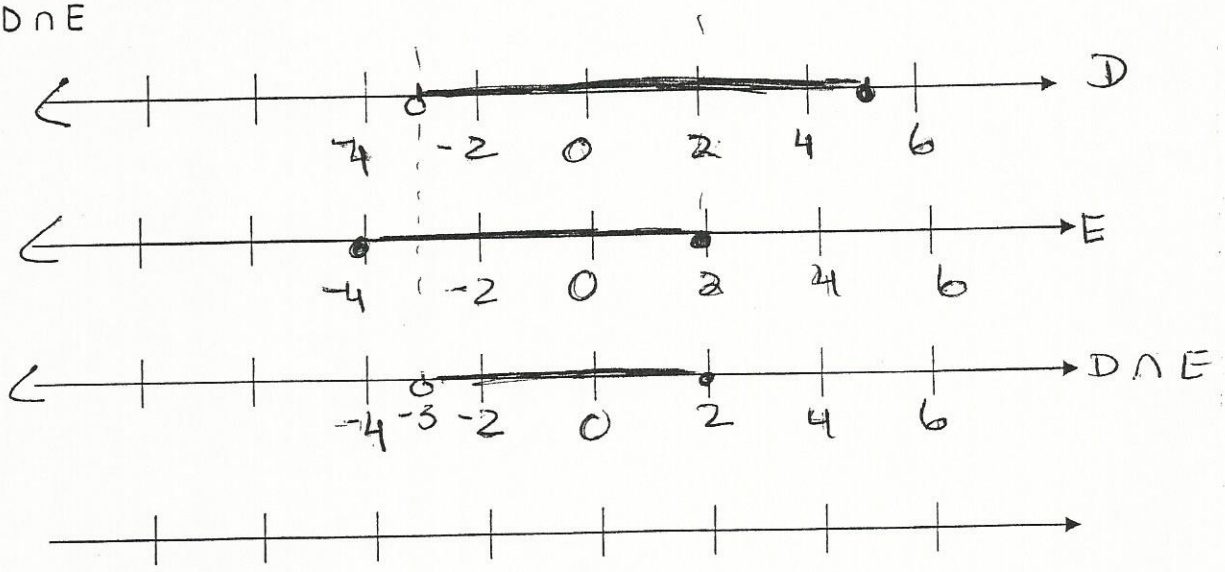
$\{x \in \mathbb{R} \mid -4 < x < 3\}$

5. a) Graph the following:

D=]-3, 5]

E = $\{x \in \mathbb{R} \mid -4 \leq x \leq 2\}$

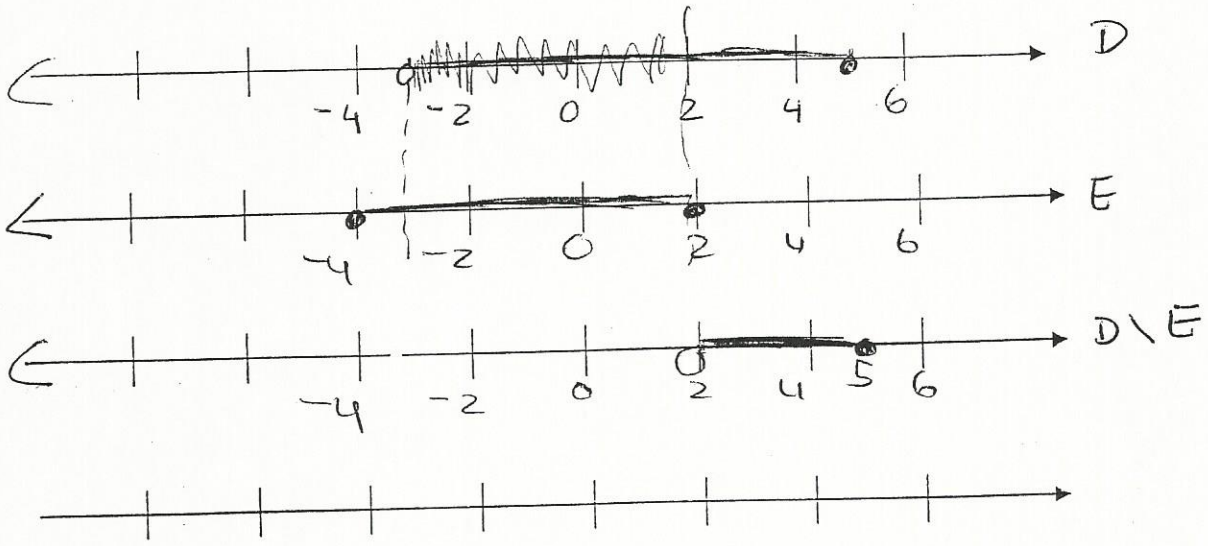
D ∩ E



Give your answer in interval notation: $]-3, 2]$

Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid -3 < x \leq 2\}$

b) Using the same information in 5a), graph $D \setminus E$



Give your answer in interval notation: $]2, 5]$

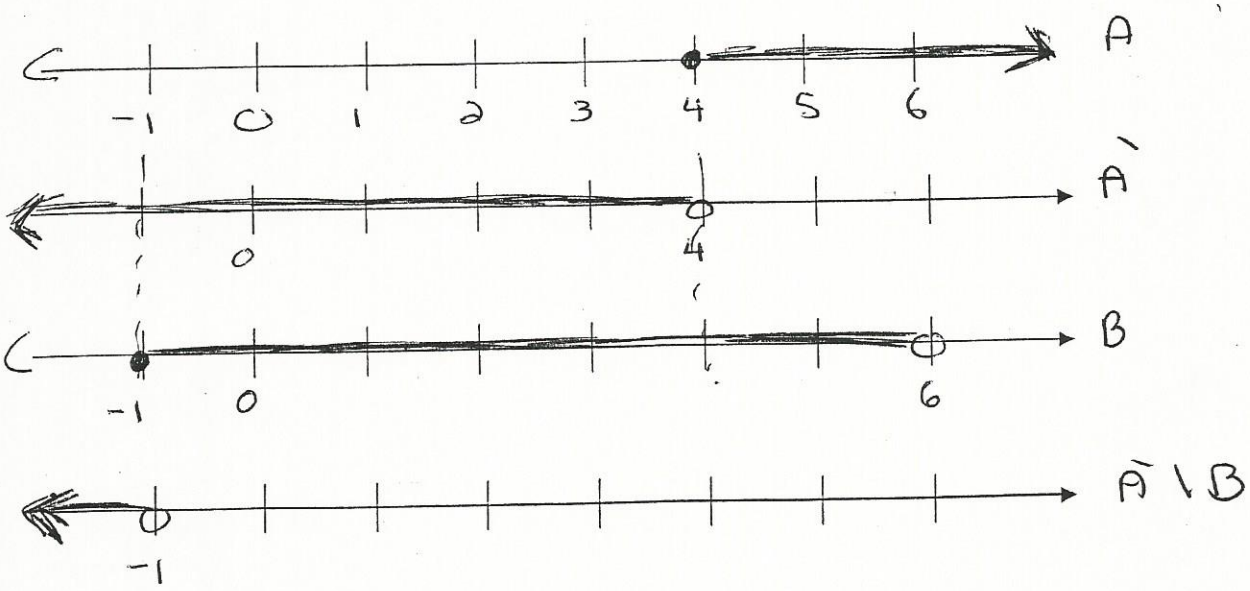
Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid 2 < x \leq 5\}$

6. Given the following sets:

$A = \{x \in \mathbb{R} \mid x \geq 4\}$

$B = \{x \in \mathbb{R} \mid -1 \leq x < 6\}$

Perform the following operations: $A' \setminus B$



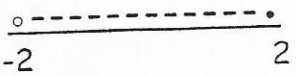
Give your answer in interval notation: $(-\infty, -1)$

Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid x < -1\}$

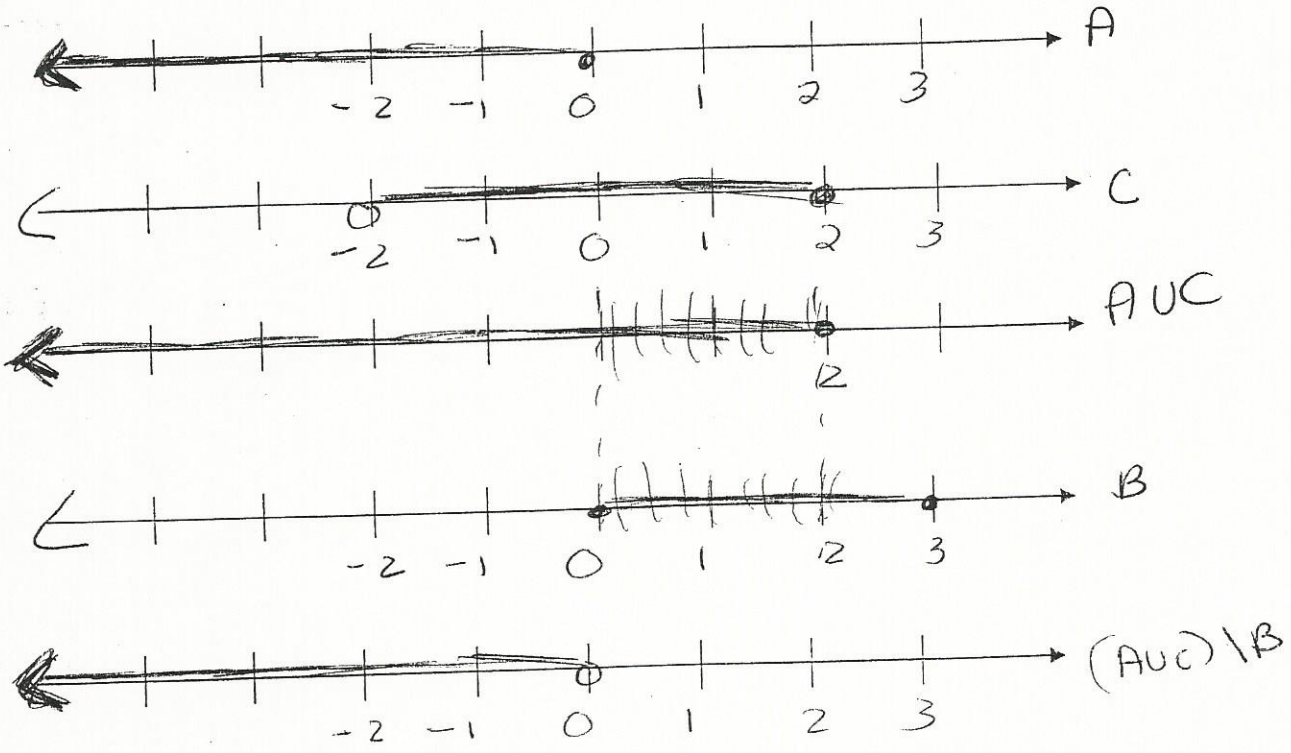
7. Given the following intervals:

$A = (-\infty, 0]$

$B = [0, 3]$

$C =$ 

Perform the following operations: $(A \cup C) \setminus B$



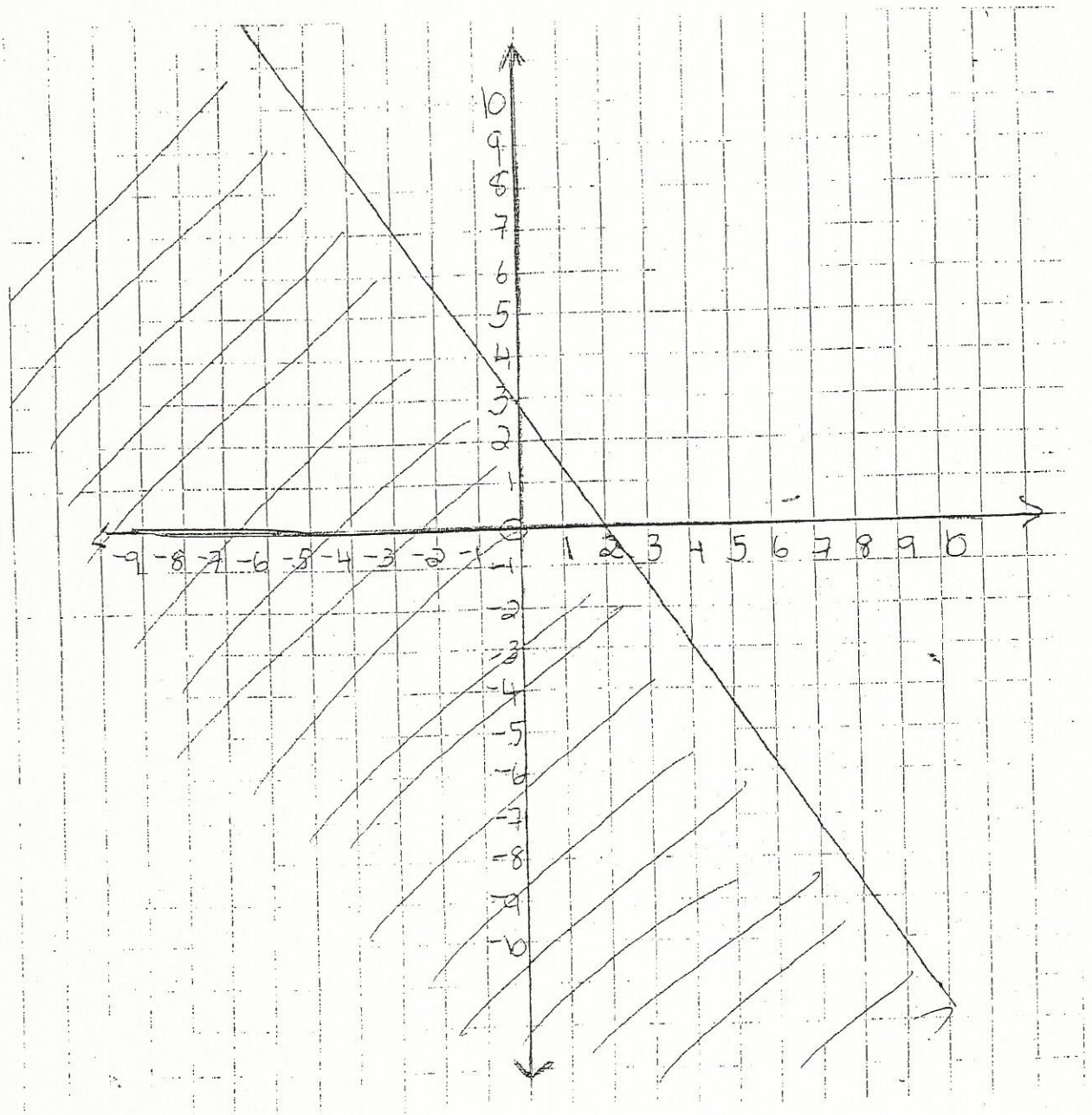
Give your answer in interval notation: $-\infty, 0[$

Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid x < 0\}$

Relations

y-int: 0, 3
x-int: 2, 0

8. Use set-builder notation to define the relation below:

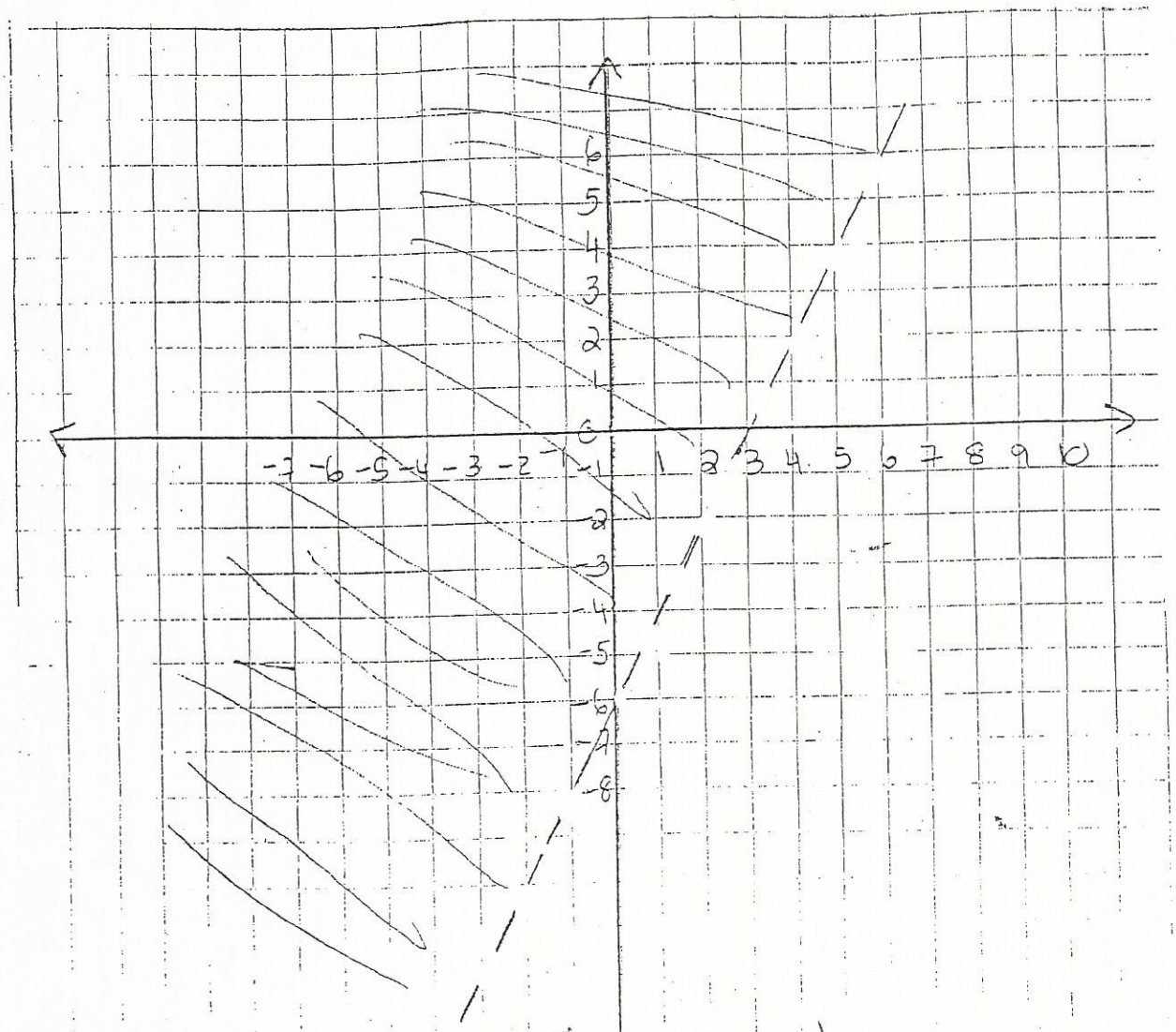


$y = mx + b$
$0 = m(2) + 3$
$0 = 2m + 3$
$\frac{-2m}{-2} = \frac{3}{-2}$
$m = -3/2$

$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y \leq -3/2x + 3\}$

9. Use set-builder notation to define the relation below:

y-int: 0, −6
x-int: 3, 0



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y > 2x - 6\}$

$y = mx + b$

$0 = m(3) - 6$

$0 = 3m - 6$

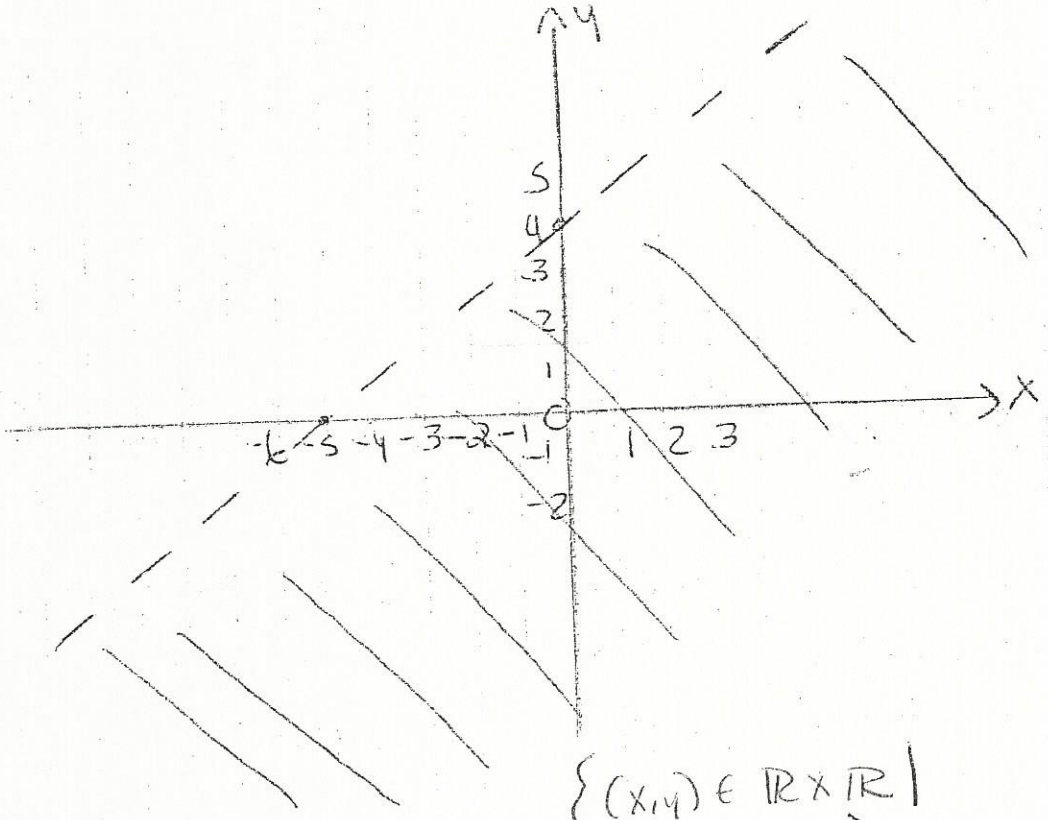
$\frac{-3m}{-3} = \frac{-6}{-3}$

$m = 2$

10. Use set-builder notation to define the relation below:

y-int: 0, 4

x-int: -5, 0



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y < 4/5x + 4\}$

$y = mx + b$

$0 = m(-5) + 4$

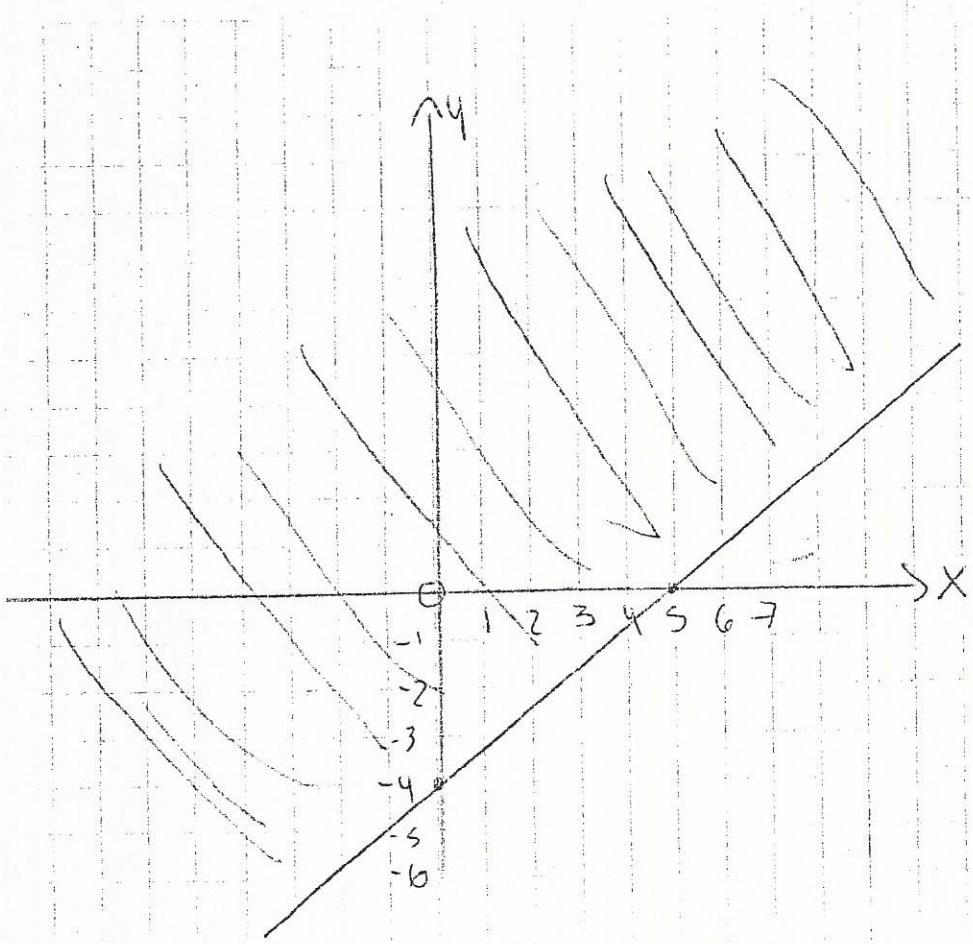
$0 = -5m + 4$

$\frac{5m}{5} = \frac{4}{5}$

$m = 4/5$

11. Use set-builder notation to define the relation below:

y-int: 0, -4
x-int: 5, 0

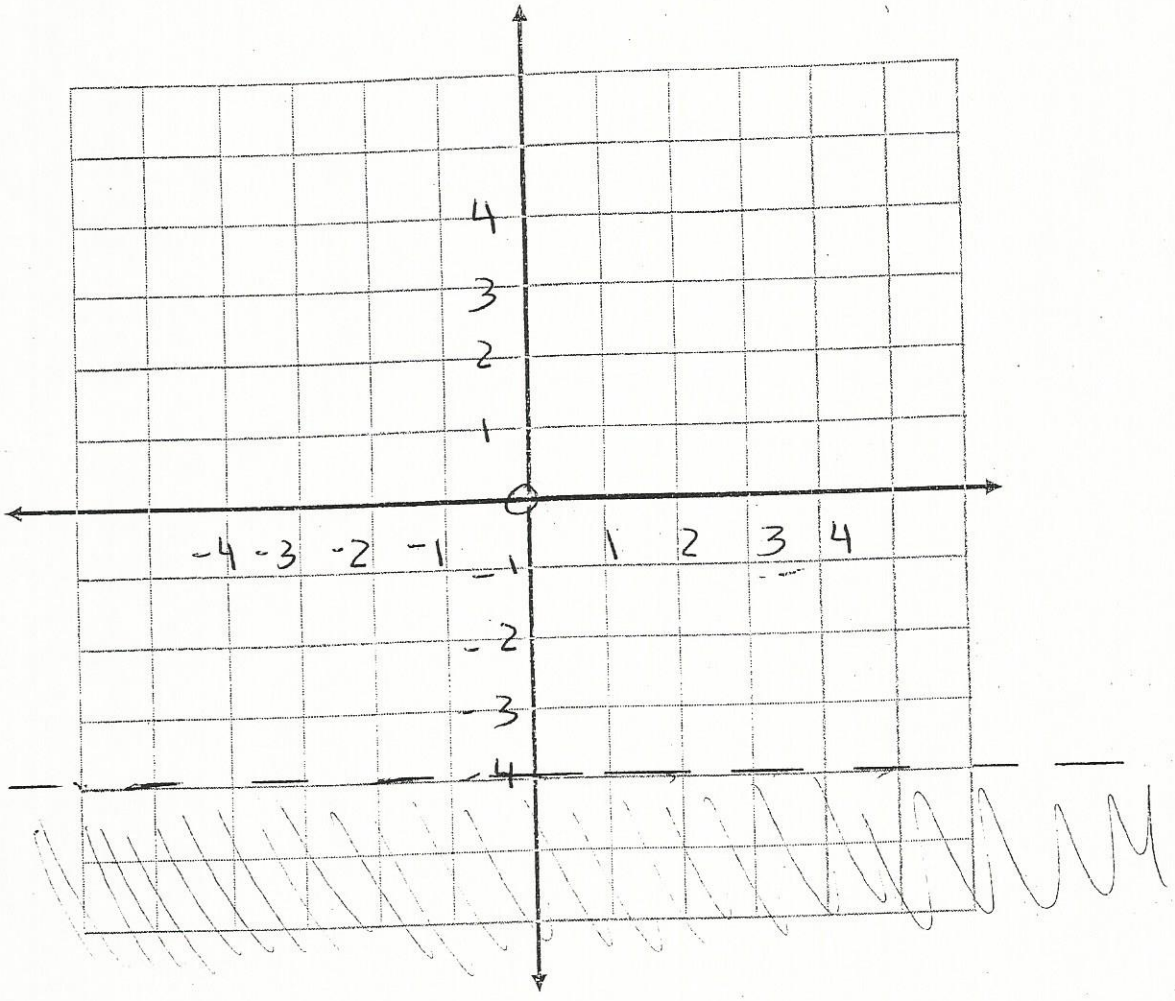


$y = mx + b$
$0 = m5 + -4$
$0 = 5m - 4$
$\frac{-5m}{-5} = \frac{-4}{-5}$
$m = \frac{4}{5}$

$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y \geq \frac{4}{5}x - 4\}$

12. Graph the following relation in the Cartesian plane below:

$$S = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid \frac{-y}{2} > 1 - \frac{y}{4}\right\}$$



Domain: $\mathbb{R}$

Range: $(-\infty, -4)$

$$+\frac{1}{1\times 4}y - \frac{1}{2\times 2}y > 1$$

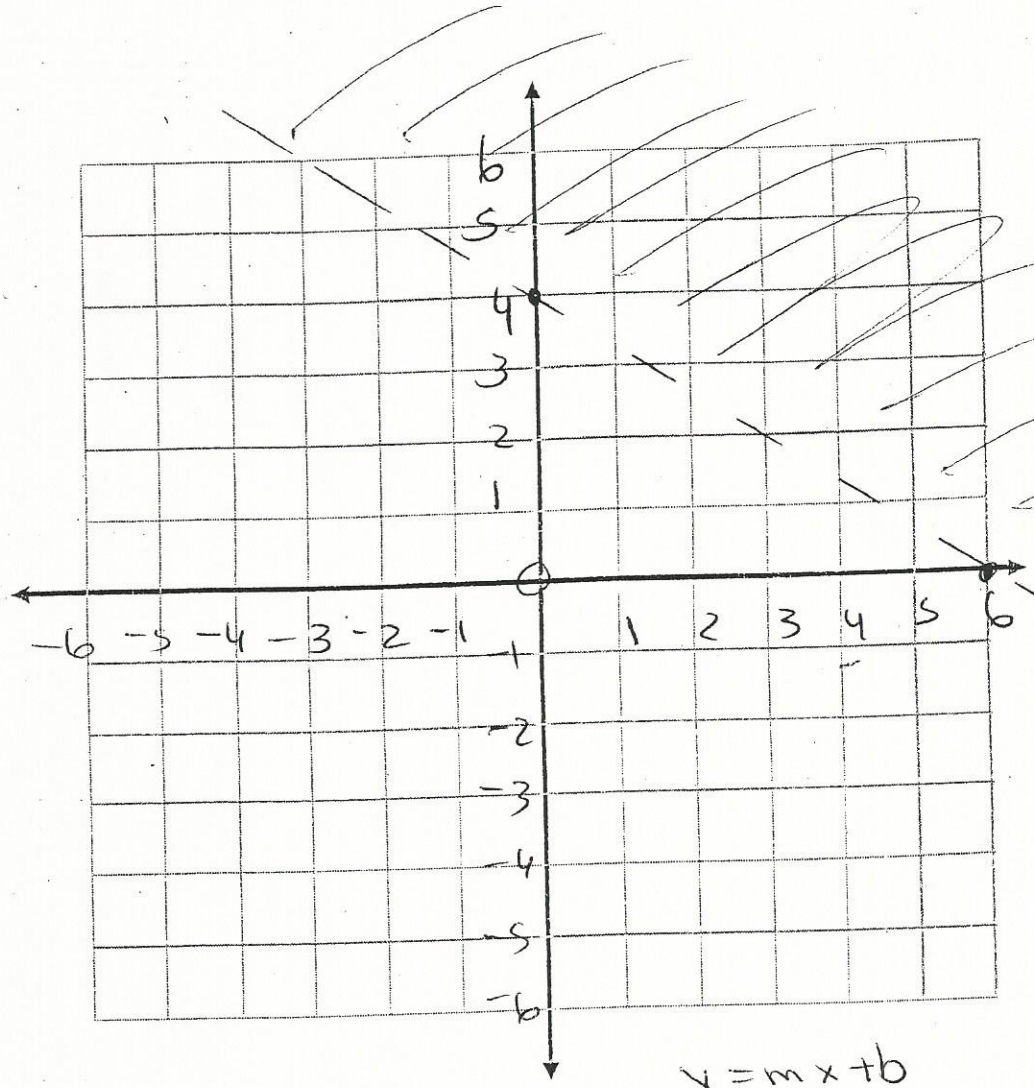
$$\frac{+1-2}{4}y > 1$$

$$\left(-\frac{4}{1}\right)\frac{-1}{4}y > 1\left(-\frac{4}{1}\right)$$

$$y < -4$$

13. Graph the following relation in the Cartesian plane below:

$$S = \left\{(x,y) \in \mathbb{R} X \mathbb{R} \mid \frac{x}{6} - 1 > \frac{-y}{4}\right\}$$



$y = mx + b$

$\left(\frac{4}{1}\right)\frac{1}{4}y > \left(-\frac{1}{6}x + 1\right)\frac{4}{1}$

$y > -\frac{4}{6}x + 4$

$y > -\frac{2}{3}x + 4$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

y-int: (0, 4)

x-int: (6, 0)

x-int

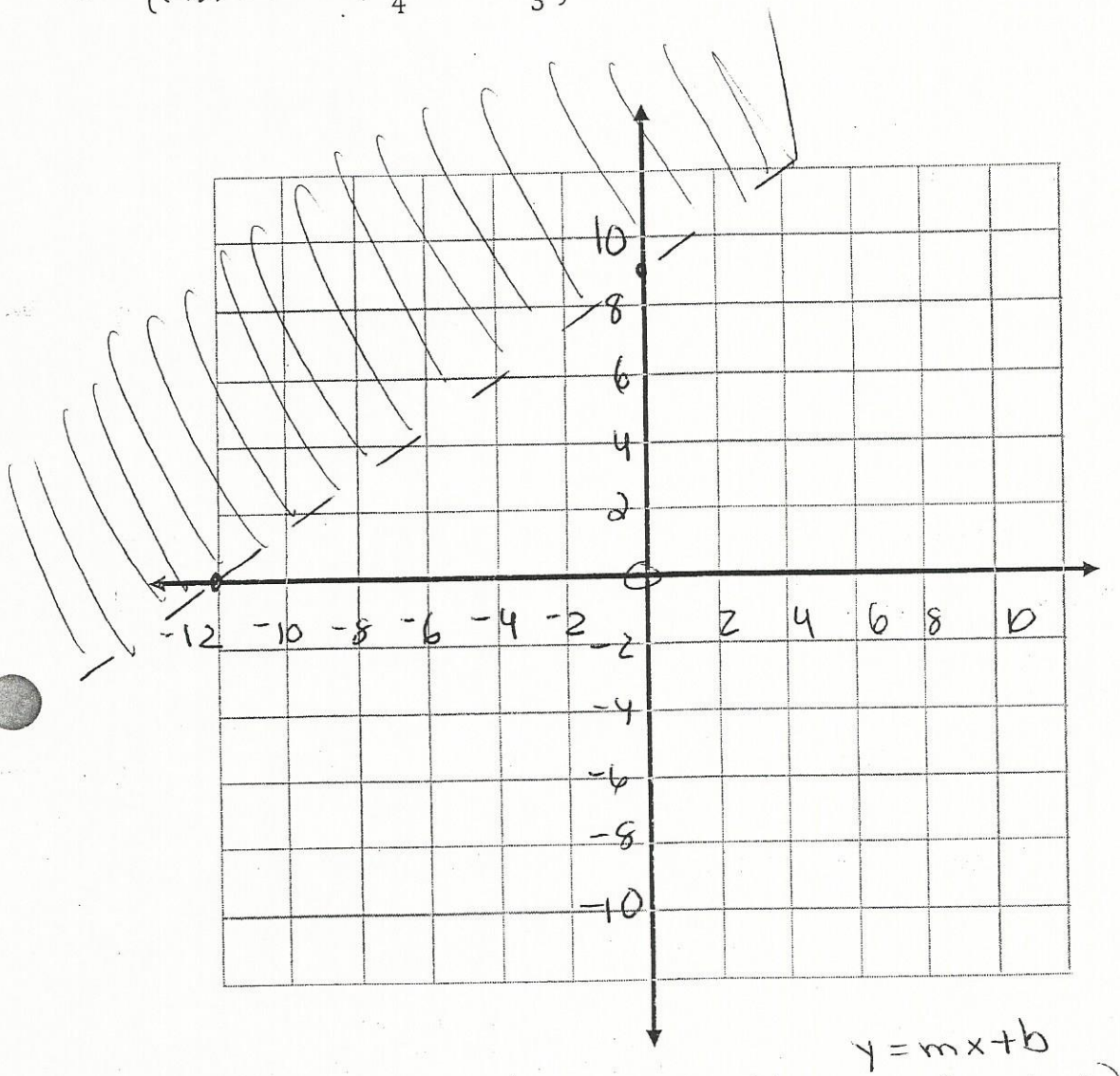
$y = -\frac{2}{3}x + 4$

$\left(\frac{3}{2}\right)\frac{2}{3}x = 4\left(\frac{3}{2}\right)$

$x = \frac{12}{2} = 6$

14. Graph the following relation in the Cartesian plane below:

$$S = \left\{(x,y) \in \mathbb{R} X \mathbb{R} \mid \frac{-x}{4} - 3 > \frac{-y}{3}\right\}$$



$y = mx + b$

$\left(\frac{3}{1}\right)\frac{1}{3}y > \left(\frac{1}{4}x + 3\right)\frac{3}{1}$

$y > \frac{3}{4}x + 9$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

y-int: $(0, 9)$

x-int: $(-12, 0)$

x-int

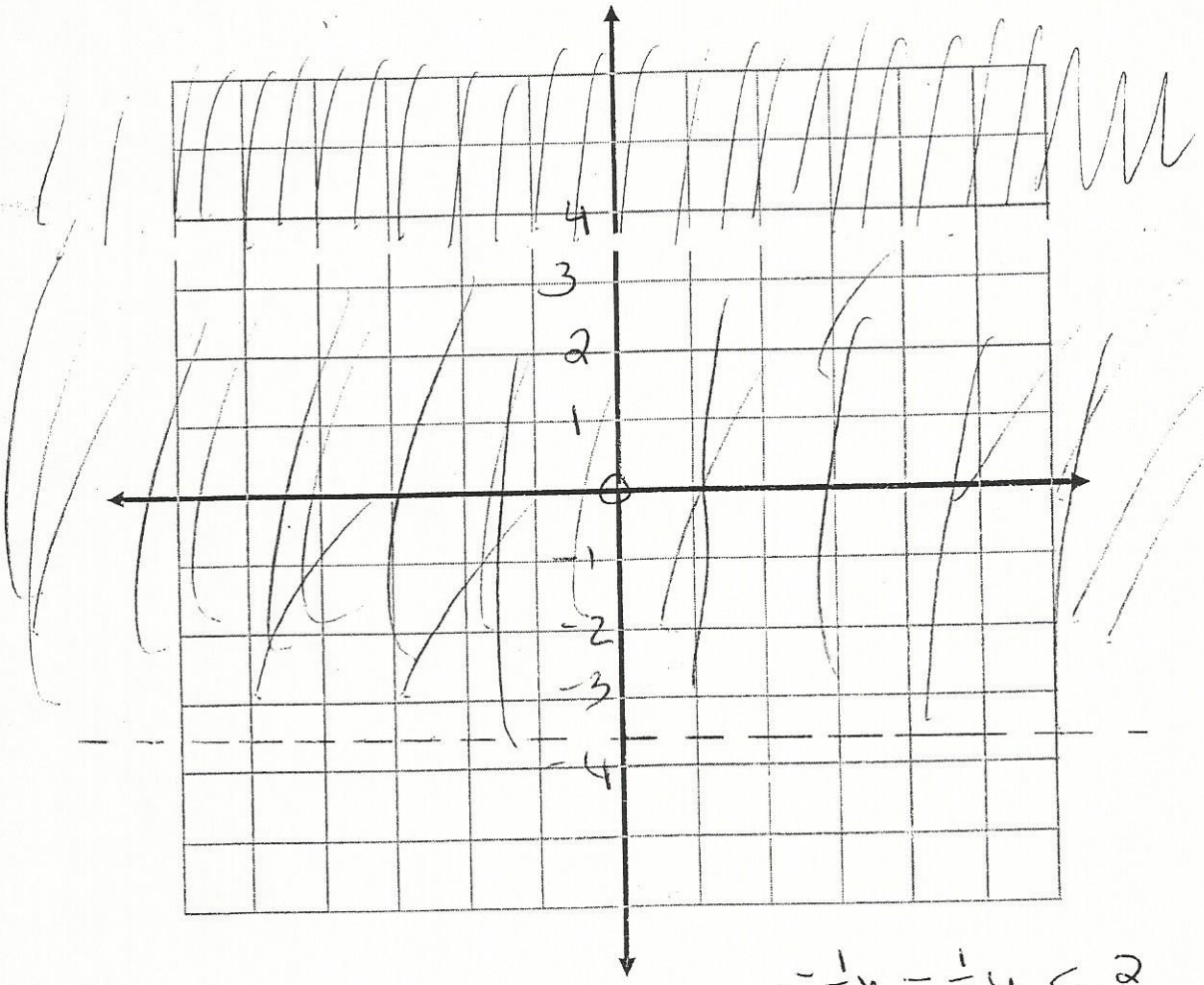
$y = \frac{3}{4}x + 9$

$\left(-\frac{4}{3}\right) - \frac{3}{4}x = 9\left(-\frac{4}{3}\right)$

$x = -\frac{36}{3} = -12$

15. Graph the following relation in the Cartesian plane below:

$S = \left\{(x,y) \in \mathbb{R}\, X\, \mathbb{R} \mid \frac{-y}{3} < 2 + \frac{y}{5}\right\}$



$-\frac{1}{3}y - \frac{1}{5}y < 2$

$\frac{-3-5}{15}y < 2$

$\left(-\frac{15}{8}\right) - \frac{8}{15}y < 2\left(-\frac{15}{8}\right)$

$y > -\frac{30}{8}$

$y > -3.75$

Domain: $\mathbb{R}$

Range: $]-3.75, \infty$

Functions

Independent and Dependent Variables

State whether the following statements are true or false:

1. Listening to music increases productivity in the workplace.
The independent variable is productivity. T (F)

2. A researcher wants to study the effects of sleep deprivation on physical coordination.
The dependent variable is physical coordination. (T) F

3. A study indicates that antioxidants found in blueberries may slow down the process of aging.
The independent variable is the blueberries. (T) F

4. Students who spend two hours a night studying have higher test scores.
The dependent variable is the time spent studying. T (F)

5. A botanist measures the growth rate of plants under full sunlight for 8 hours a day.
The independent variable is growth rate. T (F)

6. A student gets a new job that pays $11.50 per hour.
The dependent variable is the rate per hour. (T) F

7. Time spent studying causes a change in test scores. T (F)
The independent variable is the test scores.

8. A student is observing how much water flows through a faucet at different openings.
The independent variable is the flow of water. T (F)

Functions

Intervals

1. A function is described by the following rule: $f(x) = \frac{-2x}{3} + 5$

a) Determine over which interval this function is positive.

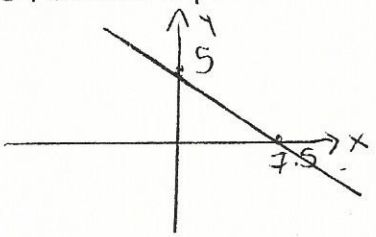
$x = -\infty, -7.5]$

$y = -\frac{2}{3}x + 5$

$\left(\frac{3}{2}\right)\frac{2}{3}x = 5\left(\frac{3}{2}\right)$

$x = 15/2$

$x = 7.5$



b) Determine the rate of change.

$m = -\frac{2}{3}$

2. A function is described by the following rule: $f(x) = \frac{3x}{2} - 4$

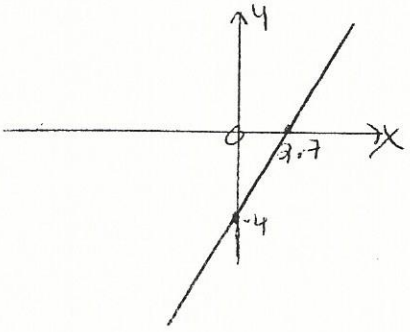
a) Determine over which interval this function is negative.

$x = -\infty, \frac{8}{3}]$

$y = \frac{3}{2}x - 4$

$\left(\frac{-2}{3}\right) -\frac{3}{2}x = -4\left(-\frac{2}{3}\right)$

$x = \frac{8}{3}$ or 2.7



b) Determine the rate of change.

$m = \frac{3}{2}$

3. A function is described by the following rule: $f(x) = -x^2 + 9$

a) Determine over which interval this function is positive.

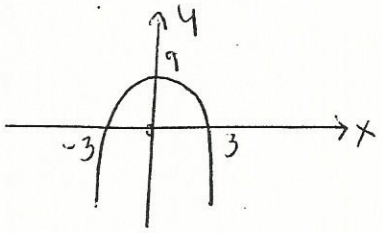
$X = [-3, 3]$

Zeros

$9 - x^2 = 0$

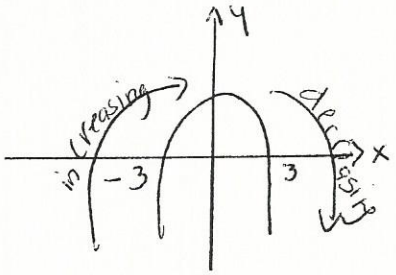
$(3-x)(3+x) = 0$

$x_1 = 3 \quad x_2 = -3$



b) Determine over which interval this function is decreasing.

$[0, \infty$



4. A function is described by the following rule: $f(x) = x^2 - 9$

a) Determine over which interval this function is negative.

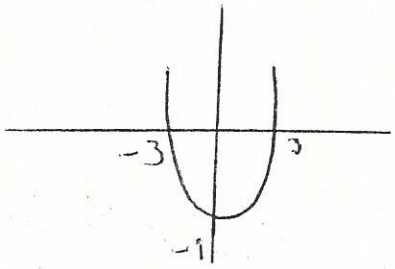
$[-3, 3]$

Zeros

$x^2 - 9 = 0$

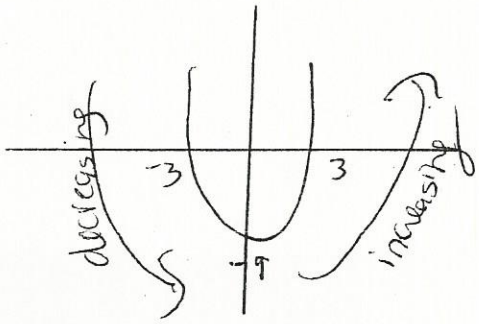
$(x-3)(x+3) = 0$

$x_1 = 3 \quad x_2 = -3$



b) Determine over which interval this function is increasing.

$[0, \infty$



Functions

Word Problems

1. Two competitors were having a fireworks display contest. Calculate the difference between the maximum heights reached by the two sets of fireworks.

Height is calculated in meters.

$h_{(1)} = -x^2 + 12x$

$h_{(2)} = -x^2 + 6x + 11$

height: $\frac{-\Delta}{4a} = \frac{-144}{4(-1)} = \frac{-144}{-4} = 36$

① $-x^2 + 12x + 0$
(a) (b) (c)

$b^2 - 4ac$
$12^2 - 4(-1)(0)$
144

$h(1) = 36m$

② $-x^2 + 6x + 11$
(a) (b) (c)

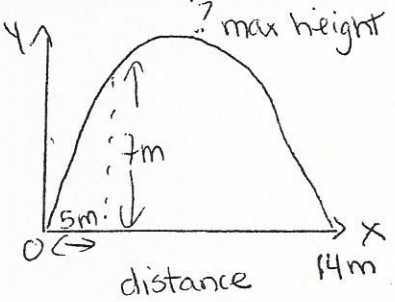
height: $\frac{-\Delta}{4a} = \frac{-80}{4(-1)} = \frac{-80}{-4} = 20$

$b^2 - 4ac$
$6^2 - 4(-1)(11)$
$36 - 4(-11)$
$36 + 44 = 80$

$h(2) = 20m$

Difference: $36 - 20 = 16m$

2. A baseball is thrown over a 7 meter high fence, where the fence is located 5 meters from the origin of the throw. The ball lands 14 meters away. If the path of the ball describes a parabola, what is the maximum height reached by the ball?



Two zeros
(0,0) (14,0) (x,y) (5,7)

$z_1 = 0$

$z_2 = 14$

① Solve for 'a'.

$f(x) = a(x-z_1)(x-z_2)$

$7 = a(5-0)(5-14)$

$7 = a(5)(-9)$

$7 = a(-45)$

$\frac{45a}{45} = \frac{7}{45}$ $a = -0.156$

② Find Max height

$y = a(x-z_1)(x-z_2)$

$y = -0.156(x^2 - 14x)$

$y = -0.156x^2 + 2.184x$

$a = -0.156$ $b = 2.184$ $c = 0$

$b^2 - 4ac$

$= (2.184)^2 - 4(-0.156)(0)$

$= 4.769856$

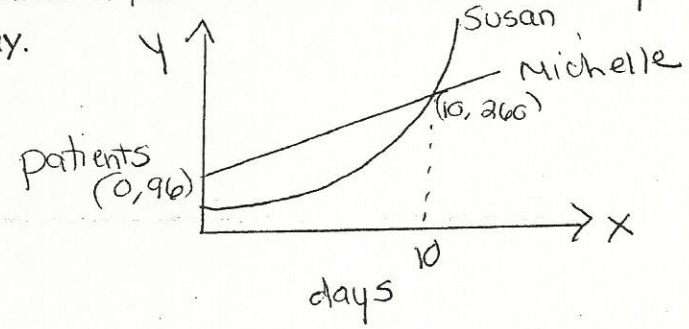
$\frac{-\Delta}{4a} = \frac{-4.77}{4(-0.156)} =$

$= \frac{-4.77}{-0.624}$

$= 7.64$

Max. height reached by the ball is 7.64m.

3. Two dermatology clinics just opened and Michelle and Susan are trying to attract new patients. Susan's clinic calculates the number of patients through the equation $f(x) = 2x^2 + 60$ where x represents the number of days. On opening day, Michelle's clinic has 96 patients. On the 10th day, the two clinics have the same number of patients. Calculate the number of patients in Michelle's clinic on the 15th day.



Susan $x = 10$ days

$y = 2x^2 + 60$

$y = 2(10)^2 + 60$

$y = 2(100) + 60$

$y = 200 + 60$

$y = 260$ patients

Michelle $x = 15$ days

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (0, 96) (10, 260)
x_1 y_1 x_2 y_2

$m = \frac{260 - 96}{10 - 0} = \frac{164}{10} = 16.4$

$y = mx + b$

$y = 16.4x + 96$

$y = 16.4(15) + 96$

$y = 246 + 96$

$y = 342$ patients.

On the 15th day, Michelle has 342 patients.