- ANSWERS -

# Math 4101: Equations and Inequalities II

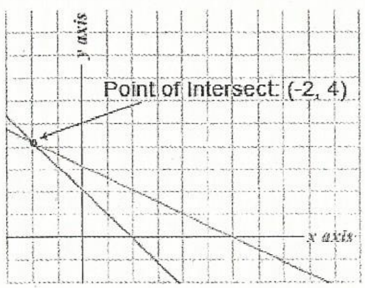
This book deals with systems of linear equations.

In this book, a system of linear equations consists of two linear (straight line) equations.

The "solution" to a system of linear equations is the (x, y) point that is common to both equations. It is the (x, y) point that "works" in the equation for each of the two straight lines. It is the intersection point.

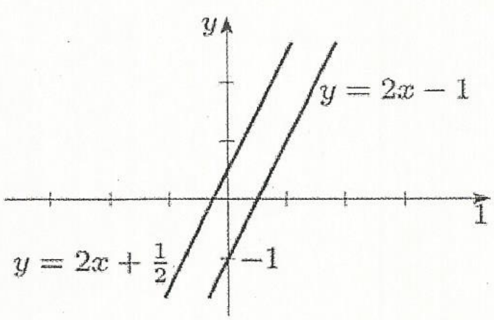
There are <u>three different scenarios concerning the intersection/solution of two straight lines.</u>

Case #1: One intersection point. The following is an example:




The solution to this system of equations is ( -2, 4)

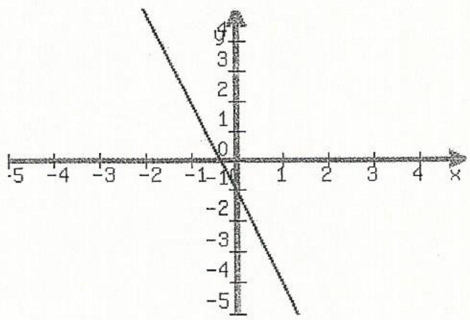
Case # 2 : No intersection points. No solution. (Parallel lines)




Solution to this system of equations:
**NO SOLUTION**

Note: the slopes are the same for these lines, but the y-intercepts are different.

Case # 3 : Infinite number of solutions. (Coinciding Lines)
The two equations, literally, are the exact same. The lines are "on top of" each other.

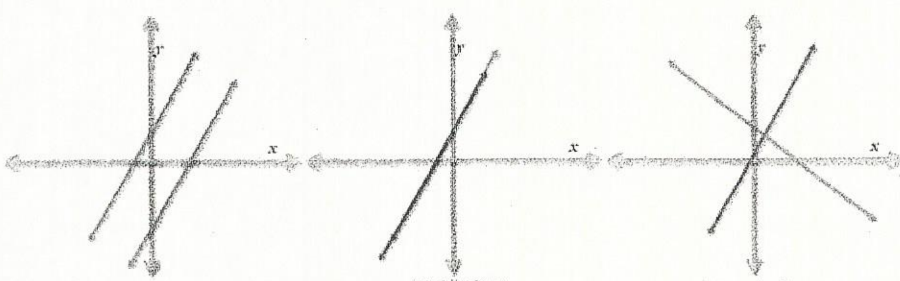



line 1: $y = -1x - 1$

line 2: $y = -1x - 1$

*The two equations are identical!

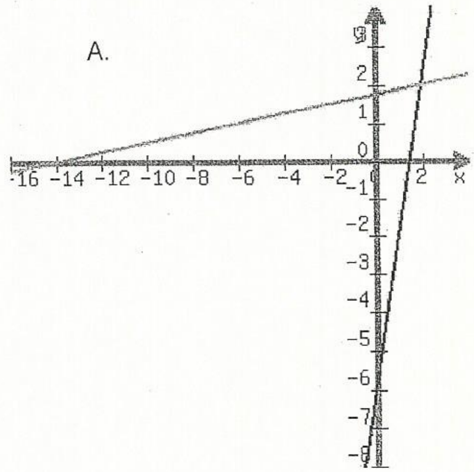
**Summary of three possibilities for a system of linear equations:**




Parallel Lines Coinciding Lines Intersecting Lines

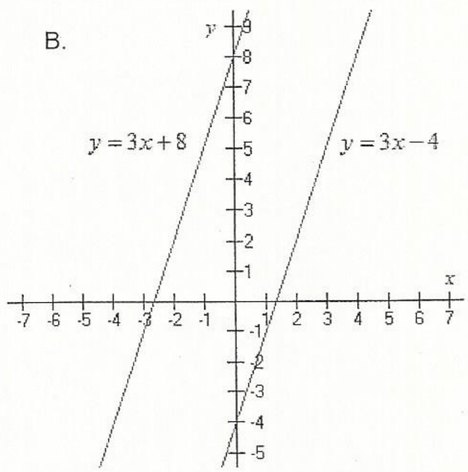
Problem 1: Find the solution/s to the following systems of equations:

A.



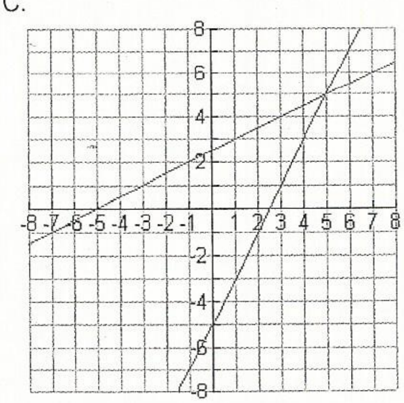

Solution: (2, 2)

B.



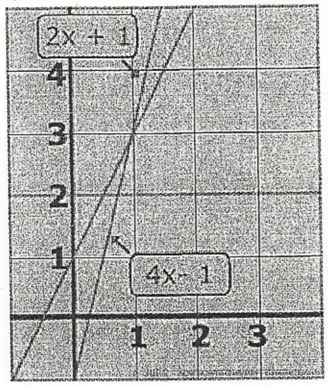

Solution: none

C.



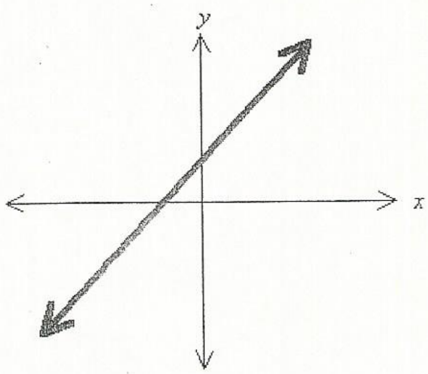

Solution: (5, 5)

D.



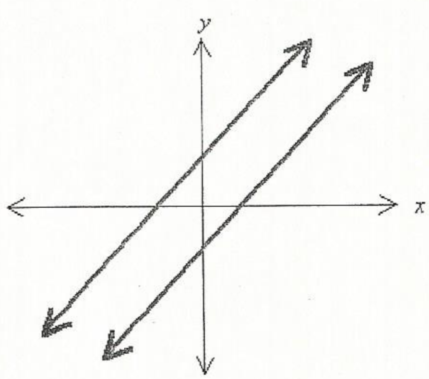

Solution: (1, 3)

E.




Solution: infinite solutions

F.




Solution: none

Question 2 Without calculations, solve the following systems of equations:

A.

| x | y |
|---|---|
| -4 | 10 |
| -2 | 6 |
| 0 | 2 |
| 2 | -2 |
| 4 | -6 |

| x | y |
|---|---|
| -3 | -4.5 |
| -2 | -4 |
| 0 | -3 |
| 2 | -2 |
| 3 | -1.5 |

Solution : (2, -2)

For these problems, since the same point is in both tables (and therefore on both lines) it must be the intersection point, (or solution).

B.

| x | y |
|---|---|
| 0 | 3 |
| 2 | 5 |
| 3 | 6 |
| 4 | 7 |

| x | y |
|---|---|
| -1 | 8 |
| 0 | 7 |
| 2 | 5 |
| 3 | 4 |

Solution : (2, 5)

C.

Equation 1

| x | y |
|---|---|
| 2 | 5 |
| −4 | −7 |
| 5 | 11 |
| −1 | −1 |

Equation 2

| x | y |
|---|---|
| 5 | 11 |
| −3 | −13 |
| 0 | −4 |
| 1 | −1 |

Solution : (5, 11)

There are two ways to visually solve a system of equations- and you have just seen these... 1. "eye-balling" a graph of the two lines to see the solution (intersection point)... and 2. noticing the point in common (intersection point/solution) in the two tables of values for the equations of the lines.

There are also two ways to mathematically solve the system of equations... using calculations. 1. To construct a table of values for each of two equations, and then graph the two lines, to see the intersection point (solution). 2. To let $y_1=y_2$ for the two equations, solve for x, then plug x into either equation to find the y. The resulting solution (x, y) will "work" in either equation.

We'll now look at each of these techniques:

① $\frac{1}{3}x + 9 = \frac{1}{2}y + 10$

$-\frac{1}{2}y = -\frac{1}{3}x + 10 - 9$

$-\frac{2}{1}\left(-\frac{1}{2}y\right) = \left(-\frac{1}{3}x + 1\right)\frac{-2}{1}$

$y = \frac{2}{3}x - 2$

**Solving a system of equaons algebraically** .

**First type: Graph the two lines**, then "eyeball" the soluon (intersecon point).

Solve the following systems of equations graphically.
Complete a table of values for each and graph.

A. 1) $\frac{x}{3} + 9 = \frac{y}{2} + 10$

2) $3y - 12 = 0$

*Graphical solution*

1

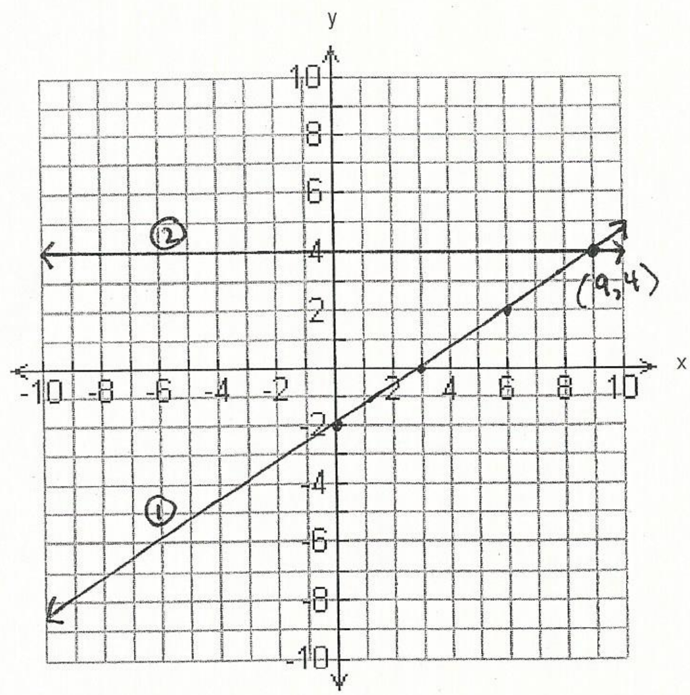
| x | y |
|---|---|
| 0 | -2 |
| 3 | 0 |
| 6 | 2 |

2

| x | y |
|---|---|
| 0 | 4 |
| 1 | 4 |
| 2 | 4 |

② $\frac{3y}{3} = \frac{12}{3}$

$y = 4$




*Ordered-pair solution* (9,4)

B)

1) $x = 2y + 8$

2) $2x + 3y + 12 = 0$

1

| x | y |
|---|---|
| 0 | -4 |
| 2 | -3 |
| 8 | 0 |

2

| x | y |
|---|---|
| 0 | -4 |
| 3 | -6 |
| -6 | 0 |

*Graphical solution*

① $x = 2y + 8$

$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{8}{-2}$

$y = \frac{1}{2}x - 4$

Let $y = 0$

$0 = \frac{1}{2}x - 4$

$2(4) = \left(\frac{1}{2}x\right)\frac{2}{1}$

$8 = x$

② $2x + 3y + 12 = 0$

$\frac{3y}{3} = -\frac{2}{3}x - \frac{12}{3}$

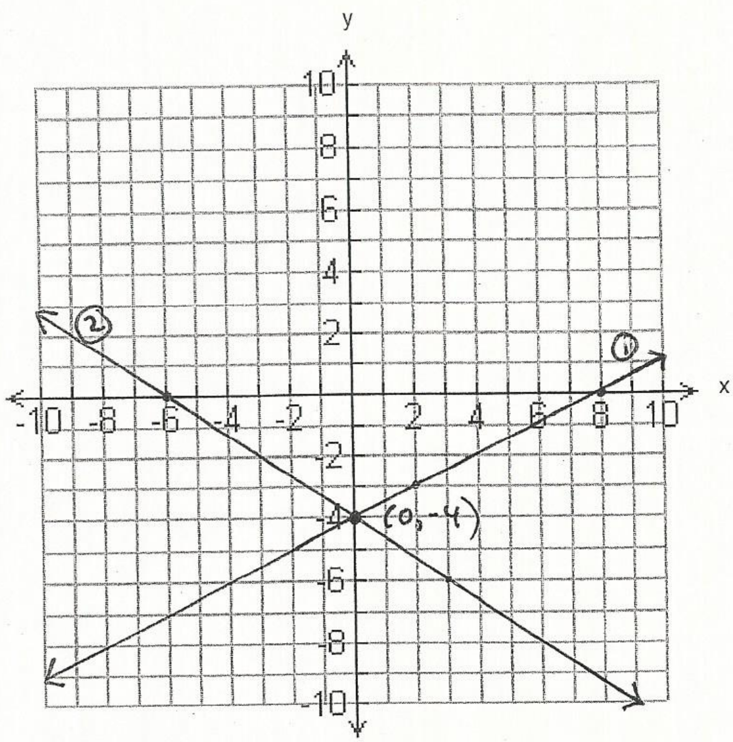
$y = -\frac{2}{3}x - 4$

Let $y = 0$

$0 = -\frac{2}{3}x - 4$

$\frac{3}{2}\left(\frac{2}{3}x\right) = \left(-\frac{4}{1}\right)\frac{3}{2}$

$x = -6$




*Ordered-pair solution* $(0, -4)$

C)

1) $4x + 5y = -2$

2) $2x = -6$

| 1 | | | 2 | |
|---|---|---|---|---|
| x | y | | x | y |
| 0 | $-\frac{2}{5} = -.4$ | | -3 | 0 |
| 5 | -4.4 | | -3 | 1 |
| -0.5 | 0 | | -3 | 2 |

*Graphical solution*

① $\frac{5y}{5} = \frac{-4x}{5} - \frac{2}{5}$

$y = -\frac{4}{5}x - \frac{2}{5}$

② $\frac{2x}{2} = \frac{-6}{2}$

$x = -3$

Let $x = 5$

$y = -\frac{4}{5}\left(\frac{5}{1}\right) - \frac{2}{5}$

$= \frac{-20}{5} - \frac{2}{5}$

$= \frac{-22}{5} = -4\frac{2}{5}$

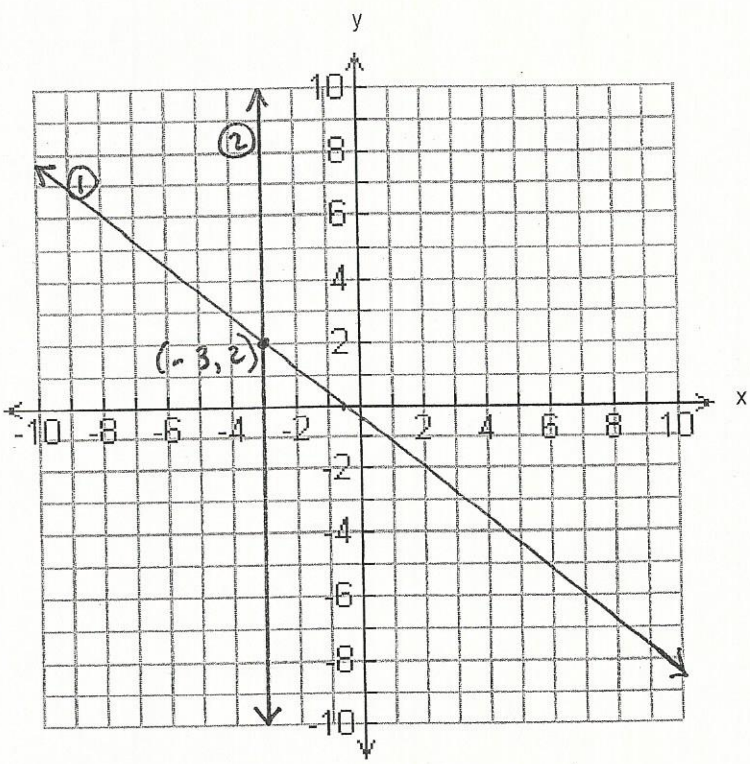
$= -4.4$

Let $y = 0$

$0 = -\frac{4}{5}x - \frac{2}{5}$

$\frac{5}{4}\left(\frac{4}{5}x\right) = \left(-\frac{2}{5}\right)\frac{5}{4}$

$x = \frac{-10}{20} = -\frac{1}{2}$ OR $-0.5$




Check $x = -3$

$4(-3) + 5y = -2$

$-12 + 5y = -2$

$5y = 12 - 2$

$\frac{5y}{5} = \frac{10}{5}$

$y = 2$

*Ordered-pair solution* $(-3, 2)$

D)

1) $\frac{3x}{4} - \frac{y}{3} = 2$

2) $4y - 16 = 9x$

1

| x | y |
|---|---|
| 0 | -6 |
| $2\frac{2}{3}$ | 0 |
| 4 | 3 |

2

| x | y |
|---|---|
| 0 | 4 |
| -4 | -5 |
| $-1\frac{7}{9}$ | 0 |

*Graphical solution*

① $\frac{3x}{4} - \frac{y}{3} = 2$

$\frac{3}{4}x - \frac{1}{3}y = 2$

$-\frac{3}{1}\left(-\frac{1}{3}y\right) = \left(-\frac{3}{4}x + 2\right)-\frac{3}{1}$

$y = \frac{9}{4}x - 6$

Let $y = 0$

$0 = \frac{9}{4}x - 6$

$\frac{4}{9}(6) = \left(\frac{9}{4}x\right)\frac{4}{9}$

$\frac{24}{9} = x$

$x = 2\frac{6}{9}$

$= 2\frac{2}{3}$

② $\frac{4y}{4} = \frac{9x}{4} + \frac{16}{4}$

$y = \frac{9}{4}x + 4$

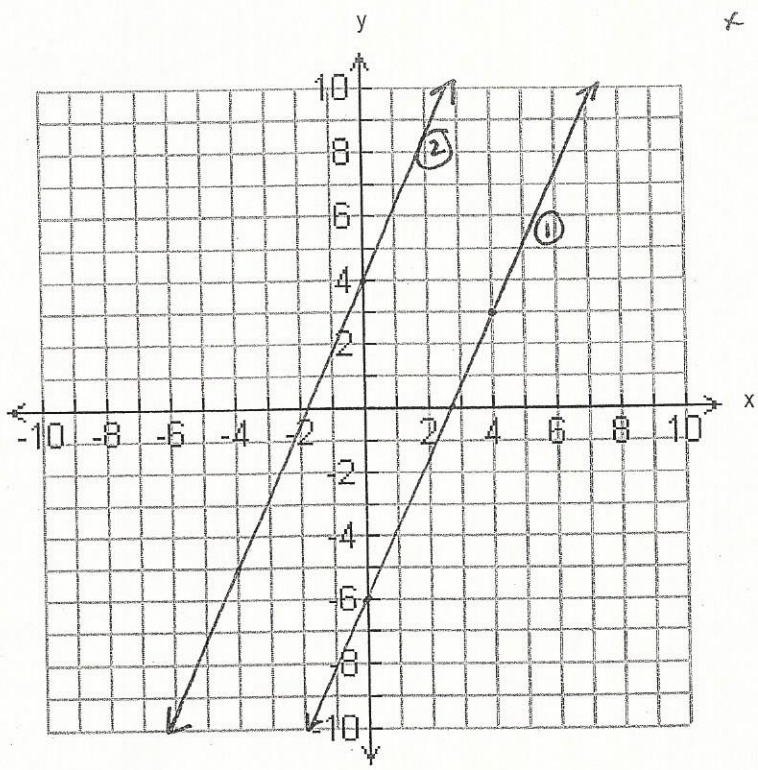
Let $y = 0$

$0 = \frac{9}{4}x + 4$

$-\frac{4}{9}\left(-\frac{9}{4}x\right) = (4)-\frac{4}{9}$

$x = -\frac{16}{9}$

$= -1\frac{7}{9}$




(The equations have the same slope = parallel lines)

Ordered-pair solution No solution (parallel lines)

E)

1) $4x + 6y - 10 = 0$

2) $y = \frac{-2x}{3} + \frac{5}{3}$

$$1 \; \overset{\text{same}}{\longleftrightarrow} \; 2$$

| x | y |
|---|---|
| 0 | $\frac{5}{3} = 1\frac{2}{3}$ |
| $2\frac{1}{2}$ | 0 |
| 6 | $-2\frac{1}{3}$ |

| x | y |
|---|---|
| | |
| | |
| | |

② $y = -\frac{2}{3}x + \frac{5}{3}$

Graphical solution

① $\frac{6y}{6} = \frac{-4x}{6} + \frac{10}{6}$

$y = -\frac{2}{3}x + \frac{5}{3}$

Let $y = 0$

$0 = -\frac{2}{3}x + \frac{5}{3}$

$\frac{3}{2}\left(\frac{2}{3}x\right) = \left(\frac{5}{3}\right)\frac{3}{2}$

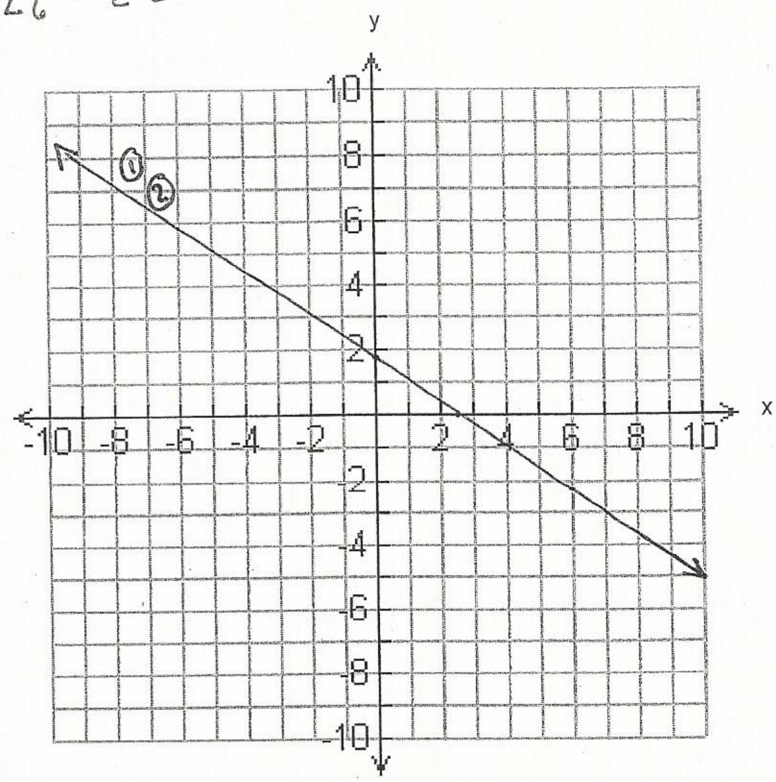
$x = \frac{15}{6} = 2\frac{3}{6} = 2\frac{1}{2}$

Let $x = 6$

$y = \left(-\frac{2}{3}\right)\left(\frac{6}{1}\right) + \frac{5}{3}$

$= -\frac{12}{3} + \frac{5}{3}$

$= -\frac{7}{3} = -2\frac{1}{3}$




Ordered-pair solution infinite number of solutions
(coinciding lines) (same line)

F)

1) x = 3 - y

2) 3x + 5y = 7

1

| x | y |
|---|---|
| 0 | 3 |
| 1 | 2 |
| 3 | 0 |

2

| x | y |
|---|---|
| 0 | $7/5 = 1\,2/5 = 1.4$ |
| 5 | $-1\,3/5 = -1.6$ |
| $2\,1/3$ | 0 |

*Graphical solution*

① $x = 3 - y$

$y = -x + 3$

Let $y = 0$

$0 = -x + 3$

$x = 3$

② $3x + 5y = 7$

$\frac{5y}{5} = -\frac{3x}{5} + \frac{7}{5}$

$y = -\frac{3}{5}x + \frac{7}{5}$

let $x = 5$

$y = \left(-\frac{3}{5}\right)\left(\frac{5}{1}\right) + \frac{7}{5}$

$= \frac{-15}{5} + \frac{7}{5} = \frac{-8}{5} =$

Let $y = 0$

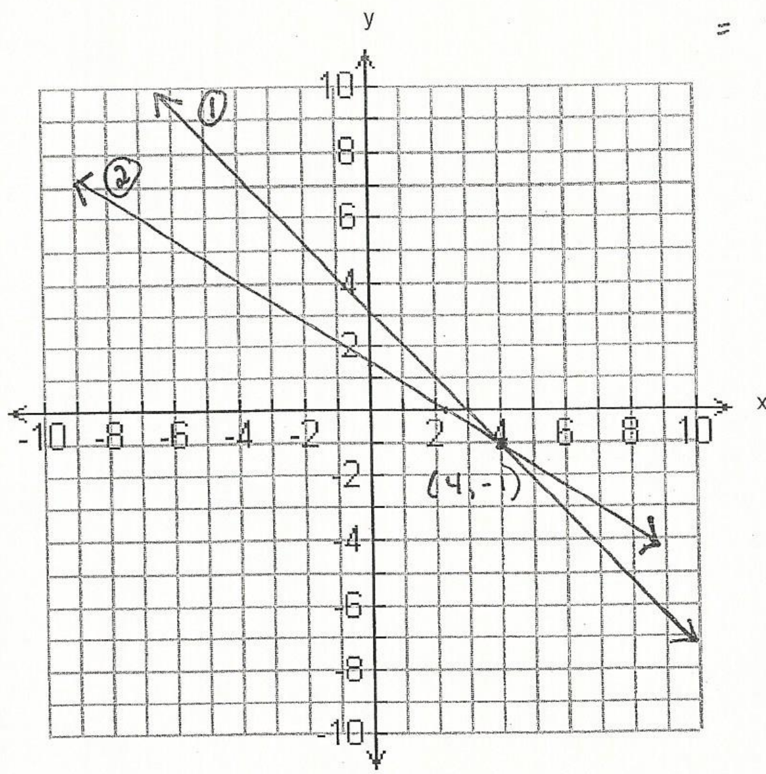
$0 = -\frac{3}{5}x + \frac{7}{5}$

$\frac{5}{3}\left(\frac{3}{5}x\right) = \left(\frac{7}{5}\right)\frac{5}{3}$

$x = \frac{35}{15}$

$= 2\frac{5}{15}$

$= 2\frac{1}{3}$




*Ordered-pair solution* (4, -1)

G)

1) $x - y - 6 = 0$

2) $\frac{3x}{5} - 1 = \frac{4}{5}$

Graphical solution

1

| x | y |
|---|---|
| 0 | -6 |
| 6 | 0 |
| 2 | -4 |

2

| x | y |
|---|---|
| 3 | 0 |
| 3 | 1 |
| 3 | 2 |

① $\frac{-y}{-1} = \frac{-x}{-1} + \frac{6}{-1}$

$y = x - 6$

Let $y = 0$

$0 = x - 6$

$6 = x$

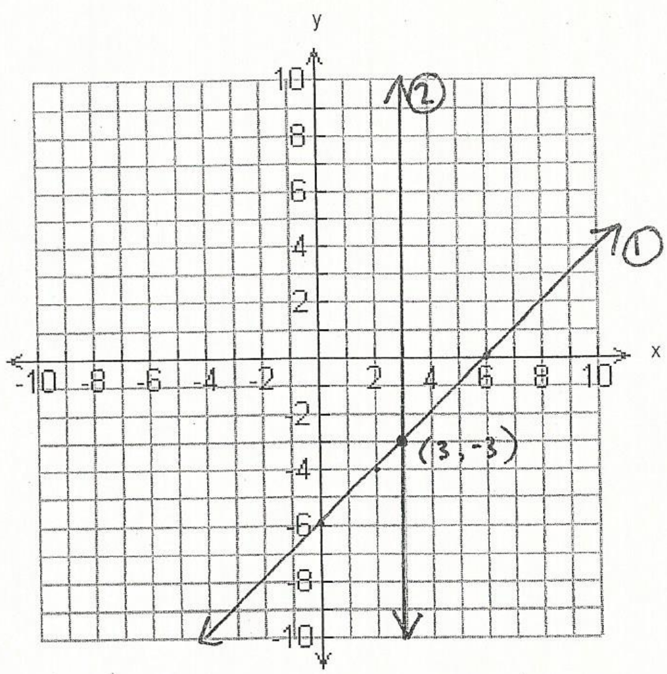
② $\frac{3}{5}x - 1 = \frac{4}{5}$

$\frac{3}{5}x = \frac{4}{5} + 1$

$\frac{3}{5}x = \frac{4}{5} + \frac{5}{5}$

$\frac{5}{3}\left(\frac{3}{5}x\right) = \left(\frac{9}{5}\right)\frac{5}{3}$

$x = 3$




Ordered-pair solution $(3, -3)$

H)

1) $x - 1 = 3 - 2y$

2) $4x - 8y = 0$

| 1 | |
|---|---|
| x | y |
| 0 | 2 |
| 4 | 0 |
| 2 | 1 |

| 2 | |
|---|---|
| x | y |
| 0 | 0 |
| 2 | 1 |
| 4 | 2 |

① $x - 1 = 3 - 2y$

$2y = -x + 3 + 1$

$\frac{2y}{2} = -\frac{x}{2} + \frac{4}{2}$

$y = -\frac{1}{2}x + 2$

Let $y = 0$

$0 = -\frac{1}{2}x + 2$

$\frac{2}{1}\left(\frac{1}{2}x\right) = (2)2$

$x = 4$

Let $x = 2$

$y = \left(-\frac{1}{2}\right)\left(\frac{2}{1}\right) + 2$

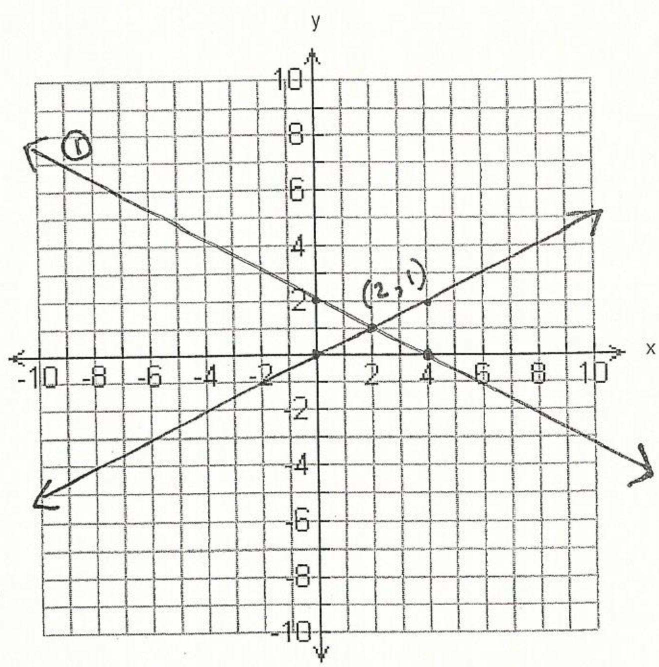
$= -1 + 2$

$= 1$

② $4x - 8y = 0$

$\frac{-8y}{-8} = \frac{-4x}{-8}$

$y = \frac{1}{2}x$




Ordered-pair solution (2,1)

I)

Same Line ∴ Same points

1) $x + 4y = 4$

2) $y = 1 - \frac{x}{4}$

1

| x | y |
|---|---|
| 0 | 1 |
| 4 | 0 |
| 8 | -1 |

2

| x | y |
|---|---|
| | |
| | |
| | |

① $x + 4y = 4$

$$\frac{4y}{4} = \frac{-x}{4} + \frac{4}{4}$$

$$y = -\frac{1}{4}x + 1$$

② $y = -\frac{1}{4}x + 1$

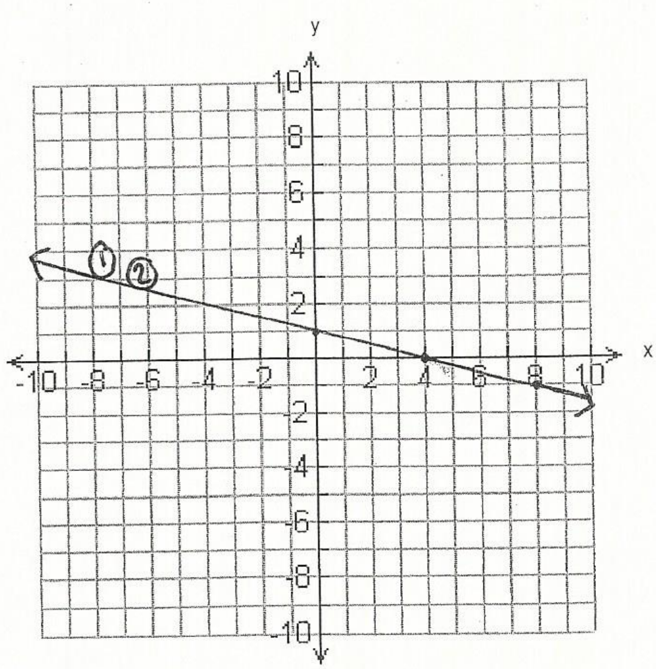
Same equation

Let $y = 0$

$$0 = -\frac{1}{4}x + 1$$

$$\frac{4}{1}\left(\frac{1}{4}x\right) = (1)4$$

$$x = 4$$




Ordered-pair solution infinite solutions (coinciding lines)

J)

1) $4x - 7y - 5 = 0$

2) $3x + 4y = 13$

1

| x | y |
|---|---|
| 0 | -5/7 |
| 5/4 | 0 |
| | |

$1\frac{1}{4} = \frac{5}{4}$

2

| x | y |
|---|---|
| 0 | 3 1/4 |
| 4 1/3 | 0 |
| 4 | 1/4 |

① $\frac{-7y}{-7} = \frac{-4x}{-7} + \frac{5}{-7}$

$y = \frac{4}{7}x - \frac{5}{7}$

Let $y = 0$

$0 = \frac{4}{7}x - \frac{5}{7}$

$\frac{7}{4}\left(\frac{5}{7}\right) = \left(\frac{4}{7}x\right)\frac{7}{4}$

$\frac{5}{4} = x$

② $\frac{4y}{4} = \frac{-3x}{4} + \frac{13}{4}$

$y = -\frac{3}{4}x + \frac{13}{4}$

OR $y = -\frac{3}{4}x + 3\frac{1}{4}$

Let $y = 0$

$0 = -\frac{3}{4}x + 3\frac{1}{4}$

$\frac{4}{3}\left(\frac{3}{4}x\right) = \left(\frac{13}{4}\right)\frac{4}{3}$

$x = \frac{13}{3}$ OR $4\frac{1}{3}$

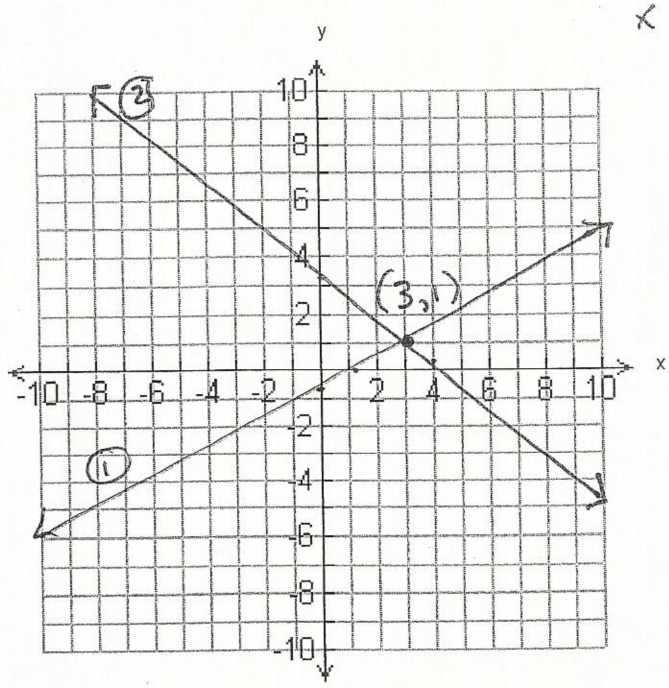
Let $x = 4$

$y = -\frac{3}{4}x + 3\frac{1}{4}$

$= \left(-\frac{3}{4}\right)\left(\frac{4}{1}\right) + 3\frac{1}{4}$

$y = -3 + 3\frac{1}{4}$

$= \frac{1}{4}$




Ordered-pair solution (3, 1)

## Second way to solve a system of equations algebraically:

Let $y_1 = y_2$ , solve for x, then plug x into either equaon to find y.

The soluon (x,y) will "work" in either equaon.

Solve the following systems of equaons using comparison (the y=y method). Show all steps to your soluon.

A. 1) $3x - y = 0$

2) $12 + 7y = 5x$

① $3x - y = 0$

$\frac{-y}{-1} = \frac{-3x}{-1}$

$\boxed{① \; y = 3x}$

② $12 + 7y = 5x$

$\frac{7y}{7} = \frac{5x}{7} - \frac{12}{7}$

$\boxed{② \; y = \frac{5}{7}x - \frac{12}{7}}$

$y_1 = y_2$

$3x = \frac{5}{7}x - \frac{12}{7}$

$3x - \frac{5}{7}x = -\frac{12}{7}$

$\frac{3 \cdot 7}{1 \cdot 7}x - \frac{5}{7}x = -\frac{12}{7}$

$\frac{21}{7}x - \frac{5}{7}x = -\frac{12}{7}$

$\frac{7}{16}\left(\frac{16}{7}x\right) = \left(-\frac{12}{7}\right)\frac{7}{16}$

$x = \frac{-12}{16} = \frac{-3}{4}$

Now plug in $x = \frac{-3}{4}$ (to one of the eqns)

$y = 3x$

$y = 3\left(\frac{-3}{4}\right)$

$= \frac{-9}{4}$

Solution: $\boxed{\left(\frac{-3}{4}, \frac{-9}{4}\right)}$

OR $-2\frac{1}{4}$

OR $-2.25$

B. 1) $\frac{5x}{3} = y - \frac{1}{2}$

2) $2x = y - \frac{1}{3}$

① $\frac{5}{3}x = y - \frac{1}{2}$

$\boxed{① \; y = \frac{5}{3}x + \frac{1}{2}}$

② $2x = y - \frac{1}{3}$

$\boxed{② \; y = 2x + \frac{1}{3}}$

$y_1 = y_2$

$\frac{5}{3}x + \frac{1}{2} = 2x + \frac{1}{3}$

$\frac{5}{3}x - \frac{2 \cdot 3}{1 \cdot 3}x = \frac{1 \cdot 2}{3 \cdot 2} - \frac{1 \cdot 3}{2 \cdot 3}$

$\frac{5}{3}x - \frac{6}{3}x = \frac{2}{6} - \frac{3}{6}$

$\frac{-3}{1}\left(-\frac{1}{3}x\right) = \left(-\frac{1}{6}\right)\frac{-3}{1}$

$x = \frac{1}{2}$

Plug in $x = \frac{1}{2}$

$y = 2x + \frac{1}{3}$

$= 2\left(\frac{1}{2}\right) + \frac{1}{3}$

$= 1 + \frac{1}{3}$

$= 1\frac{1}{3}$

Solution: $\boxed{\left(\frac{1}{2}, 1\frac{1}{3}\right)}$

OR $\frac{4}{3}$

C. 1) $4x + 5y = 10$

2) $2x + \frac{5}{2}y = 5$

① $\frac{5y}{5} = \frac{-4x}{5} + \frac{10}{5}$

$\boxed{① \ y = \frac{-4}{5}x + 2}$

② $\frac{5}{2}y = -2x + 5$

$\frac{2}{5}\left(\frac{5}{2}y\right) = \frac{2}{5}\left(\frac{-2x}{1} + \frac{5}{1}\right)$

$\boxed{② \ y = \frac{-4}{5}x + 2}$

$y_1 = y_2$

$-\frac{4}{5}x + 2 = \frac{-4}{5}x + 2$

$0 = 0$

The equations are identical (coinciding lines) therefore there are an infinite number of solutions!

D. 1) $3(x - 1) + 2(y + 3) = 8$

2) $4x - (y - 6) = 9$

① $3x - 3 + 2y + 6 = 8$

$2y = -3x + 8 + 3 - 6$

$\frac{2y}{2} = -\frac{3x}{2} + \frac{5}{2}$

$\boxed{① \ y = -\frac{3}{2}x + \frac{5}{2}}$

② $4x - y + 6 = 9$

$-y = -4x + 9 - 6$

$\frac{-y}{-1} = \frac{-4x}{-1} + \frac{3}{-1}$

$\boxed{② \ y = 4x - 3}$


$y_1 = y_2$

$-\frac{3}{2}x + \frac{5}{2} = 4x - 3$

$-\frac{3}{2}x - \frac{4 \cdot 2}{1 \cdot 2}x = -\frac{3 \cdot 2}{1 \cdot 2} - \frac{5}{2}$

$-\frac{3}{2}x - \frac{8}{2}x = \frac{-6}{2} - \frac{5}{2}$

$\frac{-2}{11}\left(-\frac{11}{2}x\right) = \left(\frac{-11}{2}\right)\frac{-2}{11}$

$x = 1$

Plug in $x = 1$:

$y = 4x - 3$

$y = 4(1) - 3$

$= 4 - 3 = 1$

$\boxed{\text{Solution: } (1, 1)}$

E. 1) $3x = 2(2y + 9)$

2) $2y = -3x$

① $4y + 18 = 3x$

$\frac{4y}{4} = \frac{3x}{4} - \frac{18}{4}$

$y = \frac{3}{4}x - \frac{9}{2}$

② $\frac{2y}{2} = \frac{-3x}{2}$

$y = \frac{-3}{2}x$

$y_1 = y_2$

$\frac{3}{4}x - \frac{9}{2} = \frac{-3}{2}x$

$\frac{3}{4}x + \frac{3 \cdot 2}{2 \cdot 2}x = \frac{9}{2}$

$\frac{3}{4}x + \frac{6}{4}x = \frac{9}{2}$

$\frac{4}{9}\left(\frac{9}{4}x\right) = \left(\frac{9}{2}\right)\frac{4}{9}$

$x = 2$

Plug $x = 2$ into eqn:

$y = -\frac{3}{2}x$

$= -\frac{3}{2}\left(\frac{2}{1}\right)$

$= -3$

Solution:

$\boxed{(2, -3)}$

F. 1) $\frac{7x}{2} + 2y = 31$

2) $\frac{x}{3} - 5y = -23$

① $\frac{7}{2}x + 2y = 31$

$-\frac{7}{2} \div \frac{2}{1}$

$= -\frac{7}{2} \cdot \frac{1}{2}$

$= \frac{-7}{4}$

$\frac{2y}{2} = \frac{-\frac{7}{2}x}{2} + \frac{31}{2}$

$y_1 = -\frac{7}{4}x + \frac{31}{2}$

② $\frac{1}{3}x - 5y = -23$

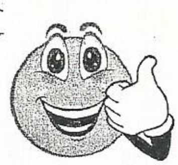
$\frac{-5y}{-5} = \frac{-\frac{1}{3}x}{-5} \frac{-23}{-5}$

$\frac{1}{3} \div -5$

$= -\frac{1}{3} \times -\frac{1}{5}$

$= \frac{1}{15}$

$y_2 = \frac{1}{15}x + \frac{23}{5}$



$y_1 = y_2$

$-\frac{7}{4}x + \frac{31}{2} = \frac{1}{15}x + \frac{23}{5}$

$-\frac{7}{4}x - \frac{1}{15}x = \frac{23}{5} - \frac{31}{2}$

$-\frac{7 \cdot 15}{4 \cdot 15}x - \frac{1 \cdot 4}{15 \cdot 4}x = \frac{23 \cdot 2}{5 \cdot 2} - \frac{31 \cdot 5}{2 \cdot 5}$

$-\frac{105}{60} - \frac{4}{60}x = \frac{46}{10} - \frac{155}{10}$

$-\frac{60}{109}\left(-\frac{109}{60}x\right) = \left(\frac{-\cancel{109}}{10}\right)\frac{-60}{\cancel{109}}$

$x = 6$

Plug in $x = 6$:

$y = -\frac{7}{4}\left(\frac{6}{1}\right) + \frac{31}{2}$

$= -\frac{42}{4} + \frac{31}{2}$

$= -\frac{21}{2} + \frac{31}{2} = \frac{10}{2} = 5$

$\boxed{\text{Solution} = (6, 5)}$

G. 1) $2x - 3y = 5$

2) $10x - 25 = 15y$

① $\dfrac{-3y}{-3} = \dfrac{-2x}{-3} + \dfrac{5}{-3}$

$y_{①} = \frac{2}{3}x - \frac{5}{3}$

② $\dfrac{15y}{15} = \dfrac{10^{\div 5}x}{15 \div 5} - \dfrac{25 \div 5}{15 \div 5}$

$y_{②} = \frac{2}{3}x - \frac{5}{3}$

Same equation!
Coinciding Lines

∴ $\boxed{\text{Infinite number of solutions}}$

---

H. 1) $\frac{1}{2}x - \frac{1}{3}y = 1$

2) $\frac{2}{3}x + \frac{1}{2}y = -2$

① $\frac{-3}{1}\left(-\frac{1}{3}y\right) = \frac{-3}{1}\left(-\frac{1}{2}x + 1\right)$

$y_1 = \frac{3}{2}x - 3$

② $\frac{2}{1}\left(\frac{1}{2}y\right) = \frac{2}{1}\left(\frac{2}{3}x - 2\right)$

$y_2 = \frac{-4}{3}x - 4$

$y_1 = y_2$

$\frac{3}{2}x - 3 = \frac{-4}{3}x - 4$

$\frac{3 \cdot 3}{2 \cdot 3}x + \frac{4 \cdot 2}{3 \cdot 2}x = -4 + 3$

$\frac{9}{6}x + \frac{8}{6}x = -1$

$\frac{6}{17}\left(\frac{17}{6}x\right) = (-1)\frac{6}{17}$

$x = -\frac{6}{17}$

Plug in x :

$y = \frac{3}{2}\left(\frac{-6}{17}\right) - 3$

$= \frac{-18}{34} - \frac{3}{1}$

$= \frac{-18}{34} - \frac{3\,(34)}{1\,(34)}$

$= \frac{-18}{34} - \frac{102}{34} = \frac{-120}{34} = \frac{-60}{17}$

I ♥ MATH

$\boxed{\text{Solution : } \left(-\frac{6}{17}, \frac{-60}{17}\right)}$

I. 1) $x = 5y - 15$

2) $y = \frac{x}{5} - 1$

① $\frac{5y}{5} = \frac{x}{5} + \frac{15}{5}$

$y_1 = \frac{1}{5}x + 3$

② $y = \frac{1}{5}x - 1$

Same slope, different y-intercepts ∴ they're parallel lines ∴

$\boxed{\text{NO SOLUTION}}$

---

J. 1) $5x + 2y = 4x + 8$

2) $4y = 20 - 3x$

① $2y = 4x - 5x + 8$

$\frac{2y}{2} = \frac{-x}{2} + \frac{8}{2}$

$y_1 = -\frac{1}{2}x + 4$

② $\frac{4y}{4} = \frac{-3x}{4} + \frac{20}{4}$

$y_2 = -\frac{3}{4}x + 5$

$y_1 = y_2$

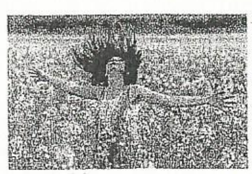
$-\frac{1}{2}x + 4 = -\frac{3}{4}x + 5$

$-\frac{1 \cdot 2}{2 \cdot 2}x + \frac{3}{4}x = 5 - 4$

$-\frac{2}{4}x + \frac{3}{4}x = 1$

$\frac{4}{1}\left(\frac{1}{4}x\right) = (1)4$

$x = 4$



Plug in $x = 4$

$y = -\frac{1}{2}\left(\frac{4}{1}\right) + 4$

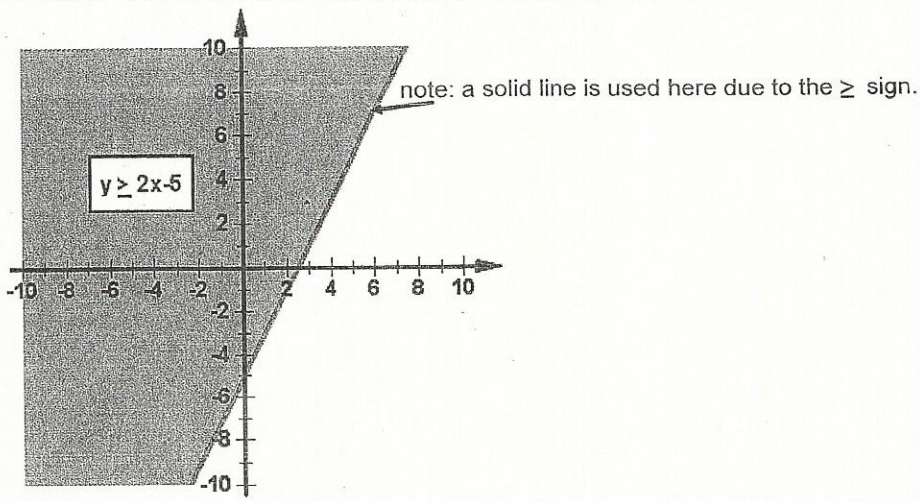
$= -2 + 4 = 2$

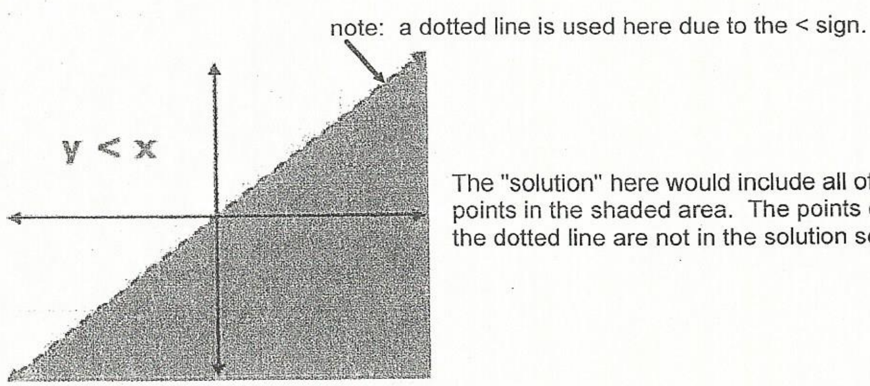
$\boxed{\text{Solution: } (4, 2)}$

*Graphing Inequalities* : An inequality contains all of the (x,y) points that meet the conditions of an equation that has a <, >, ≤ or ≥ sign in it.

Below are two examples of single inequalities which are graphed:

All of the (x,y) points in the pink area meet the conditions of the equation $y \geq 2x - 5$.
So, the "solution" is the entire area which is shaded pink, including the points on the red line (solid).




The "solution" here would include all of the points in the shaded area. The points on the dotted line are not in the solution set.

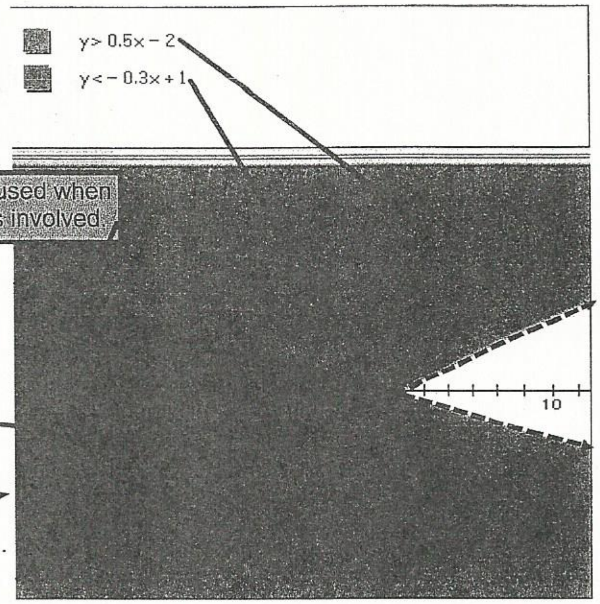
Now we'll look at graphing a <u>system</u> of inequalities.
(2 inequalities on one graph)

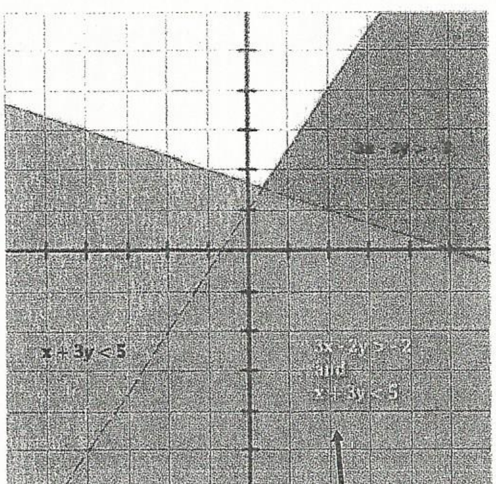
Note: A dotted line is used when the < or > sign is involved.

The (x,y) points in the overlapped-shaded area "work" in both inequality equations. That's why these points are said to be the "solution" set for this system of inequalities.

The solution is the overlapped area!

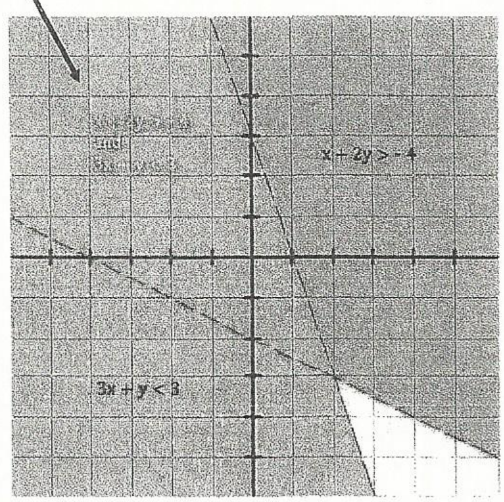
Solution is the entire rust-colored area here...



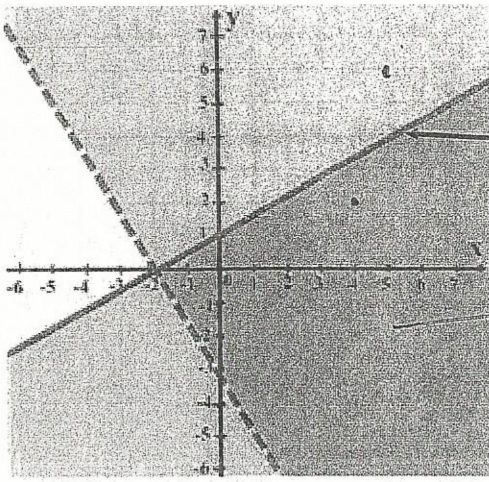




Solution area
(tan-colored)

Solution area (violet-colored)



1. Consider the following system of inequalities:



Solid line is used for $\geq$ and $\leq$ signs.

Solution area

Are the following points in the solution set?

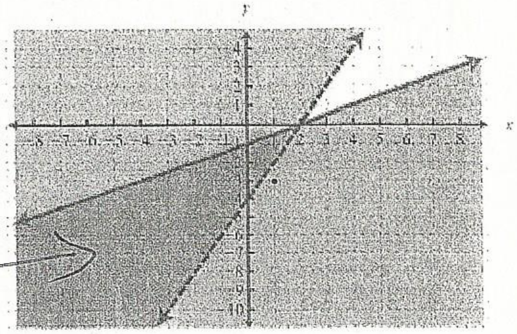
a) ( 5, 6 ) NO

b) ( 4, 2 ) YES

c) ( -2, 0 ) NO (due to dotted line)

d) ( 2, 2 ) YES (due to solid line)

2. Consider the following system of inequalities:



Solution area

Are the following points in the solution set?

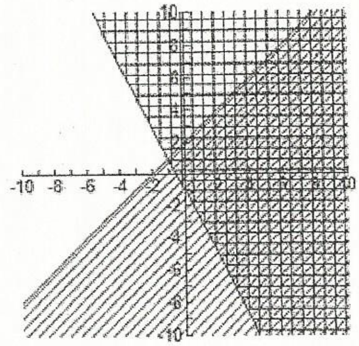
a) ( -5, -2 ) NO

b) (-2, -3 ) YES

c) ( 2, 0 ) NO (due to dotted line)

d) ( 1, -3 ) NO

3. Consider the following system of inequalities:

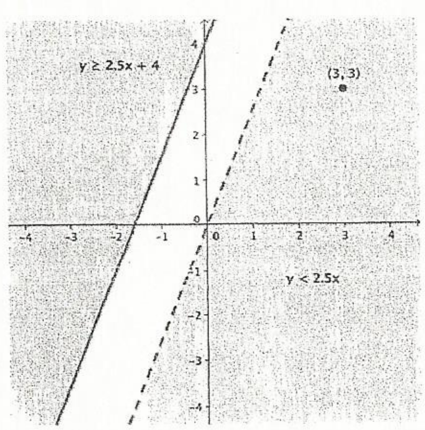


Are the following points in the solution set?

a) ( 3, 2 ) YES  c) ( 0, -1 ) YES

b) ( -2, 2 ) NO  d) ( 2, -6 ) NO

4. Consider the following system of equations:




Is ( 3, 3 ) in the solution set? NO

$\boxed{\text{NO SOLUTION}}$

( no overlapped area! )

## Graph the following systems of inequalities:

A) 1) $3x + 5y > 10$

2) $2x + y \leq 2$

① $3x + 5y > 10$

$\frac{5y}{5} > -\frac{3x}{5} + \frac{10}{5}$

dotted

$y > -\frac{3}{5}x + 2$

Find Line : $y = -\frac{3}{5}x + 2$

| x | y |
|---|---|
| 0 | 2 |
| 5 | -1 |

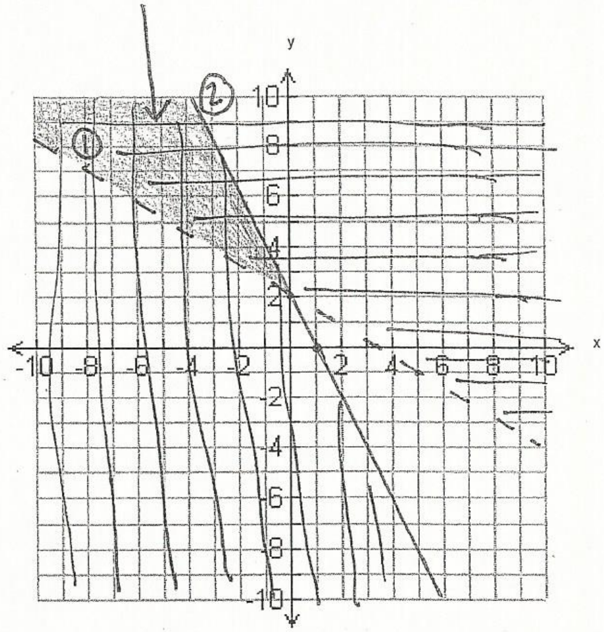
② $2x + y \leq 2$ solid Line

$y \leq -2x + 2$

Find Line : $y = -2x + 2$

| x | y |
|---|---|
| 0 | 2 |
| 1 | 0 |

Solution area (cross-hatched)



B) 1) $5x + 4y < 16$

2) $3x \geq 3$

② $\frac{3x}{3} \geq \frac{3}{3}$ Solid Line

$x \geq 1$

$x = 1$ is a vertical line

① $5x + 4y < 16$

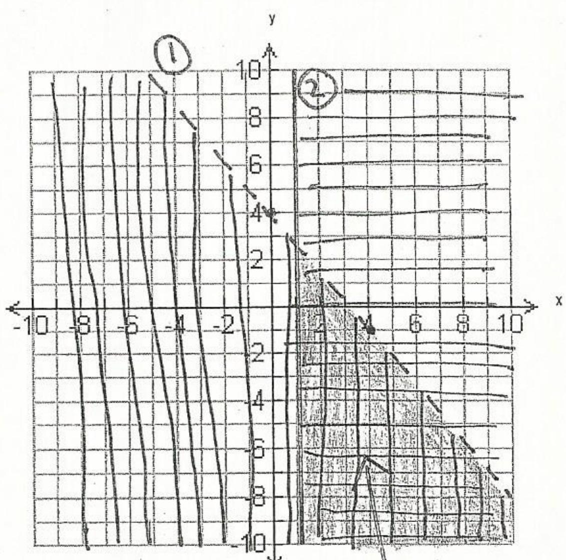
$\frac{4y}{4} < \frac{-5x}{4} + \frac{16}{4}$

dotted

$y < -\frac{5}{4}x + 4$

Find Line : $y = -\frac{5}{4}x + 4$

| x | y |
|---|---|
| 0 | 4 |
| 4 | -1 |




Solution area

C) 1) $\frac{y}{6} + \frac{2}{3} \geq \frac{-1}{2}$

2) $2x - 4y > 0$

(1) $\frac{1}{6}y + \frac{2}{3} \geq -\frac{1}{2}$

$\frac{1}{6}y \geq \frac{-1 \cdot 3}{2 \cdot 3} - \frac{2 \cdot 2}{3 \cdot 2}$

$\frac{1}{6}y \geq -\frac{3}{6} - \frac{4}{6}$

$\frac{6}{1}\left(\frac{1}{6}y\right) \geq \left(\frac{-7}{6}\right)\frac{6}{1}$

$y \geq -7$

solid line

(2) $2x - 4y > 0$

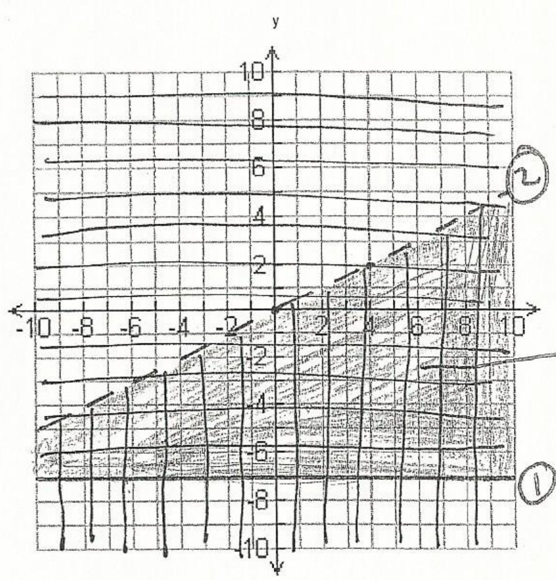
$\frac{-4y}{-4} > \frac{-2x}{-4}$

dotted line

$y < \frac{1}{2}x$

Find Line $y = \frac{1}{2}x$

| x | y |
|---|---|
| 0 | 0 |
| 4 | 2 |




Solution area

D) 1) $\frac{5x}{3} - 4 < 11$

2) $2x - 3y \le 6$

① $\frac{5}{3}x - 4 < 11$

$\frac{5}{3}x < 11 + 4$

$\frac{3}{5}\left(\frac{5}{3}x\right) < (15)\frac{3}{5}$

dotted

$x < 9$

Vertical line

② $2x - 3y \le 6$

$\frac{-3y}{-3} \le \frac{-2x}{-3} \frac{+6}{-3}$

solid

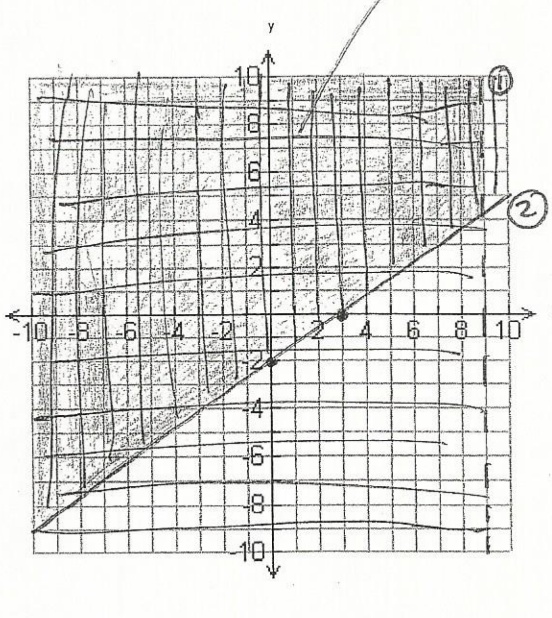
$y \ge \frac{2}{3}x - 2$

Sign flip!

Find Line for $y = \frac{2}{3}x - 2$ :

| x | y |
|---|---|
| 0 | -2 |
| 3 | 0 |

Solution area



E) 1) $12x - 4y \le 0$

2) $\frac{y}{6} + \frac{y}{3} > \frac{1}{2}$

① $12x - 4y \le 0$

$$\frac{-4y}{-4} \le \frac{-12x}{-4}$$

$$y \ge 3x$$

← solid

Sign flip!

Find Line for $y = 3x$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |

② $\frac{1}{6}y + \frac{1}{3}y > \frac{1}{2}$

$$\frac{1}{6}y + \frac{1 \cdot 2}{3 \cdot 2}y > \frac{1}{2}$$

$$\frac{1}{6}y + \frac{2}{6}y > \frac{1}{2}$$

$$\frac{3}{6}y > \frac{1}{2}$$

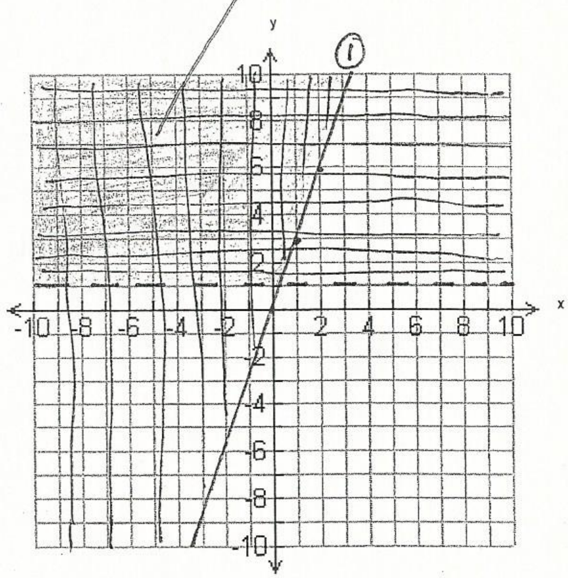
$$\frac{2}{1}\left(\frac{1}{2}y\right) > \left(\frac{1}{2}\right)\frac{2}{1}$$

$$y > 1$$

↑ dotted

horizontal line

Solution Area



F) 1) $\frac{x}{3} - \frac{y}{6} - \frac{1}{2} \geq 0$

2) $-4x - y > 0$

1 $\frac{1}{3}x - \frac{1}{6}y - \frac{1}{2} \geq 0$

$\frac{-6}{1}\left(-\frac{1}{6}y\right) \geq \left(-\frac{1}{3}x + \frac{1}{2}\right)\frac{-6}{1}$

$y \leq 2x - 3$ ← solid

Find Line for $y = 2x - 3$

| x | y |
|---|---|
| 0 | -3 |
| 3 | 3 |

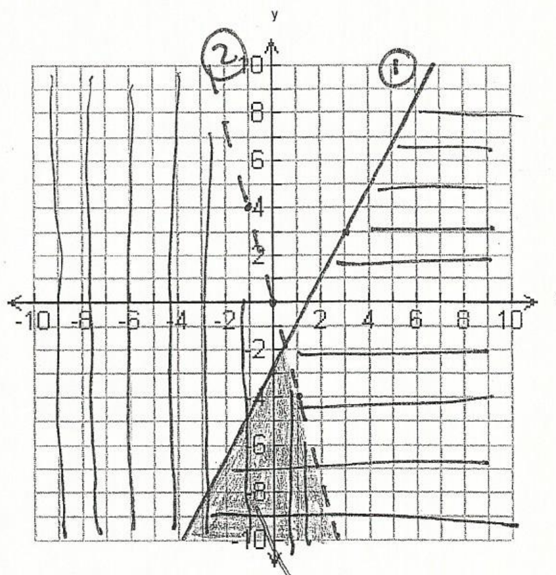
2 $-4x - y > 0$

$\frac{-y}{-1} > \frac{4x}{-1}$

$y < -4x$ ← dotted

$y = -4x$

| x | y |
|---|---|
| 0 | 0 |
| 1 | -4 |
| -1 | 4 |




Solution Area

G) 1) $3y - 2x \geq 6$

2) $2 + \frac{x}{3} - \frac{y}{2} < 0$

① $3y - 2x \geq 6$

$$\frac{3y}{3} \geq \frac{2x}{3} + \frac{6}{3}$$

solid

$$y \geq \frac{2}{3}x + 2$$

Find line for $y = \frac{2}{3}x + 2$

| x | y |
|---|---|
| 0 | 2 |
| -3 | 0 |

② $2 + \frac{1}{3}x - \frac{1}{2}y < 0$

$$-\frac{2}{1}\left(-\frac{1}{2}y\right) < \left(-\frac{1}{3}x - 2\right)-\frac{2}{1}$$

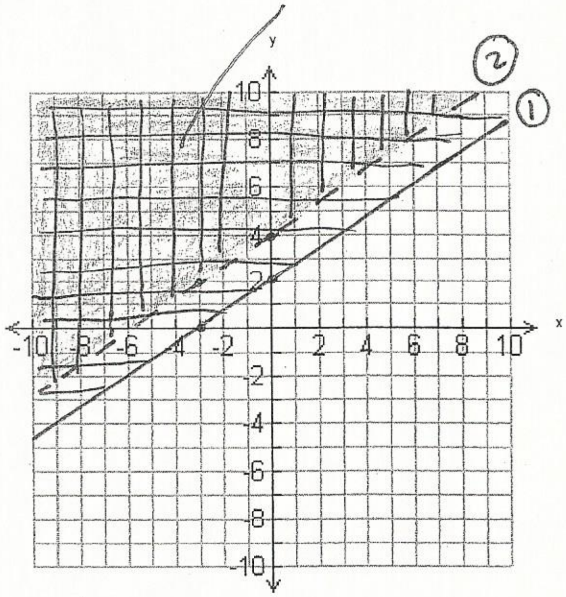
dotted

$$y > \frac{2}{3}x + 4$$

Find Line for $y = \frac{2}{3}x + 4$

| x | y |
|---|---|
| 0 | 4 |
| -3 | 2 |

Solution area



H) 1) $\frac{x}{12} - \frac{x}{3} - \frac{1}{6} > \frac{1}{3}$

2) $1 + y > 0$

1 $\frac{1}{12}x - \frac{1}{3}x - \frac{1}{6} > \frac{1}{3}$

$\frac{1}{12}x - \frac{1 \cdot 4}{3 \cdot 4}x > \frac{1 \cdot 2}{3 \cdot 2} + \frac{1}{6}$

$\frac{1}{12}x - \frac{4}{12}x > \frac{2}{6} + \frac{1}{6}$

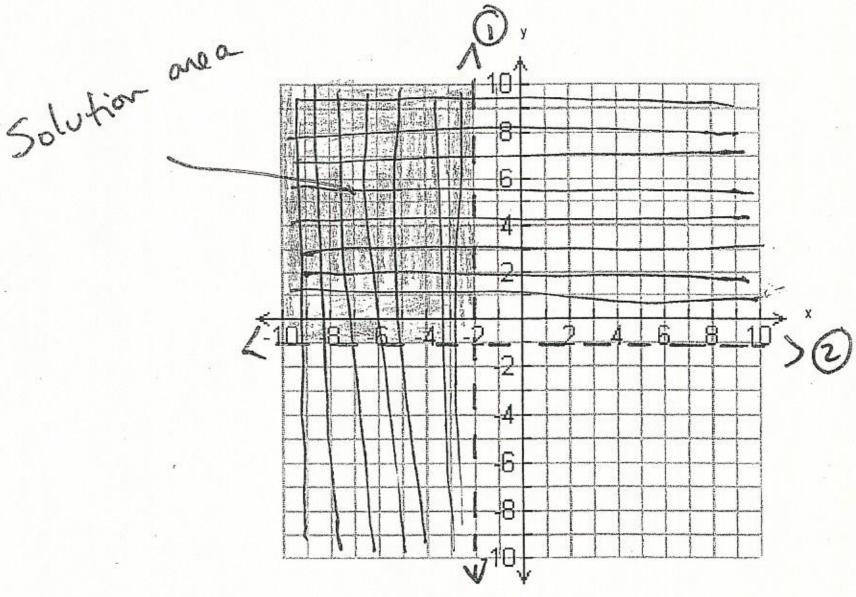
$-\frac{3}{12}x > \frac{3}{6}$

$-\frac{4}{1}\left(-\frac{1}{4}x\right) > \left(\frac{1}{2}\right)\frac{-4}{1}$

$x < -2$

2 $1 + y > 0$

$y > -1$



I) 1) $2x - y \le 6$

2) $5x + 3y < 12$

1 $2x - y \le 6$

$$\frac{-y}{-1} \le \frac{-2x}{-1} \frac{+6}{-1}$$

$$y \ge 2x - 6$$

Find line $y = 2x - 6$ :

| x | y |
|---|---|
| 0 | -6 |
| 3 | 0 |

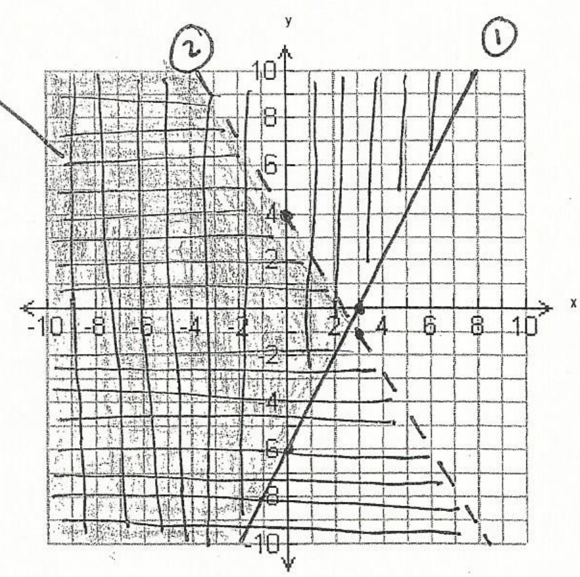
2 $5x + 3y < 12$

$$\frac{3y}{3} < \frac{-5x}{3} + \frac{12}{3}$$

$$y < -\frac{5}{3}x + 4$$

| x | y |
|---|---|
| 0 | 4 |
| 3 | -1 |

Solution area



J) 1) $-2y + 4x < 0$

2) $\frac{1}{6} + \frac{y}{2} \leq \frac{2}{3}$

1. $-2y + 4x < 0$

$$\frac{-2y}{-2} < \frac{-4x}{-2}$$

$$y > 2x$$

Find line $y = 2x$

| x | y |
|---|---|
| 0 | 0 |
| 2 | 4 |

2. $\frac{1}{6} + \frac{1}{2}y \leq \frac{2}{3}$

$$\frac{1}{2}y \leq \frac{2 \cdot 2}{3 \cdot 2} - \frac{1}{6}$$

$$\frac{1}{2}y \leq \frac{4}{6} - \frac{1}{6}$$

$$\frac{1}{2}y \leq \frac{3}{6}$$

$$\frac{2}{1}\left(\frac{1}{2}y\right) \leq \left(\frac{1}{2}\right)\frac{2}{1}$$

$$y \leq 1$$