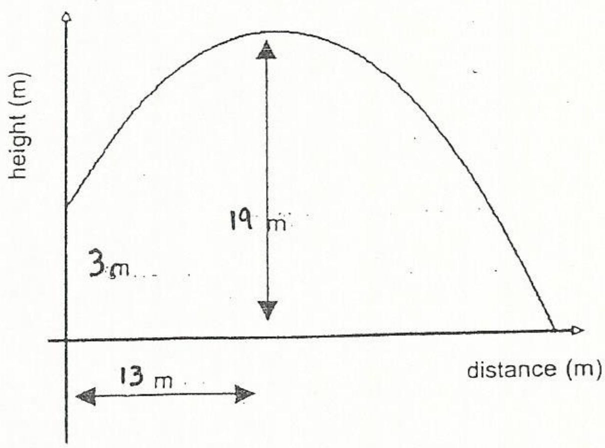
Type A Answers

1 A participant in a javelin-throwing competition throws his javelin from a height of **3** metres. The javelin's trajectory describes a parabola, and the javelin reaches a maximum height of 19 metres. Upon reaching this height, the javelin has travelled a distance of 13 metres from the starting point. Determine the length of the throw. Round off your answer to the nearest hundredth. Clearly show all your work.



Vertex:

$(13, 19)$
$h \quad k$

$(0, 3)$
$x \quad y$
$\uparrow$
$f(x)$

$f(x) = a(x-h)^2 + k$ } formula for parabola

$3 = a(0-13)^2 + 19$

$3 = a(-13)^2 + 19$

$3 = 169a + 19$

$-169a = 19 - 3$

$\frac{-169a}{-169} = \frac{16}{-169}$

$a = -0.095$

formula: $f(x) = -0.095(x-13)^2 + 19$

Need $\cancel{x}$-intercept $(y=0)$

$0 = -0.095(x-13)(x-13) + 19$

$0 = -0.095(x^2 - 26x + 169) + 19$

$0 = -0.095x^2 + 2.47x - 16.06 \quad + 19$

$0 = -0.095x^2 + 2.47x + 2.94$

$a = -0.095$

$b = 2.47$

$c = 2.94$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.47 \pm \sqrt{(2.47)^2 - 4(-0.095)(2.94)}}{2(-0.095)}$$

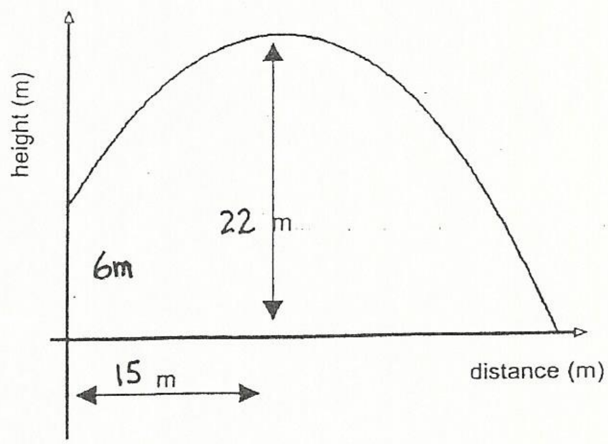
$$= \frac{-2.47 \pm \sqrt{6.1009 + 1.1172}}{-0.19}$$

$$= \frac{-2.47 \pm 2.69}{-0.19}$$

(+) $= \cancel{-1.16}$

(−) $= \boxed{27.16 \text{ m}}$

A- 2 A participant in a javelin-throwing competition throws his javelin from a height of 6 metres. The javelin's trajectory describes a parabola, and the javelin reaches a maximum height of 22 metres. Upon reaching this height, the javelin has travelled a distance of 15 metres from the starting point. Determine the length of the throw. Round off your answer to the nearest hundredth. Clearly show all your work.



Vertex:

$\underbrace{(15, 22)}_{(h, k)}$ $(0, 6)$

x y

↑

f(x)

$f(x) = a(x-h)^2 + k$ } formula for parabola

$6 = a(0-15)^2 + 22$

$6 = a(-15)^2 + 22$

$6 = 225a + 22$

$-225a = 22 - 6$

$\dfrac{-225a}{-225} = \dfrac{16}{-225}$

$a = -0.0711$

formula: $f(x) = -0.071(x-15)^2 + 22$

Need x-intercept (y = 0)

$0 = -0.071(x-15)^2 + 22$

$0 = -0.071(x-15)(x-15) + 22$

$0 = -0.071(x^2 - 30x + 225) + 22$

$0 = -0.071x^2 + 2.13x - 15.975 + 22$

$0 = -0.071x^2 + 2.13x + 6.025$ → cont.

$a = -0.071$

$b = 2.13$

$c = 6.025$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.13 \pm \sqrt{(2.13)^2 - 4(-0.071)(6.025)}}{2(-0.071)}$$

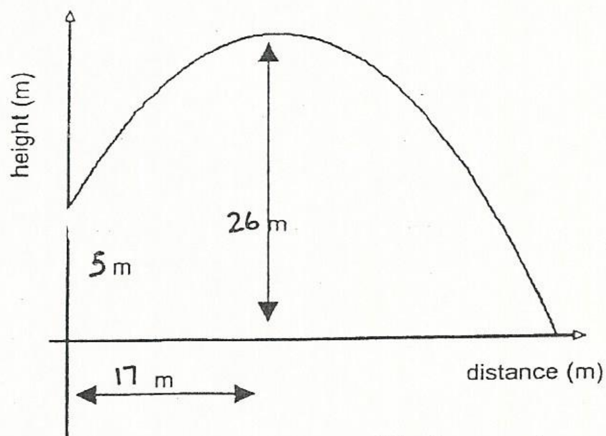
$$= \frac{-2.13 \pm \sqrt{4.5369 + 1.7111}}{-0.142}$$

$$= \frac{-2.13 \pm 2.5}{-0.142}$$

$+$ $= -2.6$ can't use

$-$ $= 32.60\text{m}$

A- (3) A participant in a javelin-throwing competition throws his javelin from a height of 5 metres. The javelin's trajectory describes a parabola, and the javelin reaches a maximum height of 26 metres. Upon reaching this height, the javelin has travelled a distance of 17 metres from the starting point. Determine the length of the throw. Round off your answer to the nearest hundredth. Clearly show all your work.



Vertex: $(17, 26)$ h K

$(0, 5)$ x y ↑ $f(x)$

$f(x) = a(x-h)^2 + K$ } formula for parabola

$5 = a(0-17)^2 + 26$

$5 = a(-17)^2 + 26$

$5 = 289a + 26$

$-289a = 26 - 5$

$\frac{-289a}{-289} = \frac{21}{-289}$

$a = -0.0727$

formula: $f(x) = -0.0727(x-17)^2 + 26$ Need x-intercept $(y=0)$

$0 = -0.073(x-17)(x-17) + 26$

$0 = -0.073(x^2 - 34x + 289) + 26$

$0 = -0.073x^2 + 2.482x - 21.097 + 26$

$0 = -0.073x^2 + 2.482x + 4.903$

→ cont'

A-3 cont'

$a = -0.073$

$b = 2.482$

$c = 4.903$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.482 \pm \sqrt{(2.482)^2 - 4(-0.073)(4.903)}}{2(-0.073)}$$

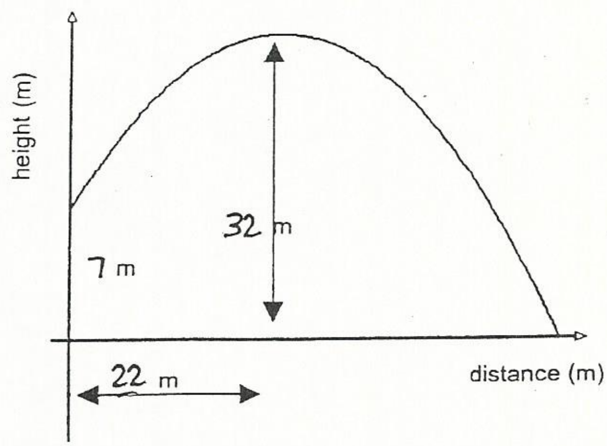
$$= \frac{-2.482 \pm \sqrt{6.1603 + 1.4317}}{-0.146}$$

$$= \frac{-2.482 \pm 2.7554}{-0.146}$$

$+ \quad = \cancel{-1.87}$

$- \quad = 35.9\,m$

A- 4 A participant in a javelin-throwing competition throws his javelin from a height of 7 metres. The javelin's trajectory describes a parabola, and the javelin reaches a maximum height of 32 metres. Upon reaching this height, the javelin has travelled a distance of 22 metres from the starting point. Determine the length of the throw. Round off your answer to the nearest hundredth. Clearly show all your work.



Vertex :

$(22, 32)$
$h \quad k$

$(0, 7)$
$x \quad y$
$\uparrow$
$f(x)$

$f(x) = a(x-h)^2 + k$ } formula for parabola

$7 = a(0-22)^2 + 32$

$7 = a(-22)^2 + 32$

$7 = 484a + 32$

$-484a = 32 - 7$

$\frac{-484a}{-484} = \frac{25}{-484}$

$a = -0.052$

formula : $f(x) = -0.052(x-22)^2 + 32$

Need x-intercept ($y=0$)

$0 = -0.052(x-22)(x-22) + 32$

$0 = -0.052(x^2 - 44x + 484) + 32$

$0 = -0.052x^2 + 2.288x - 25.168 + 32$

$0 = -0.052x^2 + 2.288x + 6.832$

A-4 cont'

$a = -0.052$

$b = 2.288$

$c = 6.832$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

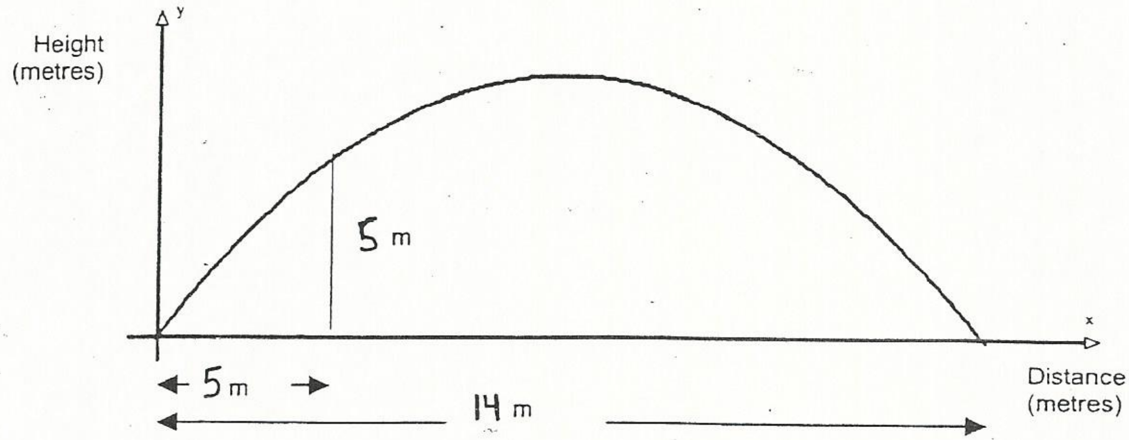
$$= \frac{-2.288 \pm \sqrt{(2.288)^2 - 4(-0.052)(6.832)}}{2(-0.052)}$$

$$= \frac{-2.288 \pm \sqrt{5.235 + 1.421}}{-0.104}$$

$$= \frac{-2.288 \pm 2.58}{-0.104}$$

$(-) = 46.8\,m$

B-1 A baseball is thrown over a 5m high fence. The fence is 6m from the origin of the throw. The ball lands 14m away. If the path of the ball describes a parabola, what is the maximum height reached by the ball?



- the two zeros are $x_1 = 0$ and $x_2 = 14$

$f(x) = a(x)(x-14)$

$f(x) = a(x^2 - 14x)$ $(x,y) = (5,5)$

$5 = a(5^2 - 14 \cdot 5)$

$5 = a(25 - 70)$

$\frac{5}{-45} = \frac{a(-45)}{-45}$

$-\frac{5}{45} = a$

$a = \frac{-1}{9}$

$f(x) = \frac{-1}{9}(x^2 - 14x)$

$f(x) = -\frac{1}{9}x^2 + \frac{14}{9}x$

At maximum point $x = 7$

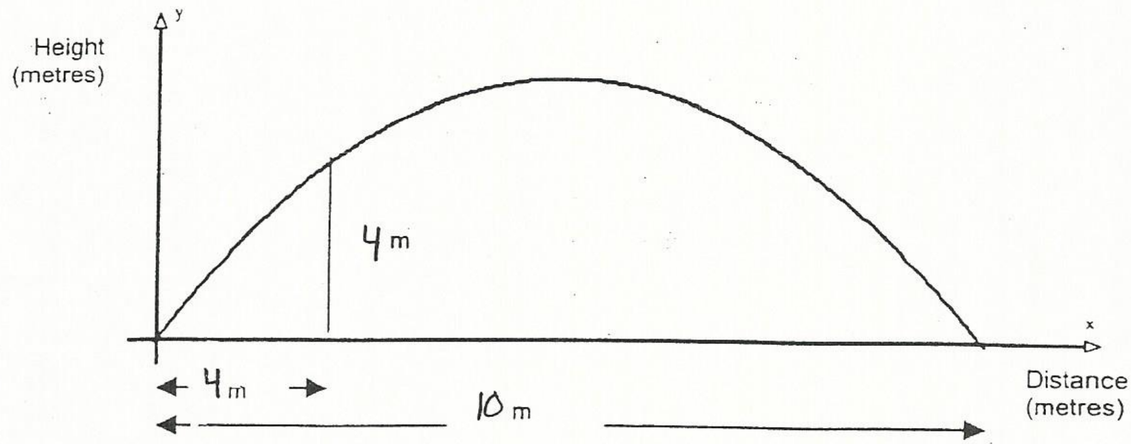
$f(x) = \frac{-1}{9}(7)^2 + \frac{14}{9}(7)$

$= -5\frac{4}{9} + 10\frac{8}{9}$

$= 5\frac{4}{9}$ or 5.44

The maximum height is $5\frac{4}{9}$m or 5.44m.

B-2 A golfer hits a ball over a 4 metre high wall. The wall is 4m from the origin of the shot. The ball lands 10m away. If the path of the ball describes a parabola, what is the maximum height reached by the ball?



- the two zeros are $x_1 = 0$ and $x_2 = 10$

$f(x) = a(x)(x-10)$

$f(x) = a(x^2 - 10x)$ $\qquad (x, y) = (4, 4)$

$4 = a(4^2 - 10 \cdot 4)$

$4 = a(16 - 40)$

$\frac{4}{-24} = \frac{a(-24)}{-24}$

$\frac{-4}{24} = a$

$a = \frac{-1}{6}$

$f(x) = -\frac{1}{6}(x^2 - 10x)$

$f(x) = -\frac{1}{6}x^2 + \frac{5}{3}x$

At maximum point $x = 5$

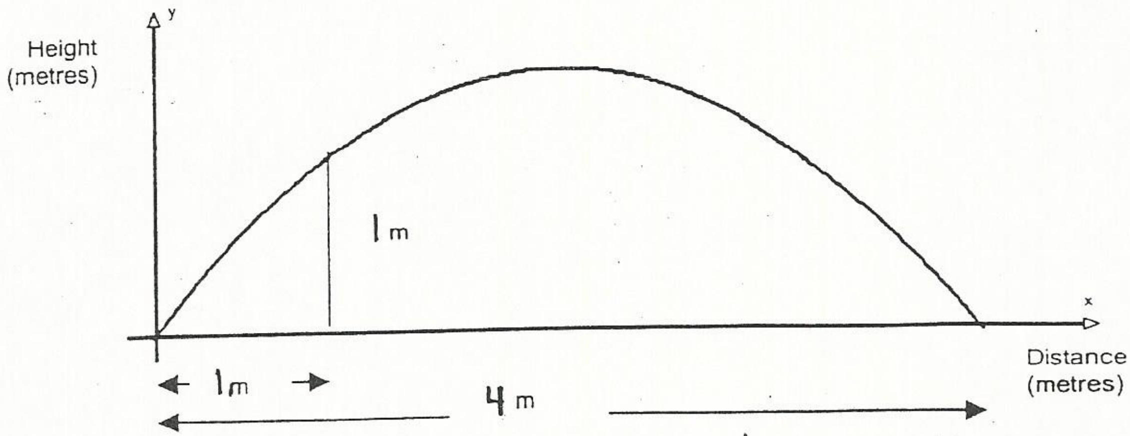
$f(x) = -\frac{1}{6}(5)^2 + \frac{5}{3}(5)$

$= -4\frac{1}{6} + 8\frac{1}{3}$

$= 4\frac{1}{6}$

The maximum height is $4\frac{1}{6}$m or 4.17m.

B-3 A dolphin is executing jumps in the main pool of an aqua park. To do this, the dolphin must jump over a rod located 1 metre from the water outlet. This rod is 1 metre high. The distance between the point where the dolphin exits the water and the point where she reenters it is 4 metres. If the dolphin's jump describes a parabola, what is the maximum height reached by the dolphin?



- the two zeros are $x_1 = 0$ and $x_2 = 4$.

$$f(x) = a(x)(x-4)$$
$$= a(x^2 - 4x) \qquad (1,1) = x, y$$

$$1 = a(1^2 - 4 \cdot 1)$$
$$1 = a(1-4)$$
$$\frac{1}{-3} = \frac{a(-3)}{-3}$$
$$-\frac{1}{3} = a$$

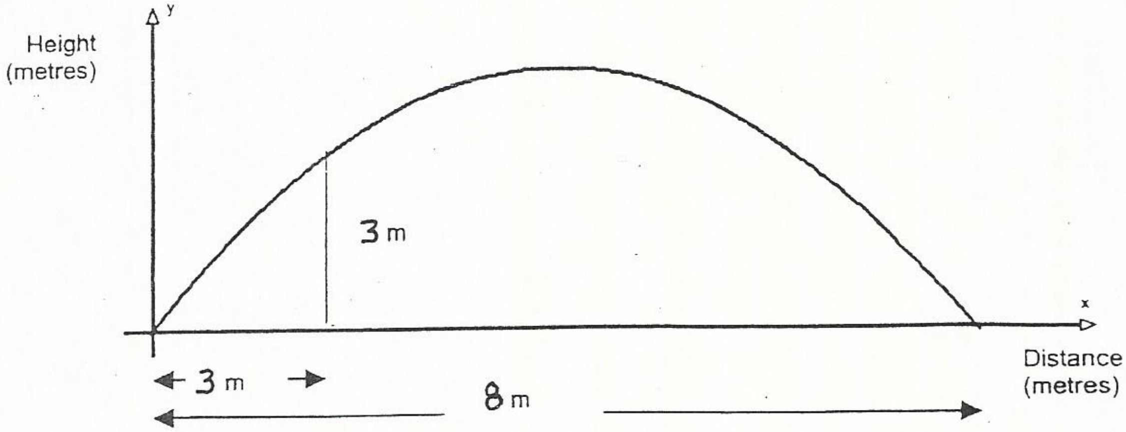
$$f(x) = -\frac{1}{3}(x^2 - 4x)$$
$$f(x) = -\frac{1}{3}x^2 + \frac{4}{3}x$$

At maximum point $x = 2$

$$f(x) = -\frac{1}{3}(2)^2 + \frac{4}{3}(2)$$
$$= -\frac{4}{3} + \frac{8}{3}$$
$$= \frac{4}{3} \text{ or } 1\tfrac{1}{3} \text{ or } 1.3\overline{3}$$

The maximum height is $\frac{4}{3}$m or $1\frac{1}{3}$m or 1.33m.

B-4 A killer whale is executing jumps in the main pool of an aqua park. To do this, the whale must jump over a rod located 3 metres from the water outlet. This rod is 3 metres high. The distance between the point where the whale exits the water and the point where she reenters it is 8 metres. If the whale's jump describes a parabola, what is the maximum height reached by the whale?



- the two zeros are $x_1 = 0$ and $x_2 = 8$

$f(x) = a(x)(x-8)$

$f(x) = a(x^2 - 8x)$ $\quad (x,y) = (3,3)$

$3 = a(3^2 - 8 \cdot 3)$

$3 = a(9 - 24)$

$\frac{3}{-15} = \frac{a(-15)}{-15}$

$\frac{-3}{15} = a$

$a = -\frac{1}{5}$

$f(x) = -\frac{1}{5}(x^2 - 8x)$

$f(x) = -\frac{1}{5}x^2 + \frac{8}{5}x$

At maximum point $x = 4$

$f(x) = -\frac{1}{5}(4)^2 + \frac{8}{5}(4)$

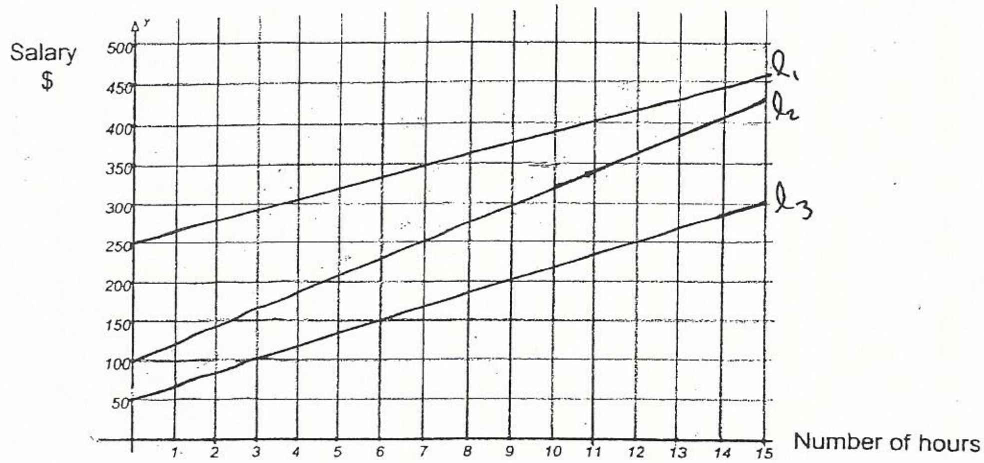
$= -3\frac{1}{5} + 6\frac{2}{5}$

$= 3\frac{1}{5}$ or 3.2

The maximum height is $3\frac{1}{5}$ m or 3.2m.

C-1 Three computer technicians received a bonus that was added to their salary. In the graph below,

- line l_1 represents Bryan's salary (including the bonus)
- line l_2 represents Trevor's salary (including the bonus)
- line l_3 represents Paul's salary (including the bonus)



If all three technicians worked 40 hours this week, which one received the highest weekly salary?

Clearly show all your work.

l_1 $(0, 250), (7, 350)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350 - 250}{7 - 0} = \frac{100}{7}$

$\boxed{y = \frac{100}{7}x + 250}$

if $x = 40$:

$y = \frac{100}{7}(40) + 250$

$= \$821.43$

l_2 $(0, 100), (7, 250)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{250 - 100}{7 - 0} = \frac{150}{7}$

$y = \frac{150}{7}x + 100$

if $x = 40$:

$y = \frac{150}{7}(40) + 100$

$= \$957.14$

l_3 $(0, 50), (3, 100)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{100 - 50}{3 - 0} = \frac{50}{3}$

$\boxed{y = \frac{50}{3}x + 50}$ if $x = 40$;

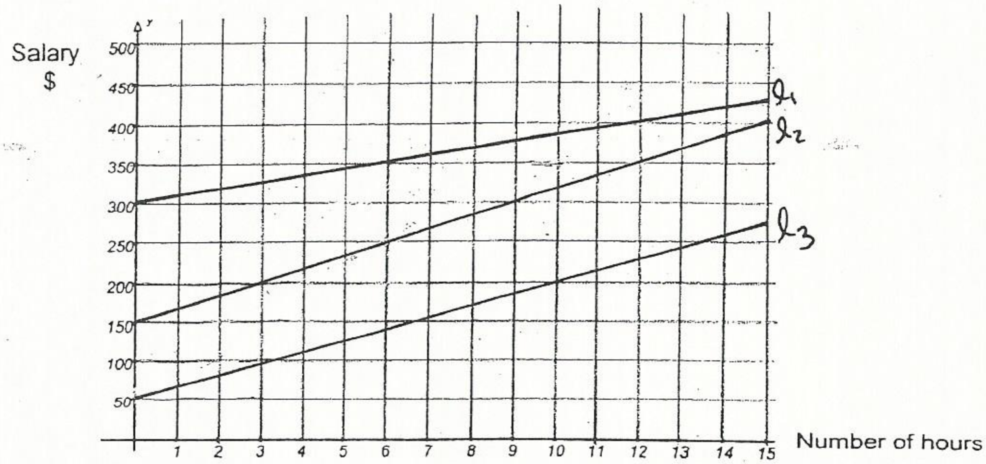
$= \frac{50}{3}(40) + 50$

$= \$716.67$

$\rightarrow$ Trevor received the highest weekly salary (\$957.14).

C- 2 Three mechanics received a bonus that was added to their salary. In the graph below,

- line l_1 represents Allans's salary (including the bonus)
- line l_2 represents Melvin's salary (including the bonus)
- line l_3 represents Matt's salary (including the bonus)



If all three mechanics worked 40 hours this week, which one received the highest weekly salary?

Clearly show all your work.

l_1 (0,300),(6,350)

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{350-300}{6-0}$

$= \frac{50}{6}$

$\boxed{y = \frac{50}{6}x + 300}$

if $x = 40$

$y = \frac{50}{6}(40) + 300$

$= \$633.33$

l_2 (0,150),(3,200)

$m = \frac{200-150}{3-0} = \frac{50}{3}$

$\boxed{y = \frac{50}{3}x + 150}$

if $x = 40$:

$y = \frac{50}{3}(40) + 150$

$= \$816.67$

l_3 (0,50),(10,200)

$m = \frac{200-50}{10-0} = \frac{150}{10} = 15$

$\boxed{y = 15x + 50}$

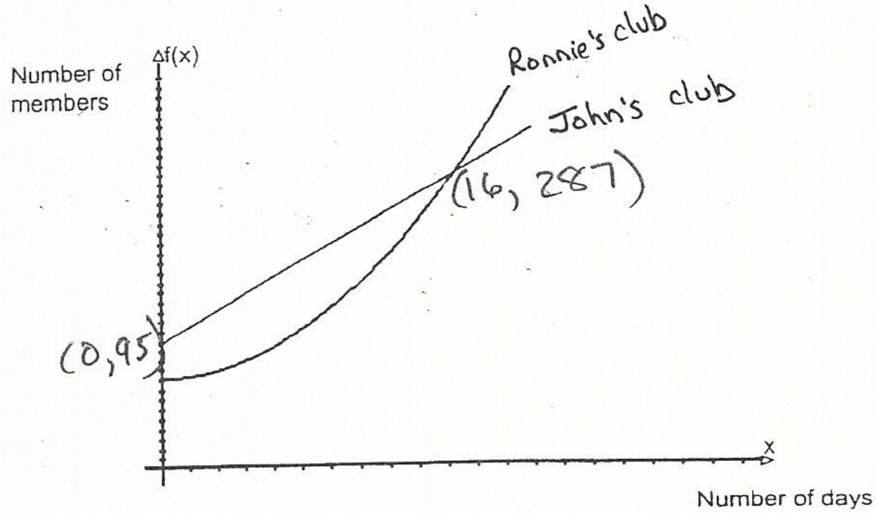
if $x = 40$:

$y = 15(40) + 50$

$= \$650.00$

Melvin earned the highest salary (\$816.67)

D- 1 **Ronnie's fitness club and John's fitness club are competing to attract new members. Ronnie's club calculates the number of members using the rule $f(x) = 0.25x^2 + 223$, where x represents the number of days. On opening day, John's club has 95 members. On the 16^{th} day the two clubs have the same number of members. The following graphs show the change in the number of members over a one-month period.**



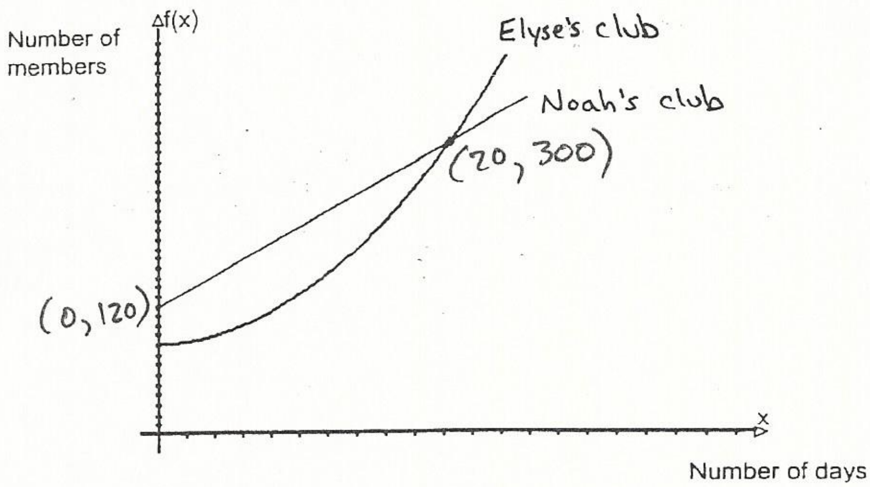
Calculate the number of members in John's club on the 27^{th} day. Clearly show all your work.

$$y = 0.25(16)^2 + 223$$
$$= 287$$

$$m = \frac{287 - 95}{16 - 0} = 12$$

$$y = 12x + 95$$
$$= 12(27) + 95 = 419$$

D-2 Elyse's fitness club and Noah's fitness club are competing to attract new members. Elyse's club calculates the number of members using the rule $f(x) = 0.4x^2 + 140$, where x represents the number of days. On opening day, Noah's club has 120 members. On the 20th day the two clubs have the same number of members. The following graphs show the change in the number of members over a one-month period.



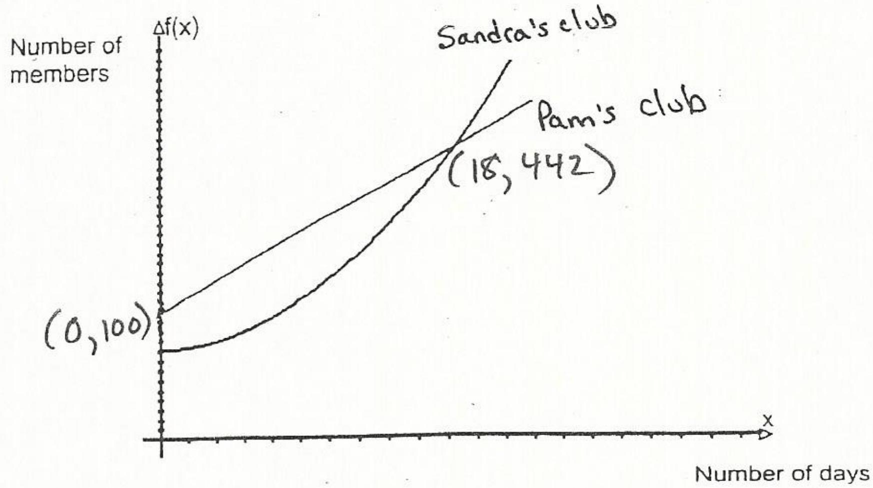
Calculate the number of members in Noah's club on the 24th day. Clearly show all your work.

$$y = 0.4(20)^2 + 140$$
$$= 300$$

$$m = \frac{300 - 120}{20 - 0} = 9$$

$$y = 9x + 120$$
$$= 9(24) + 120 = 336$$

D-3 **Sandra's fitness club and Pam's fitness club are competing to attract new members. Sandra's club calculates the number of members using the rule $f(x) = 0.75x^2 + 199$, where x represents the number of days. On opening day, Pam's club has 100 members. On the 18th day the two clubs have the same number of members. The following graphs show the change in the number of members over a one-month period.**



Calculate the number of members in Pam's club on the 30th day. Clearly show all your work.

$y = 0.75(18)^2 + 199$

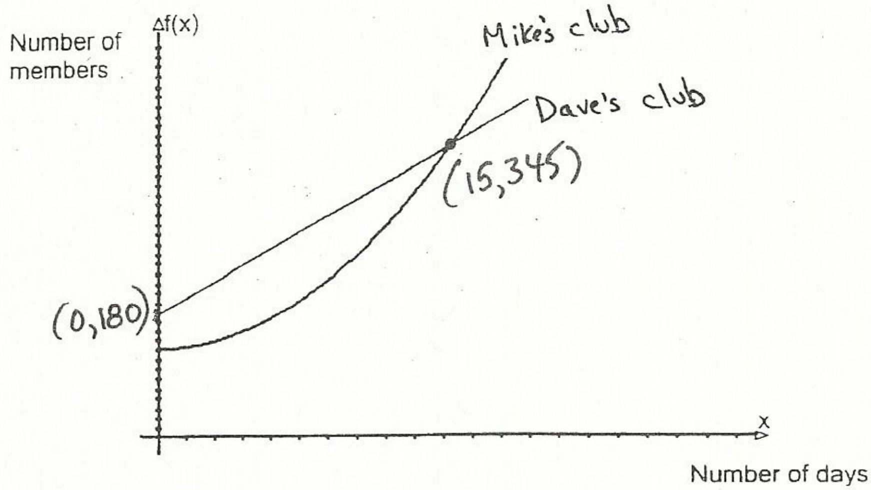
$= 442$

$m = \dfrac{442 - 100}{18 - 0} = 19$

$y = 19x + 100$

$= 19(30) + 100 = 670$

D-4 **Mike's fitness club and Dave's fitness club are competing to attract new members. Mike's club calculates the number of members using the rule $f(x) = 0.6x^2 + 210$, where x represents the number of days. On opening day, Dave's club has 180 members. On the 15th day the two clubs have the same number of members. The following graphs show the change in the number of members over a one-month period.**



Calculate the number of members in Dave's club on the 29th day. Clearly show all your work.

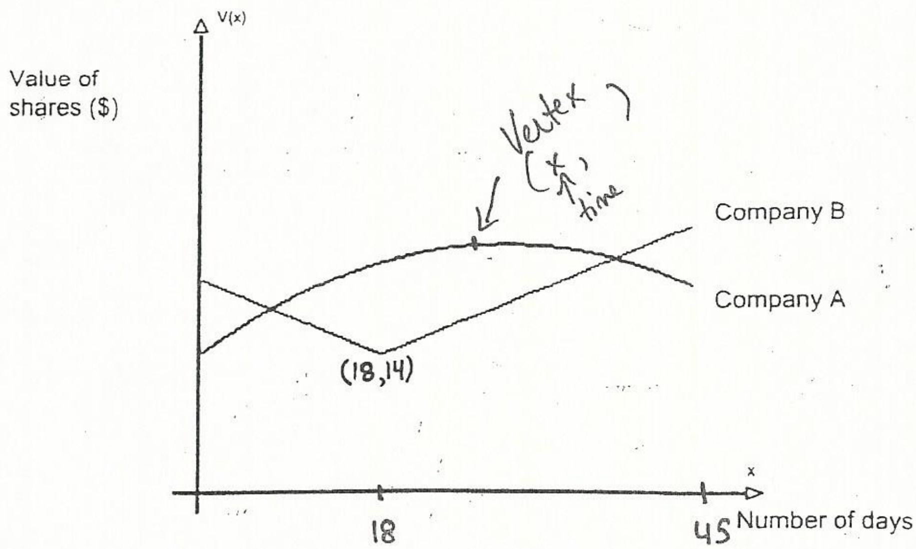
$f(x) = 0.6(15)^2 + 210 = 345$

$m = \dfrac{345 - 180}{15 - 0} = 11$

$y = 11x + 180$

$y = 11(29) + 180 = 499$

E- 1 Two companies issued their shares at the same time. The fluctuations in the value of the shares were studied over a 45–day period. First, it was established that the value of the shares of Company A varied according to the rule $V(x) = -0.01x^2 + 0.6x + 12$. Then, the fluctuations in the value of each company's shares were represented in the following graph.



Which company experienced the longest period of growth during this time?

Clearly show all your work.

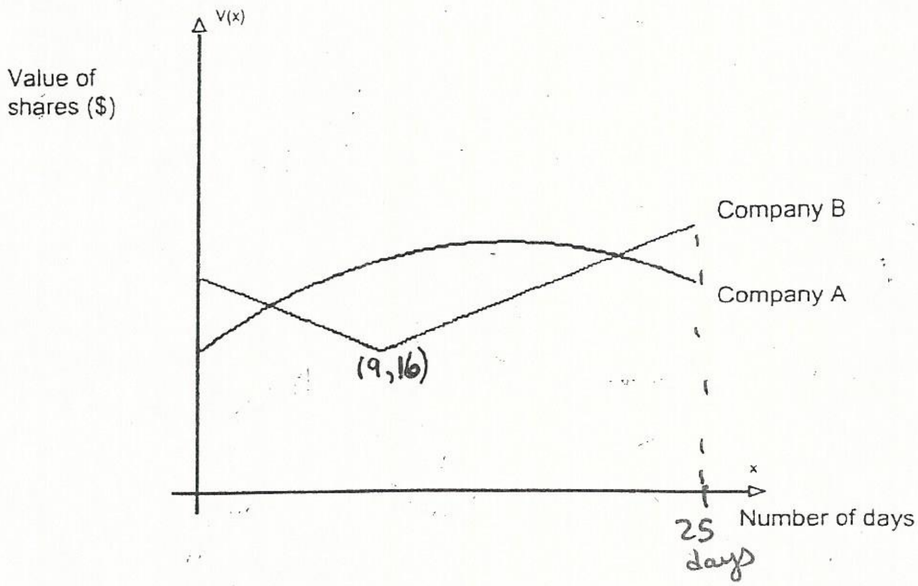
Company B growth (increasing) : 18 – 45 } 27 days

Company A growth : 0 → vertex (x value)

$$\frac{-b}{2a} = \frac{-.6}{2(-.01)} = 30 \text{ days}$$

Company A experienced the longest growth.

E- (2) Two companies issued their shares at the same time. The fluctuations in the value of the shares were studied over a 25-day period. First, it was established that the value of the shares of Company A varied according to the rule $V(x) = -0.04x^2 + 1.2x + 18$. Then, the fluctuations in the value of each company's shares were represented in the following graph.



Which company experienced the longest period of growth during this time?

Clearly show all your work.

Company B growth : 9 days → 25 days } 16 days

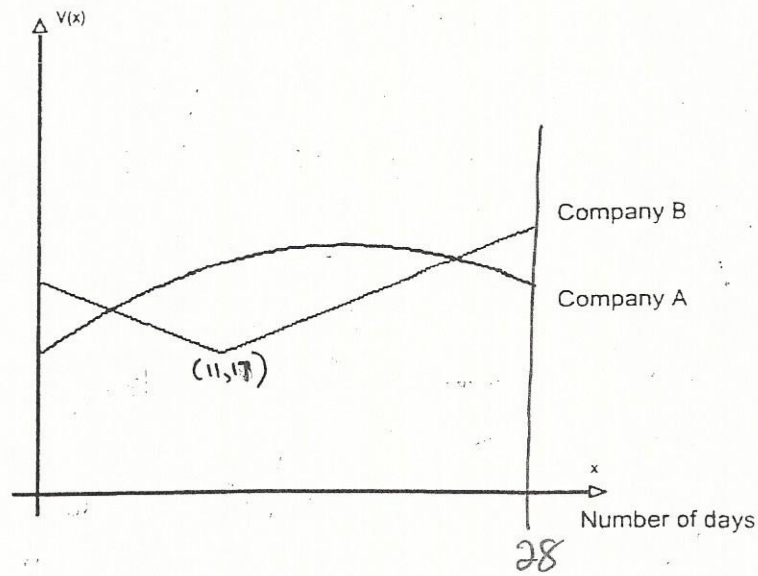
Company A growth : 0 → vertex (x-value)

$$-\frac{b}{2a} = \frac{-1.2}{2(-.04)} = \frac{-1.2}{-.08}$$

$= 15$ days

→ Company B experienced the longest growth.

E 3 Two companies issued their shares at the same time. The fluctuations in the value of the shares were studied over a **28**-day period. First, it was established that the value of the shares of Company A varied according to the rule $V(x) = -0.06x^2 + 2.16x + 12$. Then, the fluctuations in the value of each company's shares were represented in the following graph.



Which company experienced the longest period of growth during this time?

Clearly show all your work.

Company B growth : 11 days → 28 days = 17 days

Company A growth : 0 days → vertex (x value)

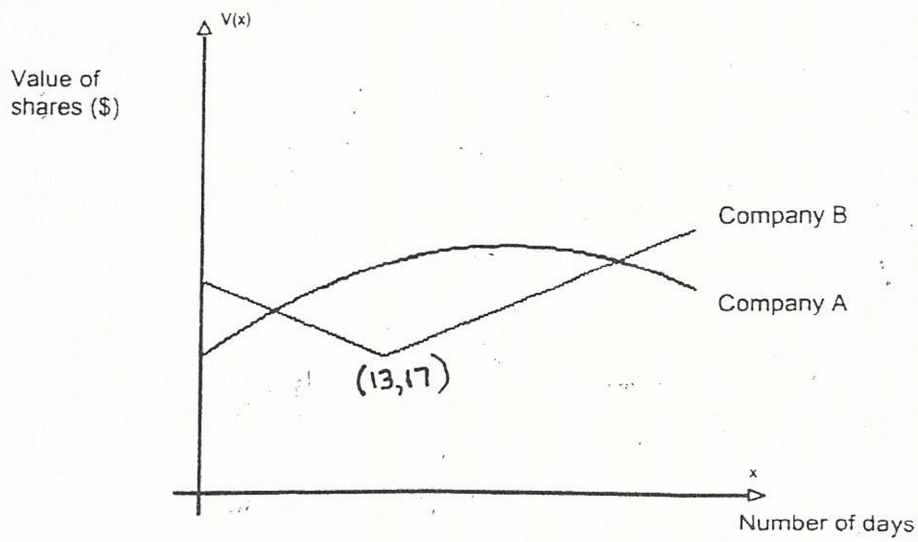
$$\frac{-b}{2a} = \frac{-2.16}{2(-.06)}$$

$$= \frac{-2.16}{-0.12}$$

$$= 18 \text{ days}$$

Company A experienced the longest growth.

E-4 Two companies issued their shares at the same time. The fluctuations in the value of the shares were studied over a 45-day period. First, it was established that the value of the shares of Company A varied according to the rule $V(x) = -0.03x^2 + 1.44x + 12$. Then, the fluctuations in the value of each company's shares were represented in the following graph.



Which company experienced the longest period of growth during this time?

Clearly show all your work.

Company B growth: 13 days → 45 days $= 32$ days

Company A growth: 0 → vertex (x-value)

$$\frac{-b}{2a} = \frac{-1.44}{2(-0.03)} = \frac{-1.44}{-0.06}$$

$= 24$ days

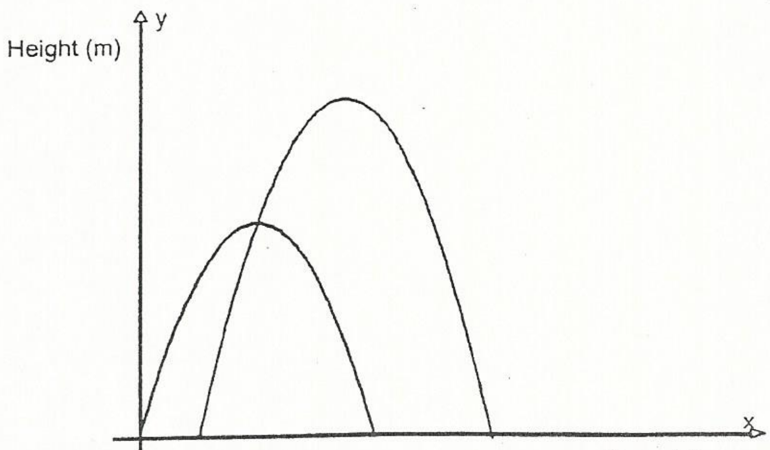
Company B experienced the longest growth.

F- 1 Two shells were launched during a fireworks display. The height in metres reached by the shells is defined by the following rules:

$h_{1(t)} = -t^2 + 6t$

$h_{2(t)} = -t^2 + 6t - 8$ where t represents the time elapsed, in seconds, since the launch.

The situation is described by the following graph:



Calculate the difference between the maximum heights reached by the two shells. Clearly show all your work.

h_1

$a = -1 \quad b = 6 \quad c = 0$

$\Delta = b^2 - 4ac$

$= 6^2 - 4(-1)(0)$

$= 36$

Vertex y value $= \frac{-\Delta}{4a}$

$= \frac{-36}{4(-1)}$

$= 9\text{ m}$

h_2

$a = -1 \quad b = 6 \quad c = -8$

$\Delta = b^2 - 4ac$

$= 36 - 4(-1)(-8)$

$= 36 - 32$

$= 4$

Vertex y value $= \frac{-\Delta}{4a}$

$= \frac{-4}{-4}$

$= 1\text{ m}$

$9\text{m} - 1\text{m} = 8\text{m}$

The difference is 8m.

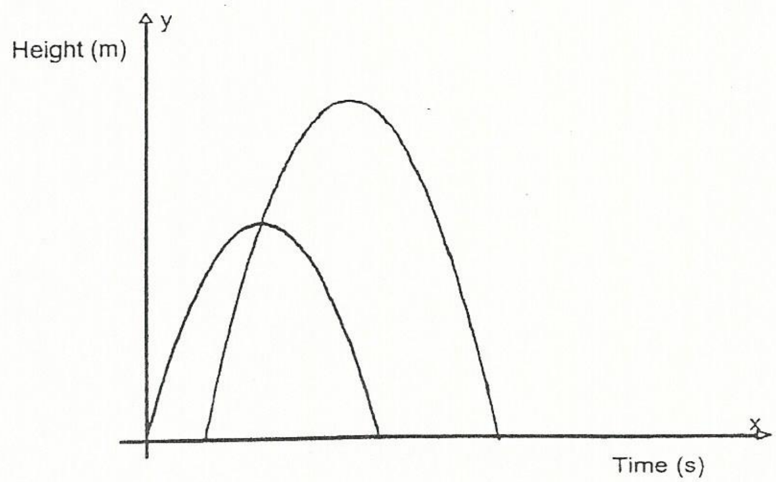
F- 2 Two shells were launched during a fireworks display. The height in metres reached by the shells is defined by the following rules:

$h_{1(t)} = -t^2 + 12t$

$h_{2(t)} = -t^2 + 18t - 14$

where t represents the time elapsed, in seconds, since the launch.

The situation is described by the following graph:



Calculate the difference between the maximum heights reached by the two shells. Clearly show all your work.

h_1

$a = -1 \quad b = 12 \quad c = 0$

$\Delta = b^2 - 4ac$

$= 12^2 - 4(-1)(0)$

$= 144$

Vertex y value $= \frac{-\Delta}{4a} = \frac{-144}{4(-1)}$

$= 36$ m

h_2

$a = -1 \quad b = 18 \quad c = -14$

$\Delta = b^2 - 4ac$

$= 18^2 - 4(-1)(-14)$

$= 324 - 56$

$= 268$

Vertex y value $= \frac{-\Delta}{4a}$

$= \frac{-268}{4(-1)}$

$= 67$ m

67m − 36m = 31m

The difference is 31m.

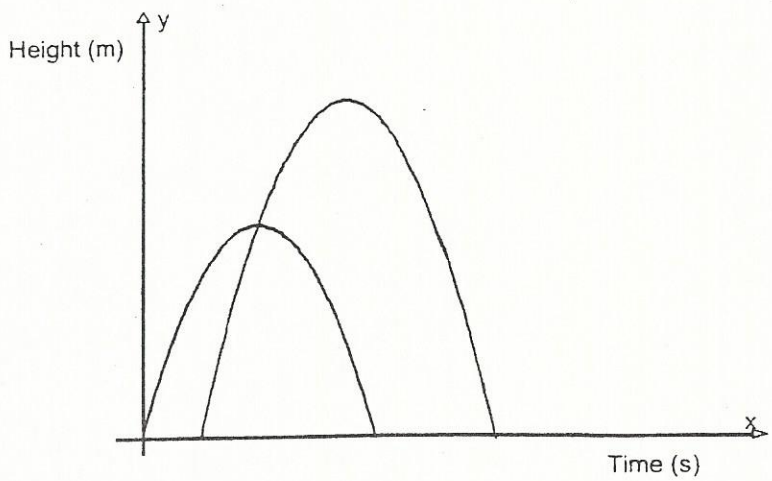
F-3 Two shells were launched during a fireworks display. The height in metres reached by the shells is defined by the following rules:

$h_{1(t)} = -t^2 + 14t$

$h_{2(t)} = -t^2 + 20t - 22$

where t represents the time elapsed, in seconds, since the launch.

The situation is described by the following graph:



Calculate the difference between the maximum heights reached by the two shells. Clearly show all your work.

h_1

$a = -1 \quad b = 14 \quad c = 0$

$\Delta = b^2 - 4ac$

$= 14^2 - 4(-1)(0)$

$= 196$

Vertex y value $= \frac{-\Delta}{4a} = \frac{-196}{4(-1)}$

$= 49m$

h_2

$a = -1 \quad b = 20 \quad c = -22$

$\Delta = b^2 - 4ac$

$= 20^2 - 4(-1)(-22)$

$= 400 - 88$

$= 312$

Vertex y value $= \frac{-\Delta}{4a} = \frac{-312}{4(-1)}$

$= 78m$

$78m - 49m = 29m$

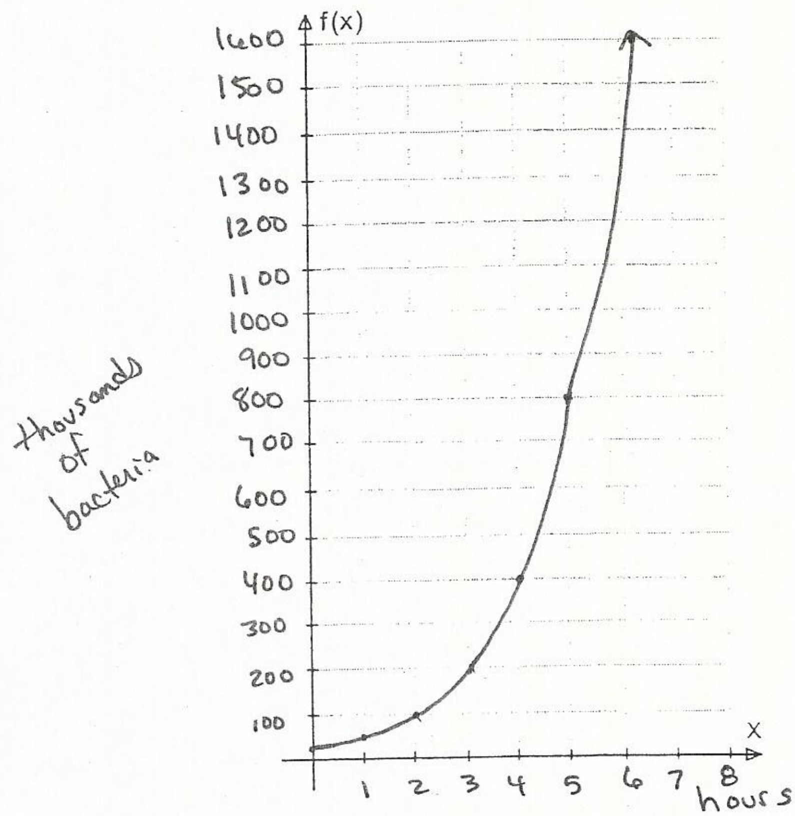
The difference is 29m.

6-1 In a laboratory, a biologist is studying a new type of bacterium. For his study, he places 25 thousand bacteria on a plate and notes that their number doubles every hour.

a) Complete the following table of values:

x (hours)	0	1	2	3	4	5	6
f(x) (thousands of bacteria)	25	50	100	200	400	800	1600

b) Graph this functional situation.



c) Is the function decreasing or increasing? Answer: increasing

Explain your answer by giving two ordered pairs. (0,25) (2,100)

2>0 , 100>25

d) What is the range of this function?

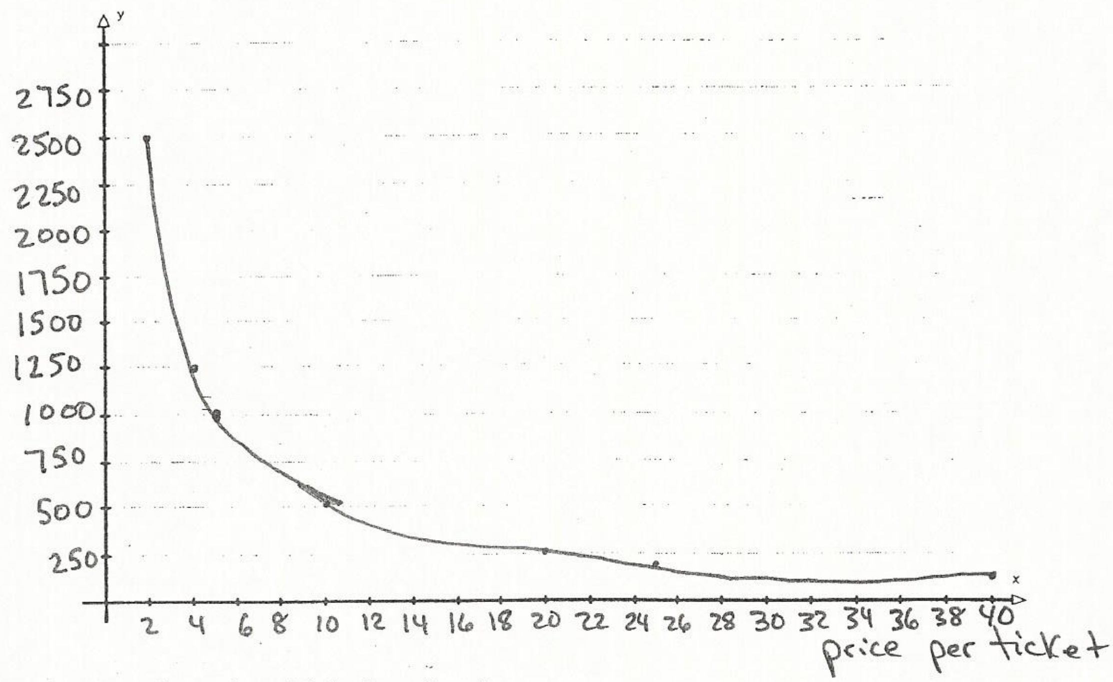
Answer: [25,000, ∞

G- ② **A high school graduating class is trying to raise $5000 for a school trip by selling raffle tickets. The students are unsure of how much to sell the tickets for. They want to ask at least $2 per ticket, but they are considering asking as much as $40 per ticket. The students considered different prices for a ticket and, for each price, the number of tickets they would have to sell.**

a) Complete the following table of values:

x (price per ticket)	2	4	5	10	20	25	40
y (number of tickets)	2500	1250	1000	500	250	200	125

b) Graph this functional situation.



c) What is the domain of this function?

Answer: [2, 40]

d) Is this function decreasing or increasing?

Answer: decreasing

Explain your answer by giving two ordered pairs: (2, 2500) (4, 1250)

2 < 4

2500 > 1250