The definition of the perpendicular bisector of a side of a triangle is a line segment that is both perpendicular to a side of a triangle and passes through its midpoint.

AD = 3.2 cm
DC = 3.2 cm
m∠EDC = 90°

An example of the perpendicular bisector of a side of a triangle.

1.   Determine the equation of the perpendicular bisector of side BC.

$(-2, 1)$ $(3, -3)$
x_1 y_1 x_2 y_2

$$m_{AB} = \frac{-3-1}{3+2} = \frac{-4}{5}$$

$$m_{xy} = +\frac{5}{4}$$

$$\text{Midpoint } BC = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+3}{2}, \frac{1-3}{2}\right) = \left(\frac{1}{2}, \frac{-2}{2}\right)$$

$$= \left(\frac{1}{2}, -1\right)$$

$$x \qquad y$$

$$y = mx + b$$

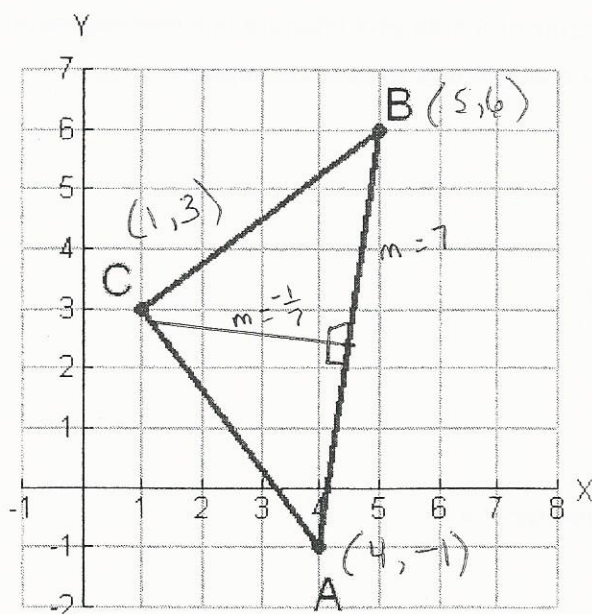$$-1 = \left(\frac{5}{4}\right)\left(\frac{1}{2}\right) + b$$

$$-1 = \frac{5}{8} + b$$

Eqn

$$y = \frac{5}{4}x - \frac{13}{8}$$

2. Find the equation of the perpendicular bisector of side AB.

$$m_{AB} = \frac{6+1}{5-4} = \frac{7}{1} = 7$$

$$\text{Midpoint } AB = \left(\frac{5+4}{2}, \frac{6-1}{2}\right)$$

$$= \left(\frac{9}{2}, \frac{5}{2}\right)$$

$$= (4.5, 2.5)$$

$$y = mx + b$$

$$2.5 = \left(-\frac{1}{7}\right)(4.5) + b$$

$$2.5 = -0.6429 + b$$

$$2.5 + 0.6429 = b$$

$$3.14 = b$$

$$4\tfrac{1}{2}$$

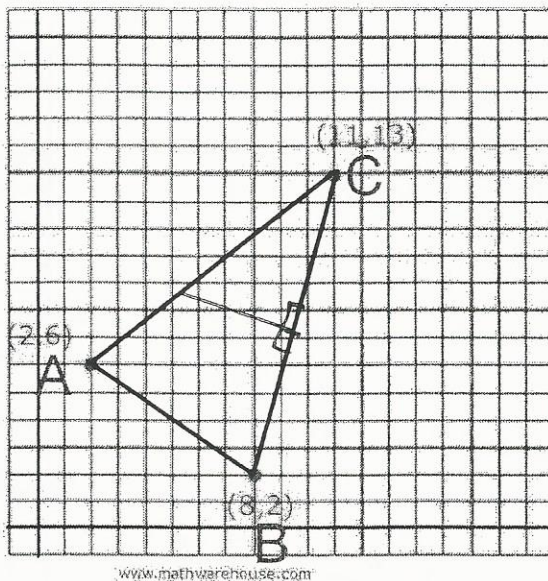$$\left(\frac{9}{2}\right)\left(\frac{-1}{7}\right) = \frac{-9}{14}$$

$$\frac{9}{14} + \frac{5}{2}$$

$$\frac{9}{14} + \frac{35}{14} = \frac{44}{14} = \frac{22}{7}$$

Ans: $y = -\frac{1}{7}x + 3.14$

3. Find the equation of the perpendicular bisector of side BC.

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$B(8,2)$　　　$C(11,13)$

$m_{BC} = \dfrac{13-2}{11-8} = \dfrac{11}{3}$ 　∴ 　$m \text{ bisector} = \dfrac{-3}{11}$

$\text{Midpoint}_{BC} = \left(\dfrac{8+11}{2}, \dfrac{15}{2}\right)$

$= \left(\dfrac{19}{2}, \dfrac{15}{2}\right)$

$y = mx + b$

$\dfrac{15}{2} = \left(\dfrac{-3}{11}\right)\left(\dfrac{19}{2}\right) + b$

$\dfrac{15}{2} = \dfrac{-57}{22} + b$

$\dfrac{15 \cdot 11}{2 \cdot 11} + \dfrac{57}{22} = b$

$\dfrac{165}{22} + \dfrac{57}{22} = b$

$\dfrac{222}{22} = b$

$10.09 = b$

Ans: ___ $y = \dfrac{-3}{11}x + 10.09$

4. Find the equation of the perpendicular bisector of side AC.

$A(-2, 3)$ $C(2, -1)$

$$m_{AC} = \frac{-1-3}{2+2} = \frac{-4}{4} = -1$$

$\therefore m_{bisector} = +1$

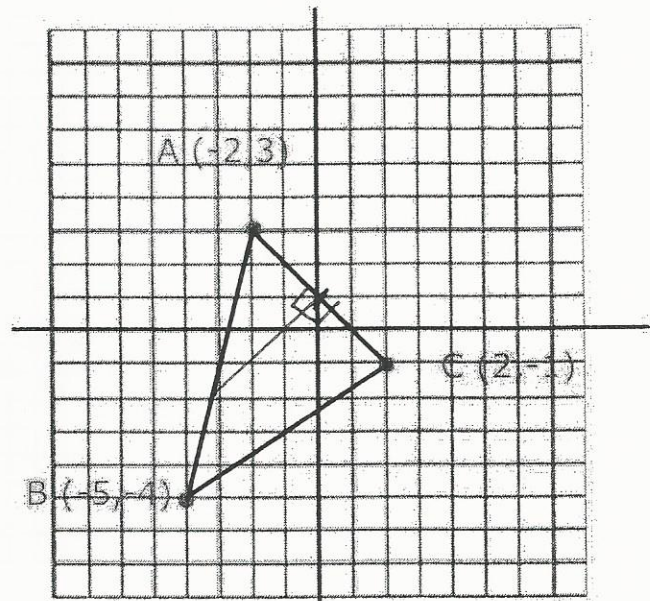$$\text{Midpoint } AC = \left(\frac{-2+2}{2}, \frac{3-1}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{2}{2}\right)$$

$$= (0, 1)$$
$\qquad x \quad y$

$y = mx + b$

$1 = (1)(0) + b$

$1 = 0 + b$

$1 = b$

Ans: ___$y = x + 1$___

5. Find the equation of the perpendicular bisector of side AC.

$A(2,1) \quad C(-3,-2)$

$m_{AC} = \dfrac{-2-1}{-3-2} = \dfrac{-3}{-5} = \dfrac{3}{5}$

$\therefore m_{bisector} = -\dfrac{5}{3}$

Midpoint $AC = \left(\dfrac{2-3}{2}, \dfrac{1-2}{2}\right)$

$= \left(\dfrac{-1}{2}, \dfrac{-1}{2}\right)$

$\quad\quad\quad x \quad\quad y$

$y = mx + b$

$-\dfrac{1}{2} = \left(\dfrac{-5}{3}\right)\left(\dfrac{-1}{2}\right) + b$

$-\dfrac{1}{2} = \dfrac{5}{6} + b$

$-\dfrac{1 \cdot 3}{2 \cdot 3} - \dfrac{5}{6} = b$

$-\dfrac{3}{6} - \dfrac{5}{6} = b$

$\dfrac{-8}{6} = b = \dfrac{-4}{3}$

Ans: $\underline{\quad y = \dfrac{-5}{3}x - \dfrac{4}{3} \quad}$
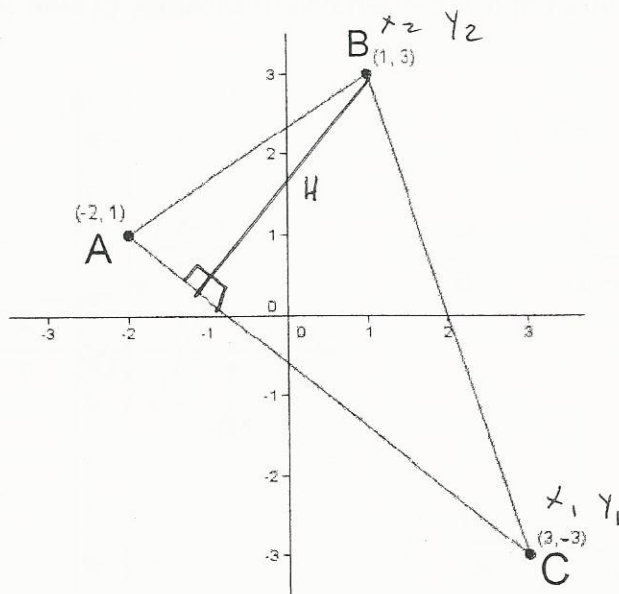
Find the Equation of Altitudes

1. Find the equation of the altitude drawn from vertex B.

$$m_{AC} = \frac{3+3}{1-3} = \frac{6}{-2} = -3$$

$$\therefore m_H = +\frac{1}{3}$$

$$y = mx + b$$

$$3 = \left(\frac{1}{3}\right)(1) + b$$

$$3 = \frac{1}{3} + b$$

$$3 - \frac{1}{3} = b \qquad = \qquad \frac{3 \cdot 3}{1 \cdot 3} - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$$

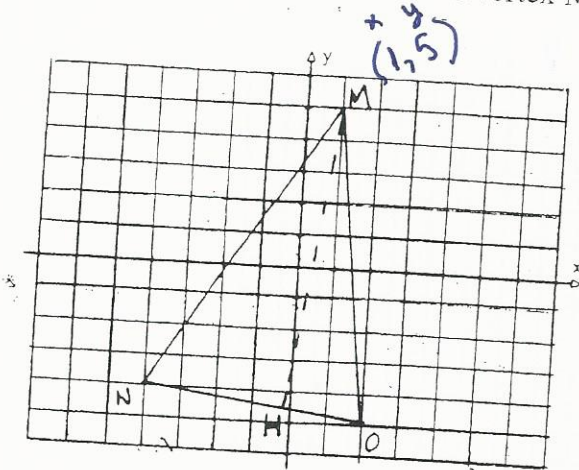$$2\frac{2}{3} = b \quad \text{OR} \quad b = \frac{8}{3}$$

Ans: $\underline{\quad y = \frac{1}{3}x + \frac{8}{3} \quad}$

Word Problems Involving Triangles

② Triangle MNO is formed by joining the following points:

M(1, 5), N(-4, -4), and O(2, -5)

What is the equation of the altitude drawn from vertex M? Clearly show all your work.

$$m_{NO} = \frac{-5+4}{2+4} = \frac{-1}{6}$$

$$m_{MH} = 6$$

Equation M·H :

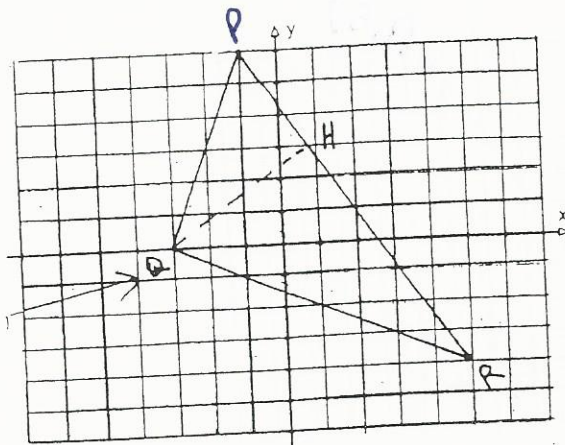$$y = mx + b$$

$$5 = (6)(1) + b$$

$$5 = 6 + b$$

$$5 - 6 = b$$

$$-1 = b$$

$$\boxed{y = 6x - 1}$$

③ Triangle PQR is formed by joining the following points:

P(-1, 6) , Q(-3, 0) , and R(5, -4)

What is the equation of the altitude drawn from vertex Q? Clearly show all your work.

$$m_{PR} = \frac{6+4}{-1-5} = \frac{10}{-6} = \frac{-5}{3}$$

$$m_{QH} = \frac{3}{5}$$

Equation QH:

$$y = mx + b$$

$$0 = \left(\frac{3}{5}\right)(-3) + b$$

$$0 = \frac{-9}{5} + b$$

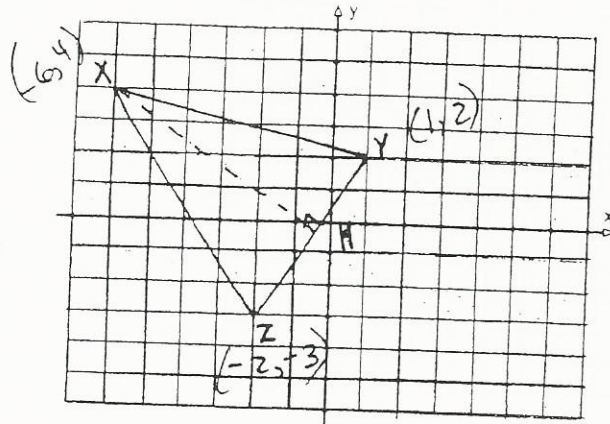$$\frac{9}{5} = b$$

$$\boxed{y = \frac{3}{5}x + \frac{9}{5}}$$

④ Triangle XYZ is formed by joining the following points:

X(-6, 4) , Y(1, 2) , and Z(-2, -3)

What is the equation of the altitude drawn from vertex X? Clearly show all your work.

$$m_{yz} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2+3}{1+2} = \frac{5}{3}$$

$$m_{XH} = -\frac{3}{5}$$

Equation XH :

$$y = mx + b$$

$$4 = \left(-\frac{3}{5}\right)\left(-6\right) + b$$

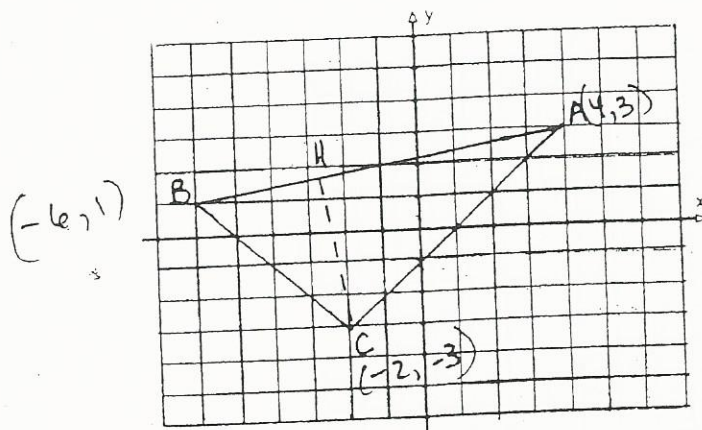$$4 = \frac{18}{5} + b$$

$$4 - \frac{18}{5} = b$$

$$\frac{2}{5} = b$$

$$\boxed{y = -\frac{3}{5}x + \frac{2}{5}}$$

(5) Triangle ABC is formed by joining the following points:

A(4, 3), B(-6, 1), and C(-2, -3)

What is the equation of the altitude drawn from vertex A? Clearly show all your work.

$$m_{AB} = \frac{3-1}{4+6} = \frac{2}{10} = \frac{1}{5}$$

$$m_{HC} = -5$$

Equation of HC:

$$y = mx + b$$
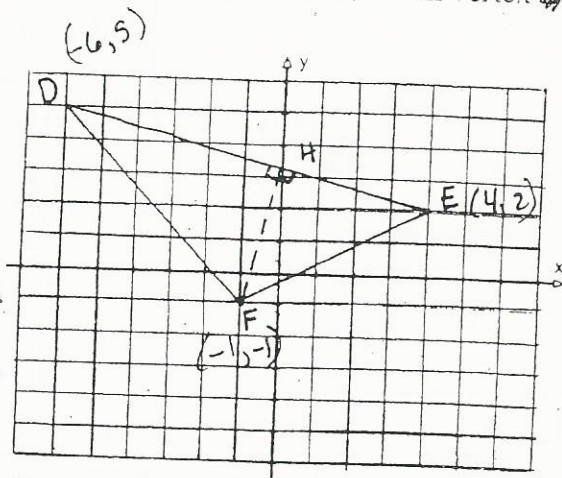
$$-3 = (-5)(-2) + b$$

$$-3 = 10 + b$$

$$-3 - 10 = b$$

$$-13 = b$$

$$\boxed{y = -5x - 13}$$

(6) Triangle DEF is formed by joining the following points:

D(-6, 5), E(4, 2), and F(-1, -1)

What is the equation of the altitude drawn from vertex ~~D~~ F? Clearly show all your work.

(-6,5)

$$m_{DE} = \frac{5-2}{-6-4} = \frac{3}{-10} = \frac{-3}{10}$$

$$m_{HF} = \frac{10}{3}$$

Equation HF :

$$y = mx + b$$

$$-1 = \left(\frac{10}{3}\right)(-1) + b$$

$$-1 = -\frac{10}{3} + b$$

$$-1 + \frac{10}{3} = b$$

$$2\frac{1}{3} = b$$
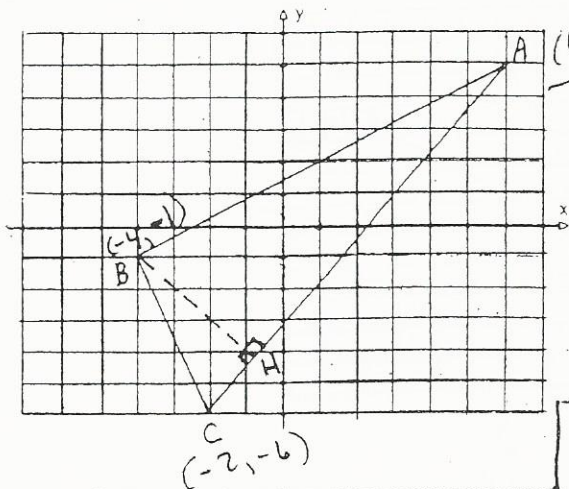
$$\boxed{y = \frac{10}{3}x + 2\frac{1}{3}}$$

Find the Area of Triangles

① Triangle ABC is formed by joining the following points:

A(6, 5), B(-4, -1), and C(-2, -6)

Calculate the area of triangle ABC. Round off your answer to the nearest unit.
Clearly show all your work.

$B = AC$

$B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6+2)^2 + (5+6)^2}$

$= \sqrt{8^2 + 11^2}$

$= \sqrt{64 + 121}$

$= \sqrt{185} = 13.6 \cup$

$m AC = \dfrac{5+6}{6+2} = \dfrac{11}{8}$

$gn\ AC \Rightarrow\ y = mx + b$

$5 = \left(\dfrac{11}{8}\right)(6) + b$

$5 = 8.25 + b$

$5 - 8.25 = b$

$-3.25 = b$

[AC] $y = \dfrac{11}{8} x - 3.25$

$m BH = \dfrac{-8}{11}$

Eqn BH $\Rightarrow\ y = mx + b$

$-1 = \left(-\dfrac{8}{11}\right)(-4) + b$

$-1 = 2\dfrac{10}{11} + b$

$-1 - 2\dfrac{10}{11} = b$

$-3.91 = b$

[BH] $y = -\dfrac{8}{11} x - 3.91$

Point of intersection between AC +

$\dfrac{11}{8} x - 3.25 = \dfrac{-8}{11} x - 3.91$

$\dfrac{11}{8} x + \dfrac{8}{11} x = -3.91 + 3.25$

$\dfrac{2.102 x}{2.102} = \dfrac{-0.66}{2.102}$

$x = -0.31$

$y = \dfrac{11}{8}(-0.31) - 3.25$

$= -3.68$

$H(-0.31, -3.68$

$h = \overline{BH}$: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 + 0.31)^2 + (-1 + 3.68)^2}$

$= \sqrt{13.62 + 7.18}$
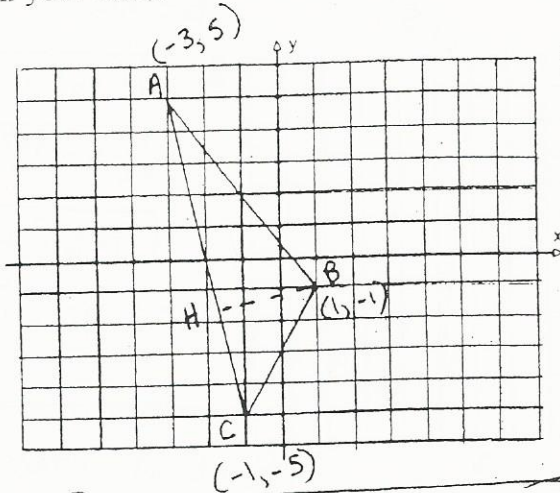
$= 4.56 \cup$

Area of ABC =

$\dfrac{B \cdot h}{2} = \dfrac{(13.6)(4.56)}{2}$

$= \boxed{31}$

(2) Triangle ABC is formed by joining the following points:

A(-3, 5), B(1, -1), and C(-1, -5)

Calculate the area of triangle ABC. Round off your answer to the nearest unit.
Clearly show all your work.

$B = AC$

$B = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3+1)^2+(5+5)^2}$

$= \sqrt{(-2)^2+10^2}$

$= \sqrt{4+100}$

$= \sqrt{104} = 10.2\smile$

(-3,5)
A
B
(1,-1)
H
C
(-1,-5)

$m_{AC} = \dfrac{5+5}{-3+1} = \dfrac{10}{-2} = -5$

Eqn AC ⇒ $y = mx+b$
$5 = (-5)(-3)+b$
$5 = 15+b$
$5-15 = b$
$-10 = b$

$\boxed{AC}:\ y = -5x-10$

$m_{BH} = \frac{1}{5}$

Eqn BH ⇒ $y = mx+b$
$-1 = \left(\frac{1}{5}\right)(1)+b$
$-1 = \frac{1}{5}+b$
$-1-\frac{1}{5} = b$
$-1.2 = b$

$\boxed{BH}\ y = \frac{1}{5}x - 1.2$

Point of intersection b/t $\overline{AC} + \overline{BH}$

$-5x-10 = \frac{1}{5}x-1.2$
$-5x-\frac{1}{5}x = -1.2+10$
$\dfrac{-5\frac{1}{5}x}{-5\frac{1}{5}} = \dfrac{8.8}{-5\frac{1}{5}}$
$x = -1.69$

$y = -5x-10$
$= -5(-1.69)-10$
$= -1.54$ H(-1.69, -1.54)

$h = \overline{BH} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1+1.69)^2+(-1+1.54)^2}$

$= \sqrt{(2.69)^2+(0.54)^2}$

$= \sqrt{7.24+.2916}$
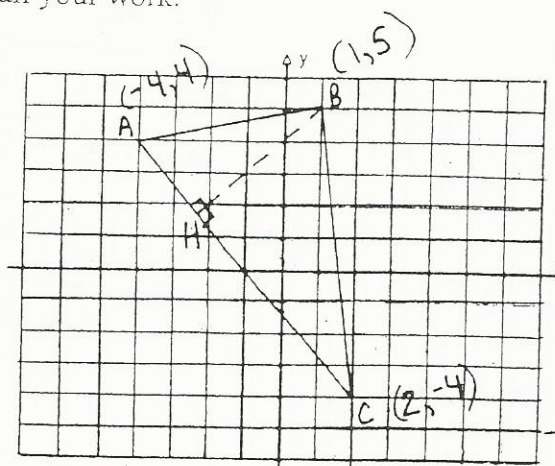
$= 2.74$

Area of ABC =

$\dfrac{B\cdot h}{2} = \dfrac{(10.2)(2.74)}{2}$

$\boxed{14}$

(3) Triangle ABC is formed by joining the following points:

A(-4, 4) , B(1, 5) , and C(2, -4)

Calculate the area of triangle ABC. Round off your answer to the nearest unit. Clearly show all your work.

$B = AC$

$$B = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-4-2)^2 + (4+4)^2}$$

$$= \sqrt{(-6)^2 + (8)^2}$$

$$= \sqrt{36+64} = \sqrt{100}$$

$$= 10 \, u$$

$m_{AC} = \dfrac{4+4}{-4-2} = \dfrac{8}{-6} = -\dfrac{4}{3}$

$m_{BH} = \dfrac{3}{4}$

Point of intersection between AC & BH :

Eqn AC $\Rightarrow$ $y = mx + b$

$4 = \left(-\dfrac{4}{3}\right)(-4) + b$

$4 = 5\dfrac{1}{3} + b$

$4 - 5\dfrac{1}{3} = b$

$-1\dfrac{1}{3} = b$

$\boxed{AC}$ $y = -\dfrac{4}{3}x - 1\dfrac{1}{3}$

Eqn BH $\Rightarrow$

$y = mx + b$

$5 = \left(\dfrac{3}{4}\right)(1) + b$

$5 = \dfrac{3}{4} + b$

$5 - \dfrac{3}{4} = b$

$4\dfrac{1}{4} = b$

$\boxed{BH}$ $y = \dfrac{3}{4}x + 4.25$

$-\dfrac{4}{3}x - 1\dfrac{1}{3} = \dfrac{3}{4}x + 4.25$

$-\dfrac{4}{3}x - \dfrac{3}{4}x = 4.25 + 1\dfrac{1}{3}$

$\dfrac{-2\dfrac{1}{12}x}{-2\dfrac{1}{12}} = \dfrac{5.58}{-2\dfrac{1}{12}}$

$x = -2.68$

$y = \dfrac{3}{4}x + 4.25$

$= \dfrac{3}{4}(-2.68) + 4.25$

$= 2.24$

$(-2.68, 2.24)$

$h = \overline{BH} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(1+2.68)^2 + (5-2.24)^2}$

$= \sqrt{13.54 + 7.62}$
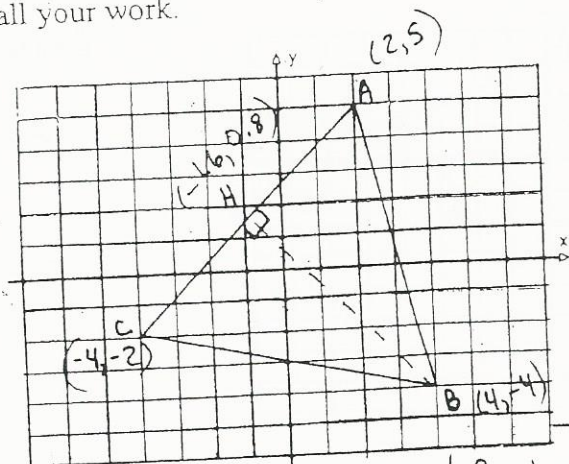
$= 4.6$

Area of ABC =

$\dfrac{B \cdot h}{2} = \dfrac{(10)(4.6u)}{2}$

$= \boxed{23 \, u^2}$

(4) Triangle ABC is formed by joining the following points:

A(2, 5), B(4, -4), and C(-4, -2)

Calculate the area of triangle ABC. Round off your answer to the nearest unit.
Clearly show all your work.

$B = AC$

$B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2+4)^2 + (5+2)^2}$

$= \sqrt{6^2 + 7^2}$

$= \sqrt{36 + 49}$

$= 9.22 \; u$

$m_{AC} = \dfrac{5+2}{2+4} = \dfrac{7}{6}$

AC: $y = mx + b$

$5 = \left(\dfrac{7}{6}\right)(2) + b$

$5 = 2\frac{1}{3} + b$

$5 - 2\frac{1}{3} = b$

$2\frac{2}{3} = b$

C $y = \frac{7}{6}x + 2\frac{2}{3}$

$m_{BH} = \dfrac{-6}{7}$

Eqn BH: $y = mx + b$

$-4 = \left(\dfrac{-6}{7}\right)(4) + b$

$-4 = -3\frac{3}{7} + b$

$-4 + 3\frac{3}{7} = b$

$-\dfrac{4}{7} = b$

[BH] $y = -\frac{6}{7}x - \frac{4}{7}$

Point of intersection between AC & BH

$\frac{7}{6}x + 2\frac{2}{3} = -\frac{6}{7}x - \frac{4}{7}$

$\frac{7}{6}x + \frac{6}{7}x = -\frac{4}{7} - 2\frac{2}{3}$

$\dfrac{2.02x}{2.02} = \dfrac{-3.24}{2.02}$

$x = -1.604$

$y = \frac{7}{6}(-1.604) + 2\frac{2}{3}$

$= 0.80$ $(-1.6, 0.8)$

$h = \overline{BH} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1.6 - 4)^2 + (0.8 + 4)^2}$

$= \sqrt{31.36 + 23.04}$

$= 7.4$

Area of ABC =

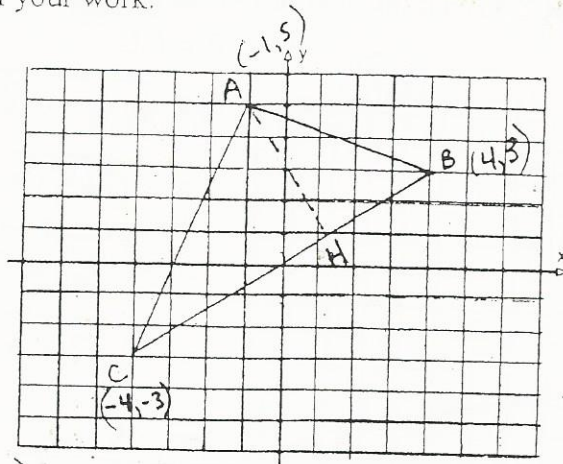$\dfrac{B \cdot h}{2} = \dfrac{(9.22)(7.4)}{2}$

$= 34$

⑤ Triangle ABC is formed by joining the following points:

A(-1, 5), B(4, 3), and C(-4, -3)

Calculate the area of triangle ABC. Round off your answer to the nearest unit. Clearly show all your work.

Area of triangle
$$= \frac{B \cdot h}{2}$$
$$= \frac{10 \cdot 4.6}{2} = \boxed{23.}$$

base of triangle (B):
$$\overline{BC} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4+4)^2+(3+3)^2}$$
$$= \sqrt{8^2+6^2}$$
$$= \sqrt{64+36} = 10 \text{ units} =$$

$$m = \frac{3+3}{4+4} = \frac{6}{8} = \frac{3}{4}$$

$$m_{AH} = \frac{-4}{3}$$

equation AH ⟹
$$y = mx+b$$
$$5 = \left(-\frac{4}{3}\right)(-1)+b$$
$$5 = \frac{4}{3}+b$$
$$5-\frac{4}{3} = b$$
$$3\frac{2}{3} = b$$

EQN
AH: $y = -\frac{4}{3}x + 3\frac{2}{3}$

equation BC ⟹
$$y = mx+b$$
$$3 = \left(\frac{3}{4}\right)(4)+b$$
$$3 = 3 +b$$
$$3-3 = b$$
$$0 = b$$

Eqn BC:
$$y = \frac{3}{4}x$$

$$-\frac{4}{3}x + 3\frac{2}{3} = \frac{3}{4}x$$
$$-\frac{4}{3}x - \frac{3}{4}x = -3\frac{2}{3}$$
$$-2\frac{1}{12}x = -3\frac{2}{3}$$
$$\overline{-2\frac{1}{12}} \quad \overline{-2\frac{1}{12}}$$
$$x = 1\frac{19}{25} \text{ or } 1.76$$

Point of intersection between $\overline{AH}$ + $\overline{BC}$:
A = (-1, 5) ⟶ H(1.76, 1.32)

$$h = \sqrt{(1.76+1)^2+(1.32-5)^2}$$
$$= \sqrt{(2.76)^2+(-3.68)^2}$$
$$= \sqrt{7.62+13.54} = 4.6 \text{ units}$$