Math 4109 Quiz 1

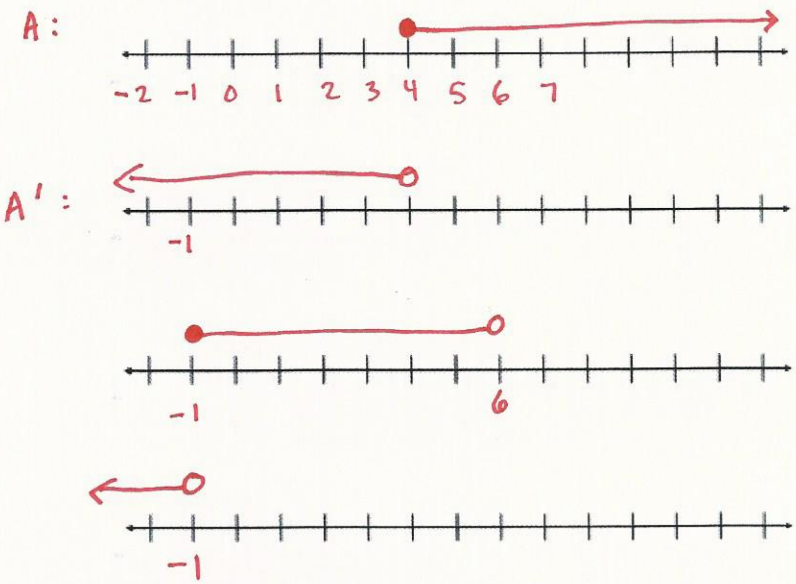
Name: Shanna

1. $A = \{x \in \mathbb{R} \mid x \geq 4\}$

   $B = \{x \in \mathbb{R} \mid \mathbf{-1 \leq x < 6}\}$

Perform the following set operations: A'\ B

Graph the detailed operation below.



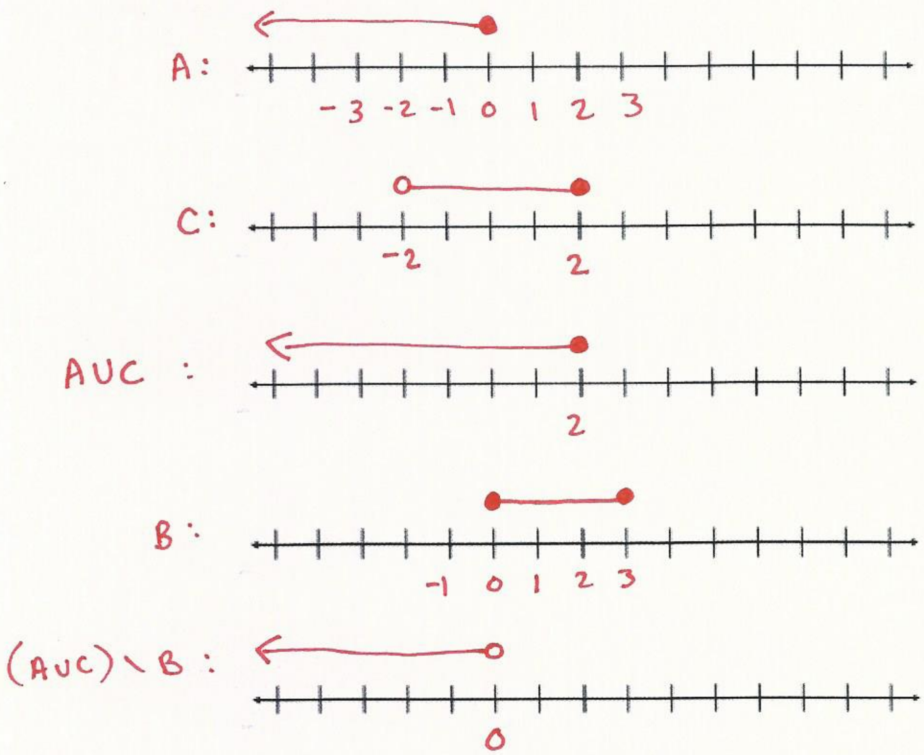
Give your answer in interval form: $-\infty, -1[$

2. A = -∞, 0]

   B = [0,3]

   C =]-2,2]

Perform the following set operations: $(A \cup C) \setminus B$

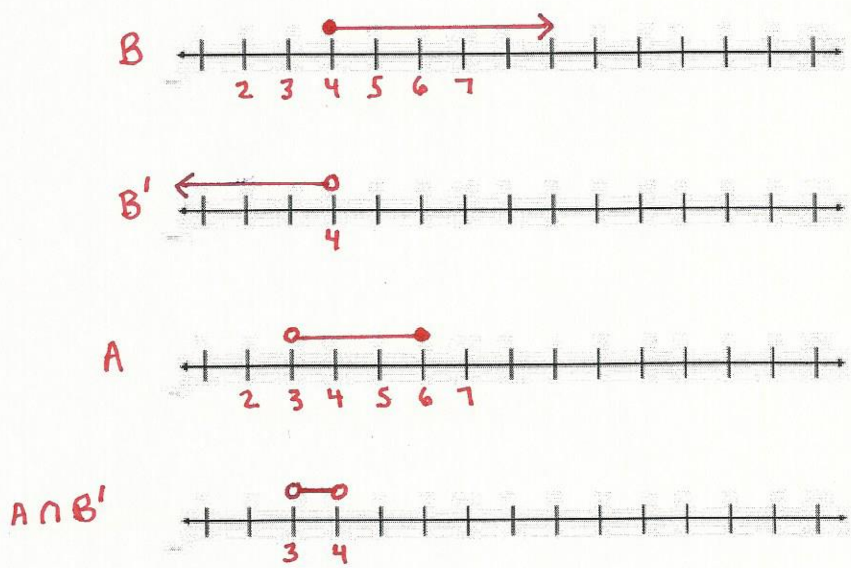


Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid x < 0\}$

3. $A = \{x \in \mathbb{R} \mid 3 < x \leq 6\}$

$B = \{x \in \mathbb{R} \mid x \geq 4\}$

Perform the following set operations: $A \cap B'$



Give your answer in interval notation:]3, 4[

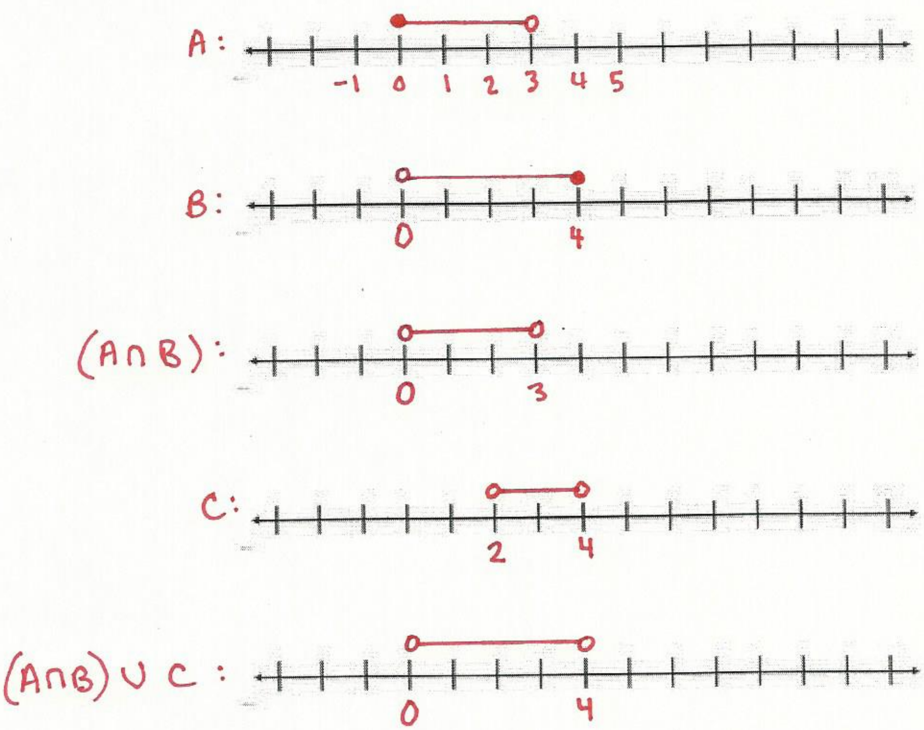
4. $A = [0,3[$

$B =]0,4]$

$C =]2,4[$

Perform the following set operations: $(A \cap B) \cup C$

Graph the detailed operation below:



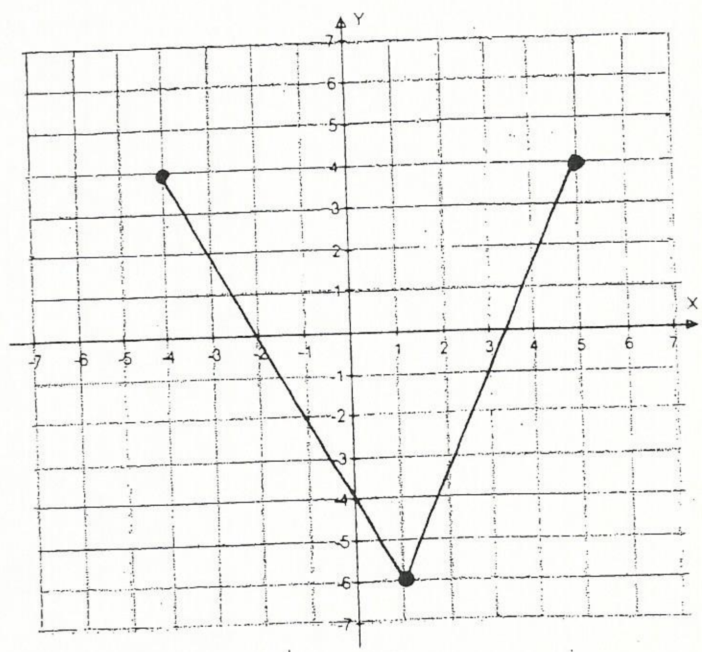
Give your answer in set-builder notation: $\{x \in \mathbb{R} \mid 0 < x < 4\}$

MTH-4109-1 Sets, Relations and Functions Quiz

Name:

Date:

(1) The following graph represents functional situation f .



Indicate whether each of the following statements is true or false.

a) The function has a minimum and two maximums. F

b) The domain is [-6,4]. F

c) The function has no axis of symmetry. T

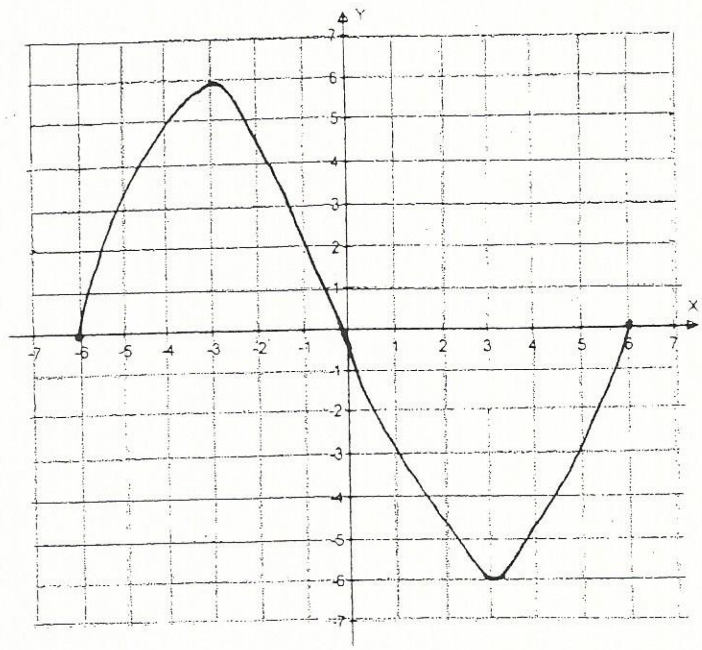
d) The y-intercept is -6 . F

MTH-4109-1 Sets, Relations and Functions Quiz

Name: Shannon

Date:

2. The following graph represents functional situation g.



Indicate whether each of the following statements is true or false.

a) Function g is both decreasing and positive over the interval $[-3, 0]$. T

b) $g(-6) = g(0) = g(6)$ T

c) The range and the domain of the function are $[-6, 6]$. T

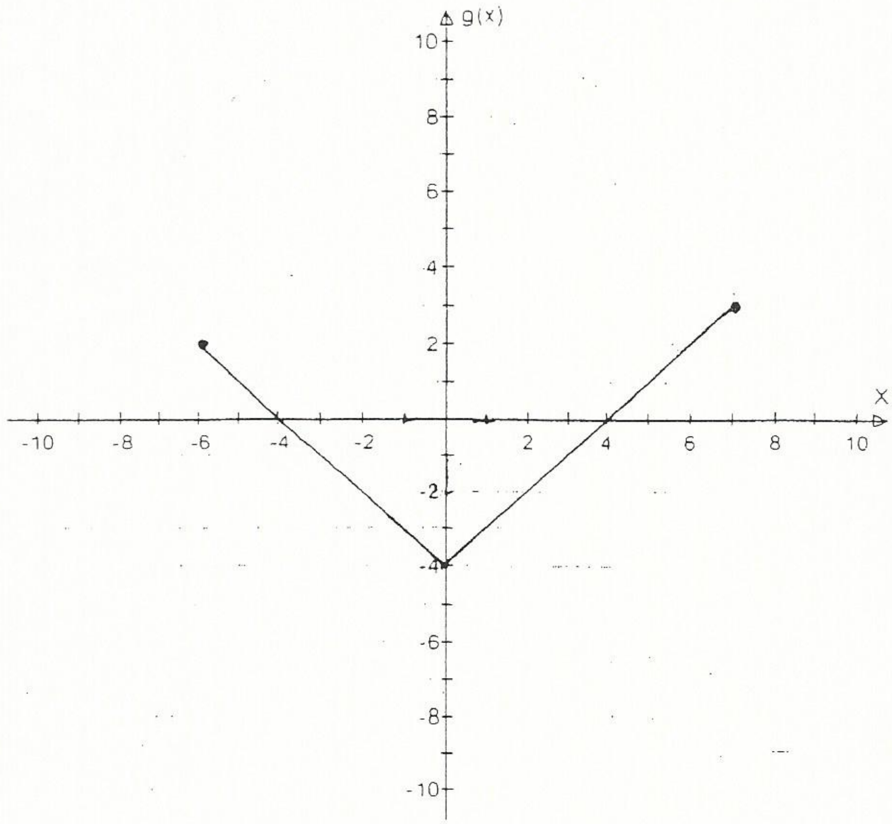
d) The values $x = -3$ and $x = 3$ are the x-intercepts of this function. F

MTH-4109-1 Sets, Relations and Functions Quiz

Name: Shannon

Date:

3 The following graph represents functional situation g.



Determine the following characteristics of this function.

a) Domain: [-6, 7]

b) Range: [-4, 3]

c) Zero(s): -4 and 4

d) The minimum of g: -4

e) The interval over which the function is both increasing and negative:

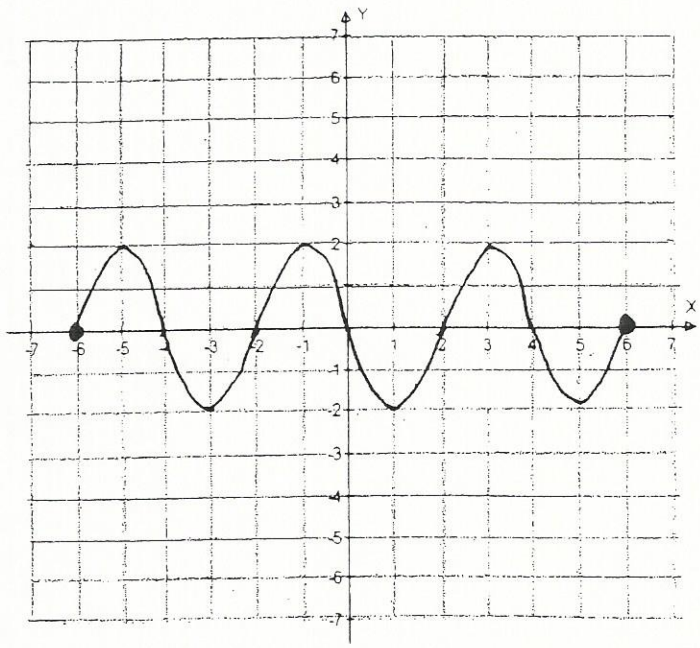
[0, 4]

MTH-4109-1 Sets, Relations and Functions Quiz

Name:

Date:

4 The graph below represents functional situation $f(x)$. Determine the following characteristics of this function.



a) Domain: $[-6, 6]$

b) Range: $[-2, 2]$

c) An interval over which the function is both decreasing and positive: $[-5, -4]$ OR $[-1, 0]$ OR $[3, 4]$

d) $f(2) =$ 0

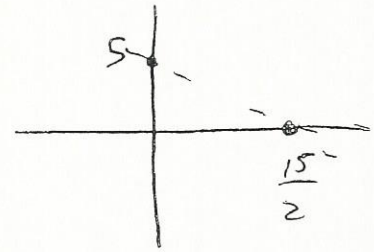
e) The maximum of $f(x)$: 2

MTH-4109-1 Sets, Relations and Functions Quiz

Name: Shannon

Date:

1. A function is described by the following rule: $f(x) = \frac{-2x}{3} + 5$.



a) Determine over which interval this function is positive.

Answer: $x \in -\infty, \frac{15}{2}]$

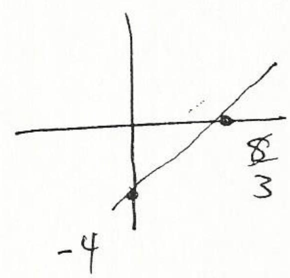
b) Determine the rate of change of this function.

Answer: $\frac{-2}{3}$

$$\left(\frac{3}{2}\right)\frac{2}{3}x = 5\left(\frac{3}{2}\right)$$

$$x = \frac{15}{2}$$

2. A function is described by the following rule: $f(x) = \frac{3x}{2} - 4$.



a) Determine over which interval this function is negative.

Answer: $x \in -\infty, \frac{8}{3}]$

b) Determine the rate of change of this function.

Answer: $\frac{3}{2}$

$$\left(\frac{-2}{3}\right)\frac{-3}{2}x = -4\left(\frac{-2}{3}\right)$$

$$x = \frac{8}{3}$$

3. A function is described by the following rule: $f(x) = -x^2 + 9$

$9 - x^2$

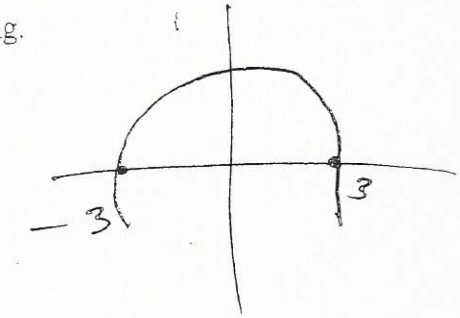
$(3 - x)(3 + x)$

a) Determine the interval over which this function is positive.

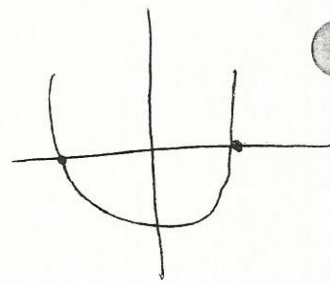
Answer: $x \in [-3, 3]$

b) Determine the interval over which this function is decreasing.

Answer: $x \in [0, \infty$



4. A function is described by the following rule: $f(x) = x^2 - 9$



a) Determine the interval over which this function is ~~positive.~~ negative

Answer: $x \in [-3, 3]$

b) Determine the interval over which this function is decreasing.

Answer: $x \in -\infty, 0]$

$(x - 3)(x + 3)$

Name:

1. Consider the following statements A - O. After examining the eight graphs which follow, print each letter next to each graph to which it corresponds. The same letter may definitely be used more than once!

A. It has a maximum.

B. It has only one zero, which is negative.

C. It is decreasing over its entire domain.

D. It is decreasing if $x \in -2, \infty]$.

E. The function is negative if $x \in [0, 2]$.

F. The equation of the axis of symmetry is $x = h$ and $h > 0$.

G. The function has two zeros, one of which is the y-intercept.

H. It has two zeros, one which is negative and one positive.

I. It is increasing over its entire domain.

J. It is increasing if $x \in -\infty, -2]$.

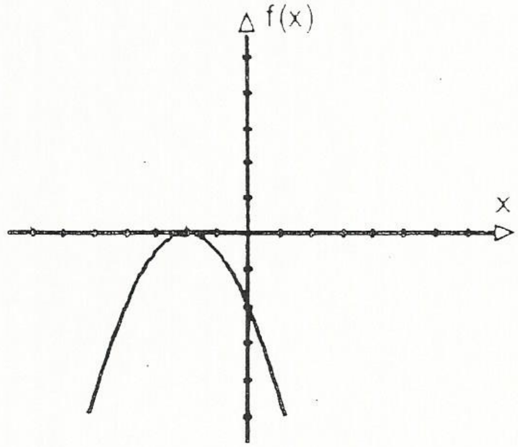
K. The range of the function is $[-3, \infty$.

L The equation of the axis of symmetry is $x = h$ and $h < 0$.

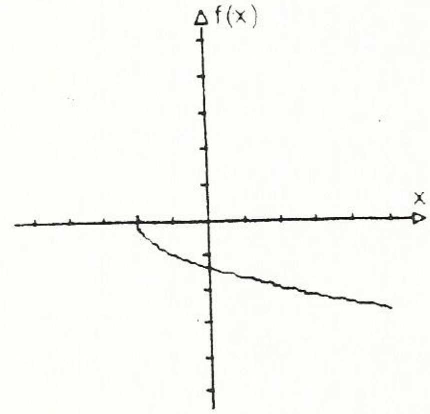
M. The equation of the axis of symmetry is $x = h$ and $h = 0$.

N. The range of this function is $-\infty, 2]$.

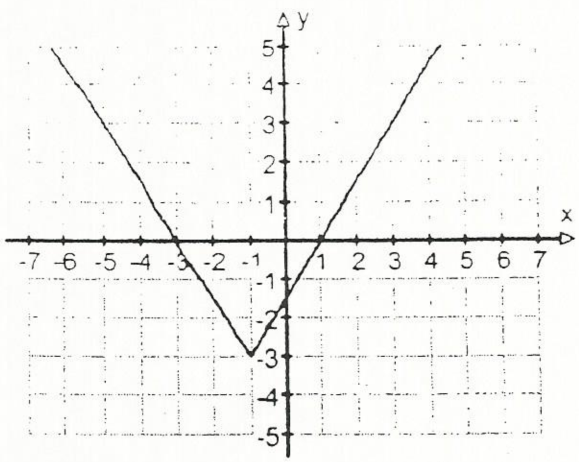
O. The domain of this function is $[-2, \infty$.



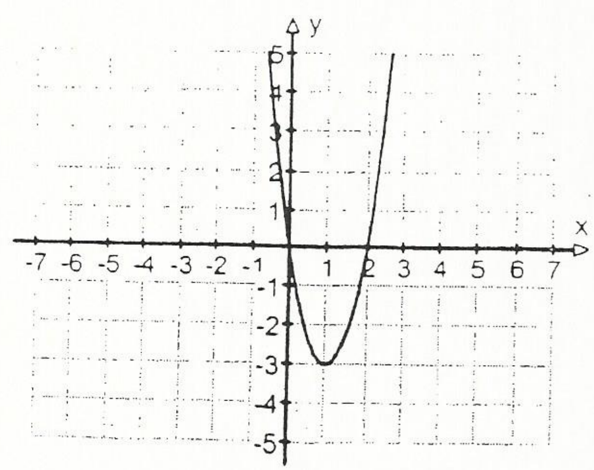
1. A, B, D, E, J, L



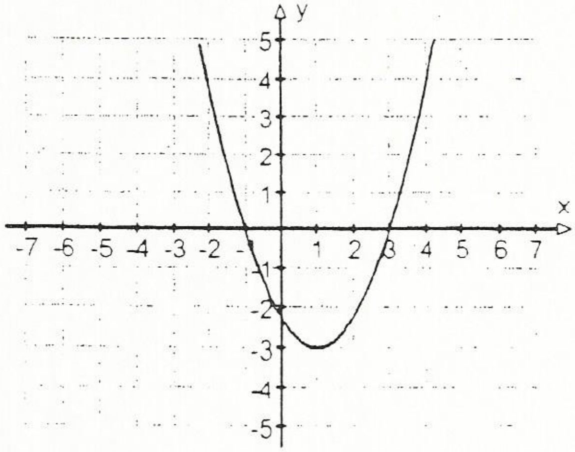
2. A, B, C, D, E, O



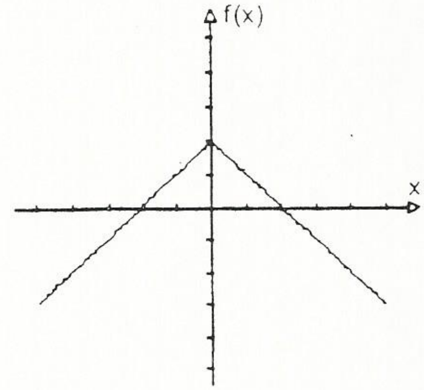
3. H, K, L



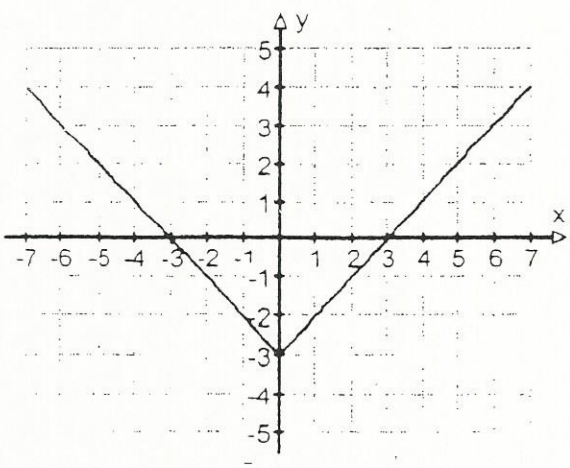
4. E, F, G, K



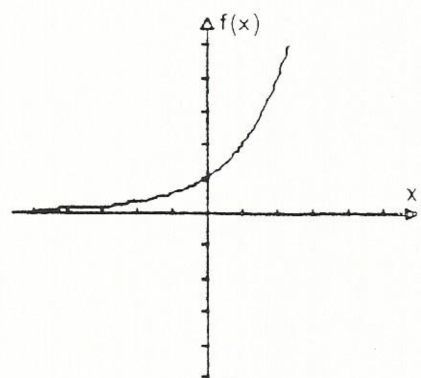
5. E, F, H, K



6. A, H, J, M, N



7. E, H, K, M



8. B, I, J

MTH-4109-1 Sets, Relations and Functions Quiz

Name:

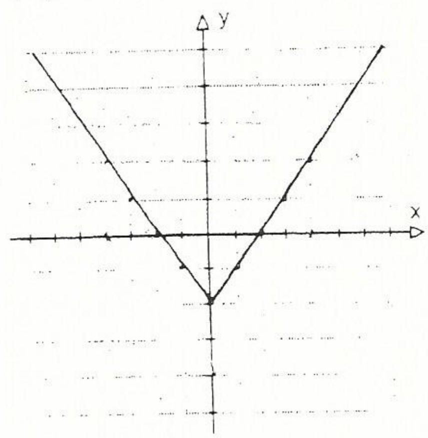
Date:

Six representations are given below.

A

$k(x) = |x| - 2$

B $j(x)$



C

x	$i(x)$
−4	2
−3	1
−2	0
−1	−1
0	−2
1	−1
2	0
3	1

D

x	$h(x)$
−4	−6
−3	−5
−2	−4
−1	−3
0	−2
1	−1
2	0
3	1

E $\frac{f(x)}{1} = x - 2$

F $g(x)$ = The image of an element is obtained by subtracting 2 from the square of this element.

Three of these representations correspond to the same function f_1 and two of them correspond to another function f_2.

Indicate which representations correspond to each function.

f_1: A, B, C

f_2: D, E

Six representations are given below.

A

$f(x) = |x| - 2$

B

x	g(x)
-3	-5
-2	-4
-1	-3
0	-2
1	-1
2	0
3	1
4	2

C

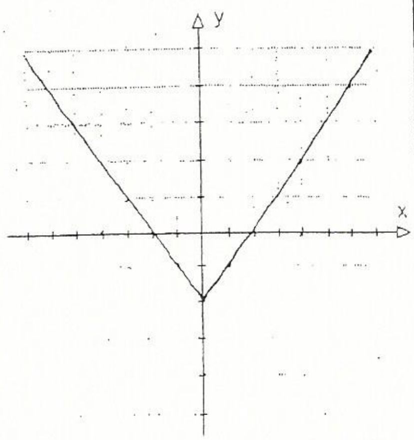
x	h(x)
-3	1
-2	0
-1	-1
0	-2
1	-1
2	0
3	1
4	2

D

$i(x)$ = The image of an element is obtained by subtracting 2 from the square of this element.

E

$j(x)$



F

$k(x) = x - 2$

Three of these representations correspond to the same function f_1 and two of them Correspond to another function f_2.

Indicate which representations correspond to each function.

A C E

f_1 : f(x), h(x), j(x)

f_2 : g(x), K(x)

B F

MTH-4109 Quiz

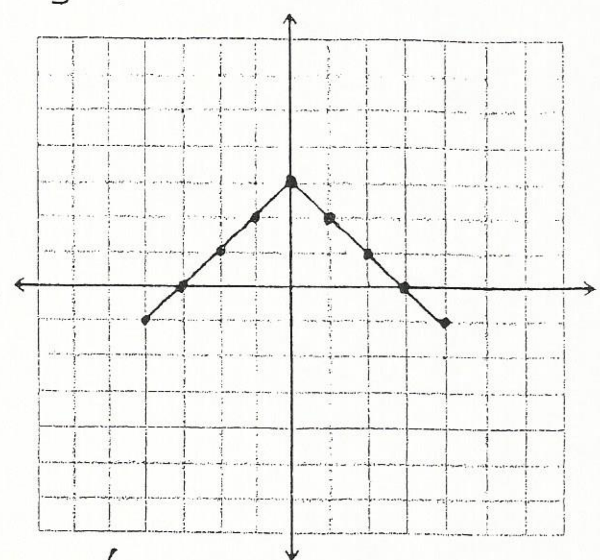
Name: Shannon

1. Six representations are given below.

A. f(x) = the image of an element is obtained by adding three to the element.

$Y = x+3$

B. $g(x)$



C. $h(x) = 3 - |x|$

D.

x	i(x)
-4	7
-3	6
-2	5
-1	4
0	3
1	2
2	1
3	0

E. $j(x) = 3 - x$

F.

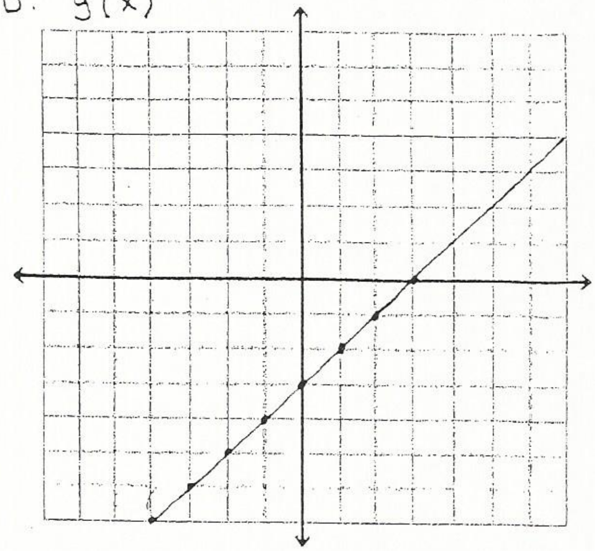
x	K(x)
-4	-1
-3	0
-2	1
-1	2
0	3
1	2
2	1
3	0
4	-1

Three of these representations correspond to the same function f_1 and two of them correspond to another function f_2.
Indicate which representations correspond to each function.

f_1 : B C F

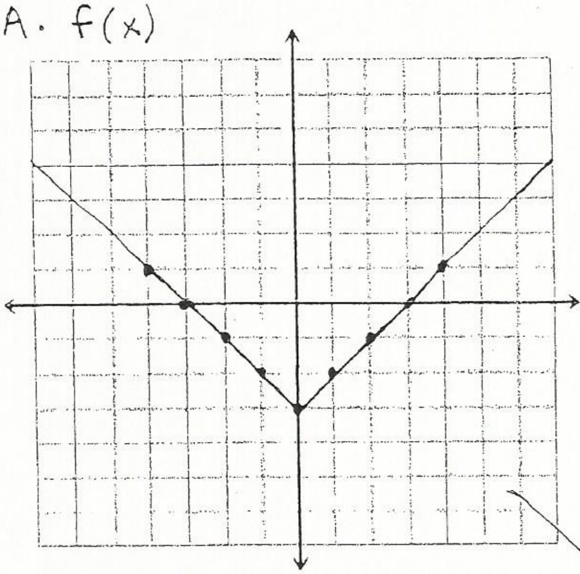
f_2 : D E

B. $g(x)$



2. Six representations are given below.

A. $f(x)$



C. $\frac{h(x)}{1} = \frac{X-3}{1}$

D. $i(x) = |x| - 3$

E.

x	$j(x)$
−4	1
−3	0
−2	−1
−1	−2
0	−3
1	−2
2	−1

F. $k(x)$ = the image of an element is obtained by subtracting three from the square of an element.

Three of these representations correspond to the same function f_1 and two of them correspond to another function f_2.
Indicate which representations correspond to each function.

f_1 : ADE

f_2 : B, C

MTH-4109-1 Sets, Relations and Functions Quiz

Name: Shanna

Date:

$-\frac{1}{2}y > 1 - \frac{1}{4}y$

$-\frac{1}{2}y + \frac{1}{4}y > 1$

$-\frac{2}{4}y + \frac{1}{4}y > 1$

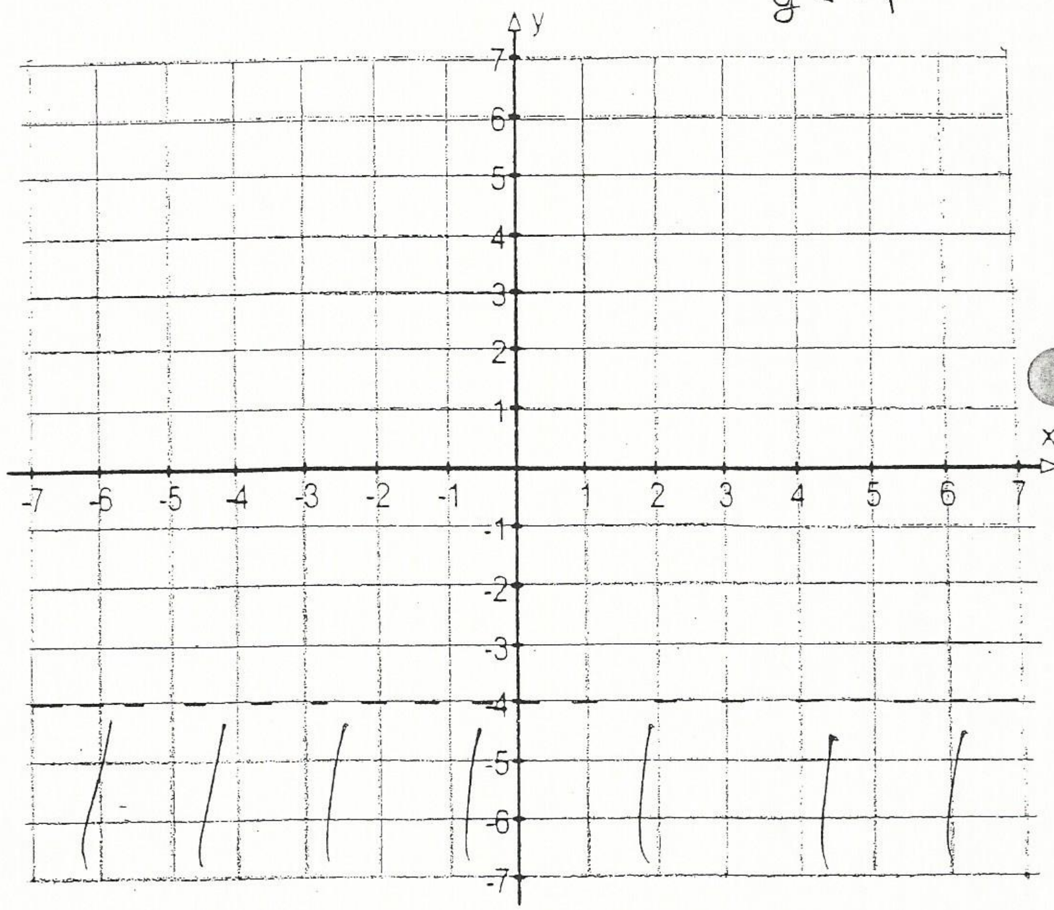
$\left(\frac{-4}{1}\right) -\frac{1}{4}y > 1(-4)$

$y < -4$

1. Graph the following relation in a Cartesian plane:

$$S = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid -\frac{y}{2} > 1 - \frac{y}{4}\right\}$$

Then determine its domain and range.

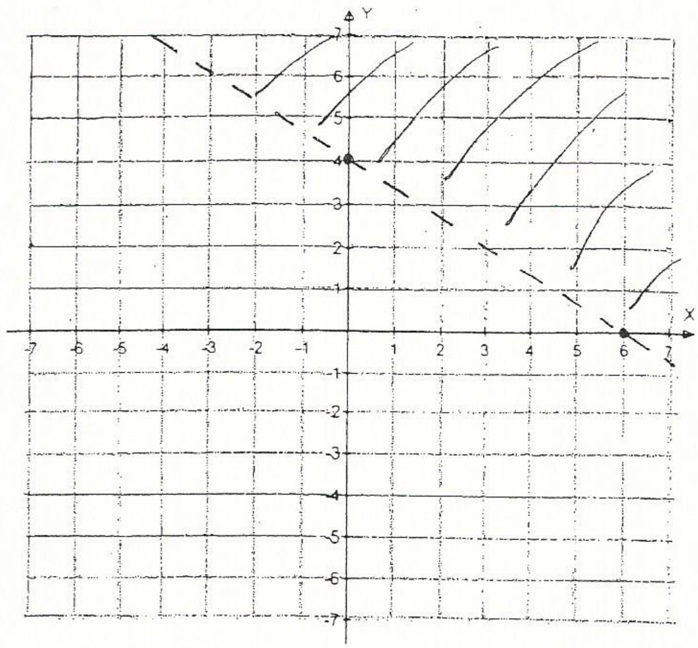


Domain: $\mathbb{R}$

Range: $-\infty, -4[$

2. Graph the following relation in a Cartesian plane:

$$R = \left\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid \frac{x}{6} - 1 > \frac{-y}{4}\right\}$$



Then determine its domain and range.

Domain: $\mathbb{R}$ Range: $\mathbb{R}$

$\frac{1}{6}x - 1 > -\frac{1}{4}y$

$\left(\frac{4}{1}\right)\frac{1}{4}y > \left(-\frac{1}{6}x + 1\right)\frac{4}{1}$

$y > -\frac{4}{6}x + 4$

$y > -\frac{2}{3}x + 4$

$0 = -\frac{2}{3}x + 4$

$\left(\frac{3}{2}\right)\frac{2}{3}x = \frac{4}{1}\left(\frac{3}{2}\right)$

$x = \frac{12}{2} = 6$

$0 > 0 + 4$

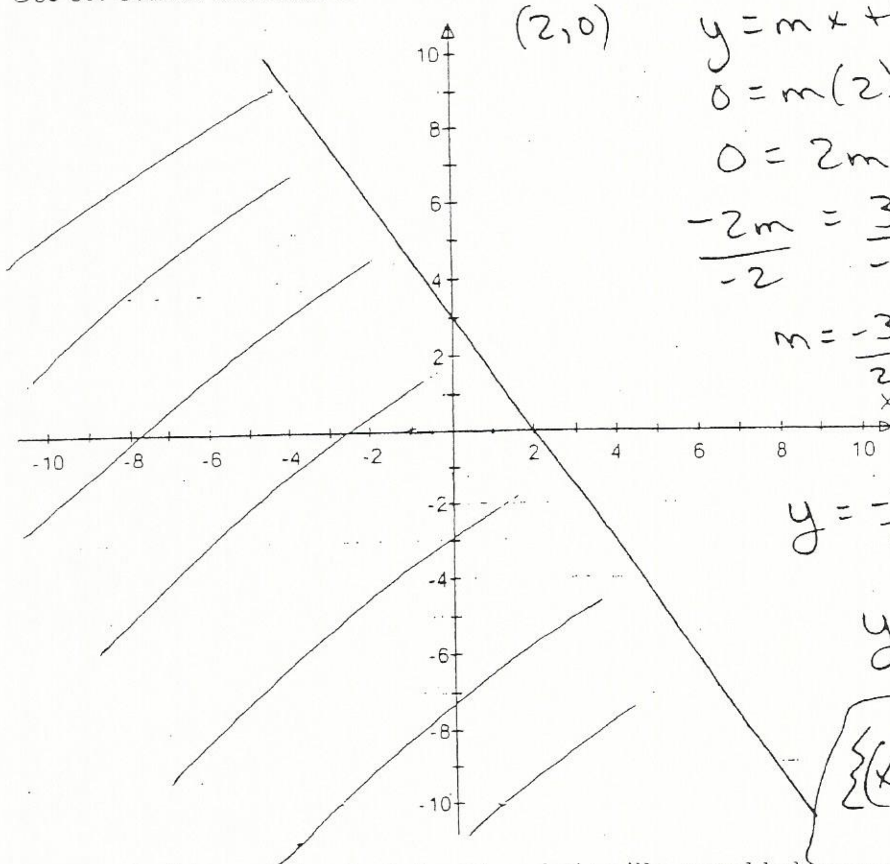
$0 > 4$ NO

MTH-4109-1 Sets, Relations and Functions Quiz

Name: Shannon

Date:

1. Use set-builder notation to define the relation illustrated below.



$(2,0)$

$y = mx + b$

$0 = m(2) + 3$

$0 = 2m + 3$

$\frac{-2m}{-2} = \frac{3}{-2}$

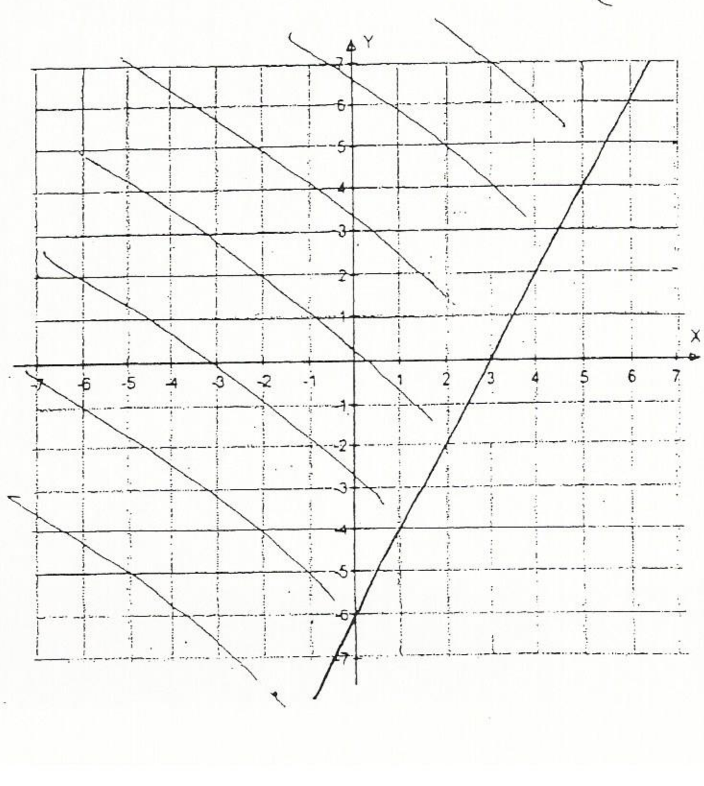
$m = \frac{-3}{2}$

$y = \frac{-3}{2}x + 3$

$y < \frac{-3}{2}x + 3$

$\{(x,y) \in \mathbb{R}\times\mathbb{R} \mid y \leq \frac{-3}{2}x \cdot$

2. Use set-builder notation to define the relation illustrated below.



$(3,0)$

$y = mx + b$

$0 = m(3) + b$

$0 = 3m + b$

$\frac{-3m}{-3} = \frac{-6}{-3}$

$m = 2$

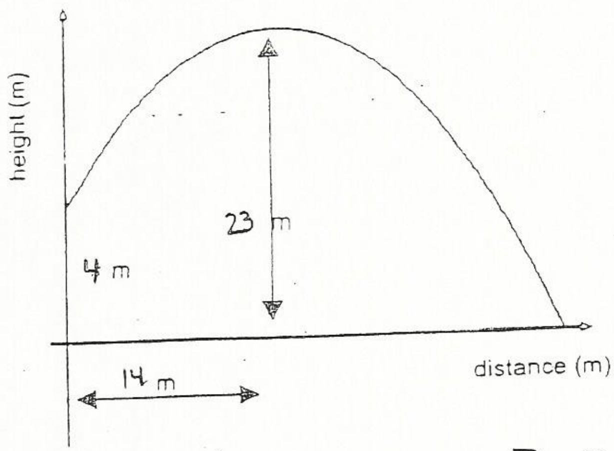
$y = 2x - 6$

$y > 2x - 6$

$\{(x,y) \in \mathbb{R}\times\mathbb{R} \mid y \geq 2x - 6$

Functions Word Problems Quiz — Answers

1) A participant in a javelin-throwing competition throws his javelin from a height of 4 metres. The javelin's trajectory describes a parabola, and the javelin reaches a maximum height of 23 metres. Upon reaching this height, the javelin has travelled a distance of 14 metres from the starting point. Determine the length of the throw. Round off your answer to the nearest hundredth. Clearly show all your work.



Vertex:

$(14, 23)$
$h \quad k$

$(0, 4)$
$x \quad y$
$\uparrow$
$f(x)$

$f(x) = a(x-h)^2 + k$ } formula for parabola

$4 = a(0-14)^2 + 23$

$4 = a(-14)^2 + 23$

$4 = 196a + 23$

$-196a = 23 - 4$

$\frac{-196a}{-196} = \frac{19}{-196}$

$a = -0.097$

formula:

$f(x) = -0.097(x-14)^2 + 23$

Need x-intercept ($y=0$)...

$0 = -0.097(x-14)(x-14) + 23$

$0 = -0.097(x^2 - 28x + 196) + 23$

$0 = -0.097x^2 + 2.716x - 19.012 + 23$

$0 = -0.097x^2 + 2.716x + 3.988$

$a = -0.097$

$b = 2.716$

$c = 3.988$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

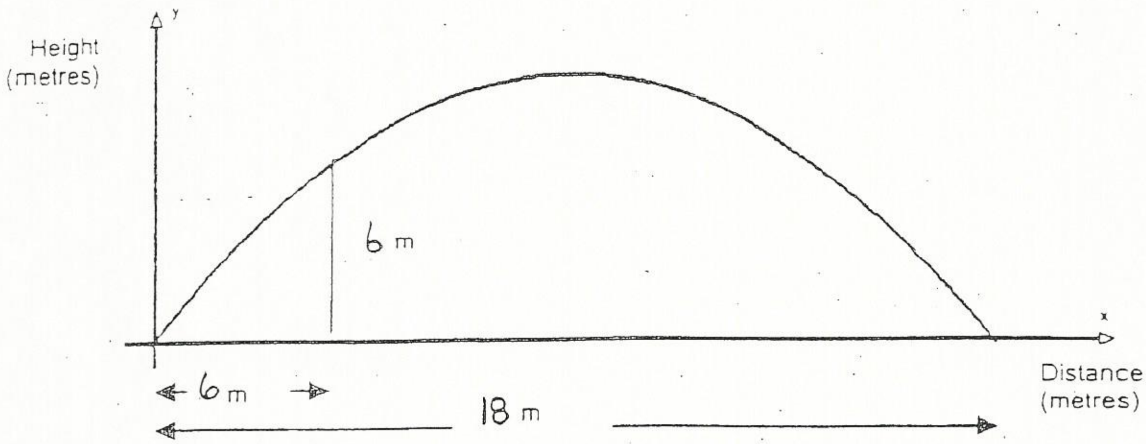
$$= \frac{-2.716 \pm \sqrt{(2.716)^2 - 4(-0.097)(3.988)}}{2(-0.097)}$$

$$= \frac{-2.716 \pm \sqrt{7.377 + 1.547}}{-0.194}$$

$$= \frac{-2.716 \pm 2.987}{-0.194}$$

$$= 29.40\text{m}$$

(2) A baseball is thrown over a 6m high fence. The fence is 6m from the origin of the throw. The ball lands 18m away. If the path of the ball describes a parabola, what is the maximum height reached by the ball?



- the two zeros are $x_1 = 0$ and $x_2 = 18$.

$f(x) = a(x)(x-18)$

$f(x) = a(x^2 - 18x)$ $\quad (x, y) = (6, 6)$

$6 = a(6^2 - 18 \cdot 6)$

$6 = a(36 - 108)$

$\frac{6}{-72} = \frac{a(-72)}{-72}$

$-\frac{6}{72} = a$

$a = -\frac{1}{12}$

$f(x) = -\frac{1}{12}(x^2 - 18x)$

$f(x) = -\frac{1}{12}x^2 + 1\frac{1}{2}x$

At maximum point $x = 9$

$f(x) = -\frac{1}{12}(9)^2 + 1\frac{1}{2}(9)$

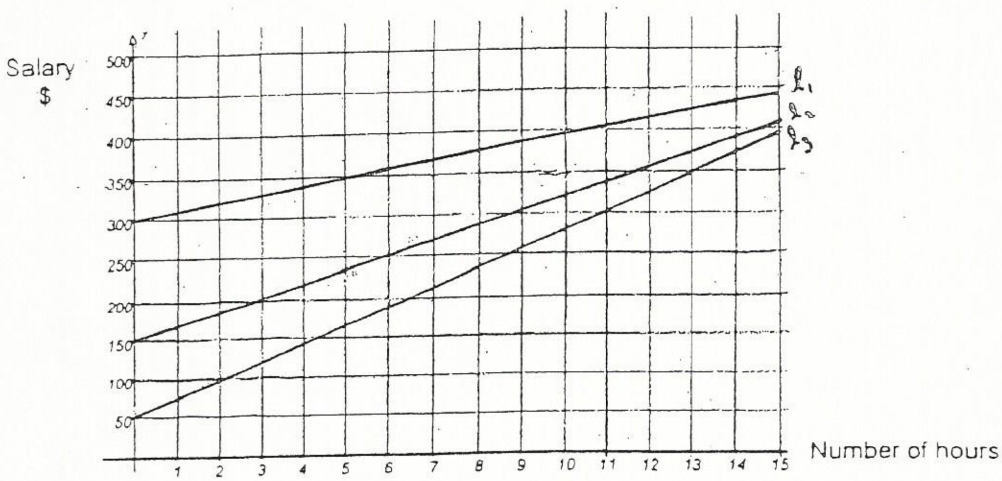
$= -6\frac{3}{4} + 13\frac{1}{2}$

$= 6\frac{3}{4}$ or 6.75

The maximum height is $6\frac{3}{4}$ m or 6.75m.

(3) Three accountants received a bonus that was added to their salary. In the graph below,

- line ℓ_1 represents Alka's salary (including the bonus)
- line ℓ_2 represents Michele's salary (including the bonus)
- line ℓ_3 represents Melissa's salary (including the bonus)



If all three accountants worked 45 hours this week, which one received the highest weekly salary?

Clearly show all your work.

ℓ_1 $(0,300),(5,350)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350-300}{5-0} = \frac{50}{5} = 10$

$\boxed{y = 10x + 300}$

if $x = 45$:

$y = 10(45) + 300$
$= 450 + 300$
$= \$750$

ℓ_2 $(0,150),(3,200)$

$m = \frac{200-150}{3-0} = \frac{50}{3}$

$\boxed{y = \frac{50}{3}x + 150}$

if $x = 45$:

$y = \frac{50}{3}(45) + 150$

$\$ = 900$

ℓ_3 $(0,50),(2,100)$

$m = \frac{100-50}{2-0} = \frac{50}{2} = 25$

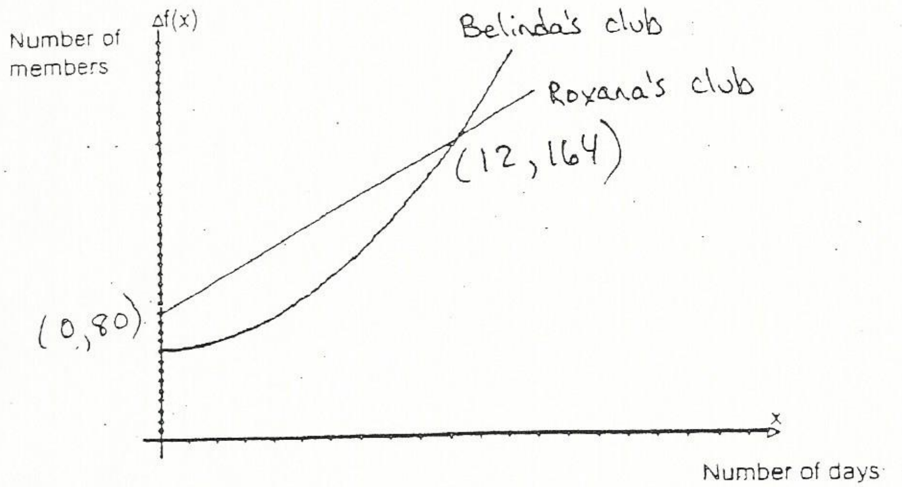
$\boxed{y = 25x + 50}$

if $x = 45$:

$y = 25(45) + 50$
$= 1125 + 50$
$= \$1175$

* Melissa earned the highest salary (\$1175).

(4) Belinda's fitness club and Roxana's fitness club are competing to attract new members. Belinda's club calculates the number of members using the rule $f(x) = 0.5x^2 + 92$, where x represents the number of days. On opening day, Roxana's club has 80 members. On the 12th day the two clubs have the same number of members. The following graphs show the change in the number of members over a one-month period.



Calculate the number of members in Roxana's club on the 28th day. Clearly show all your work.

$$y = 0.5(12)^2 + 92$$
$$= 164$$

$$m = \frac{164 - 80}{12 - 0} = 7$$

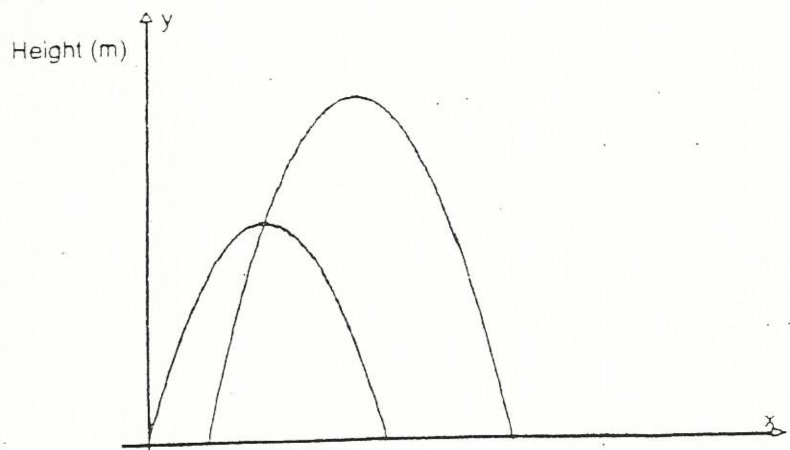
$$y = 7x + 80$$
$$= 7(28) + 80 = 276$$

Two shells were launched during a fireworks display. The height in metres reached by the shells is defined by the following rules:

$h_{1(t)} = -t^2 + 10t$

$h_{2(t)} = -t^2 + 16t - 20$ where t represents the time elapsed, in seconds, since the launch.

The situation is described by the following graph:



Calculate the difference between the maximum heights reached by the two shells. Clearly show all your work.

h_1

$a = -1 \quad b = 10 \quad c = 0$

$\Delta = b^2 - 4ac$

$= 10^2 - 4(-1)(0)$

$= 100$

Vertex y value $= \frac{-\Delta}{4a} = \frac{-100}{4(-1)}$

$= 25m$

h_2

$a = -1 \quad b = 16 \quad c = -20$

$\Delta = b^2 - 4ac$

$= 16^2 - 4(-1)(-20)$

$= 256 - 80$

$= 176$

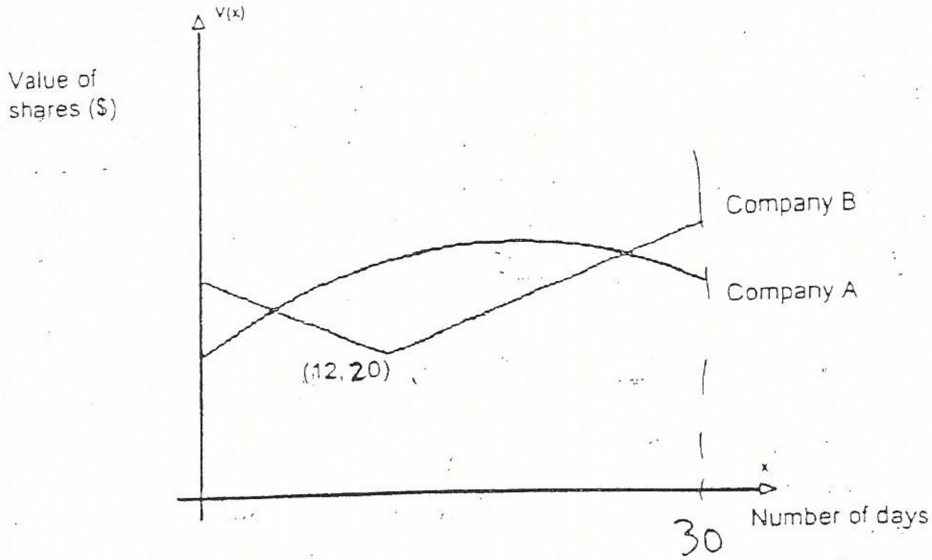
Vertex y value $= \frac{-\Delta}{4a} = \frac{-176}{4(-1)}$

$= 44m$

$44m - 25m = 19m$

The difference is 19m.

6) Two companies issued their shares at the same time. The fluctuations in the value of the shares were studied over a 30-day period. First, it was established that the value of the shares of Company A varied according to the rule $V(x) = -0.05x^2 + 2.1x + 11$. Then, the fluctuations in the value of each company's shares were represented in the following graph.



Which company experienced the longest period of growth during this time?

Clearly show all your work.

Company B growth: day 12 → day 30
= 18 days.

Company A growth: time 0 → vertex (x-value)

$$\frac{-b}{2a} = \frac{-2.1}{2(-0.05)}$$

$$= \frac{-2.1}{-0.1}$$

= 21 days

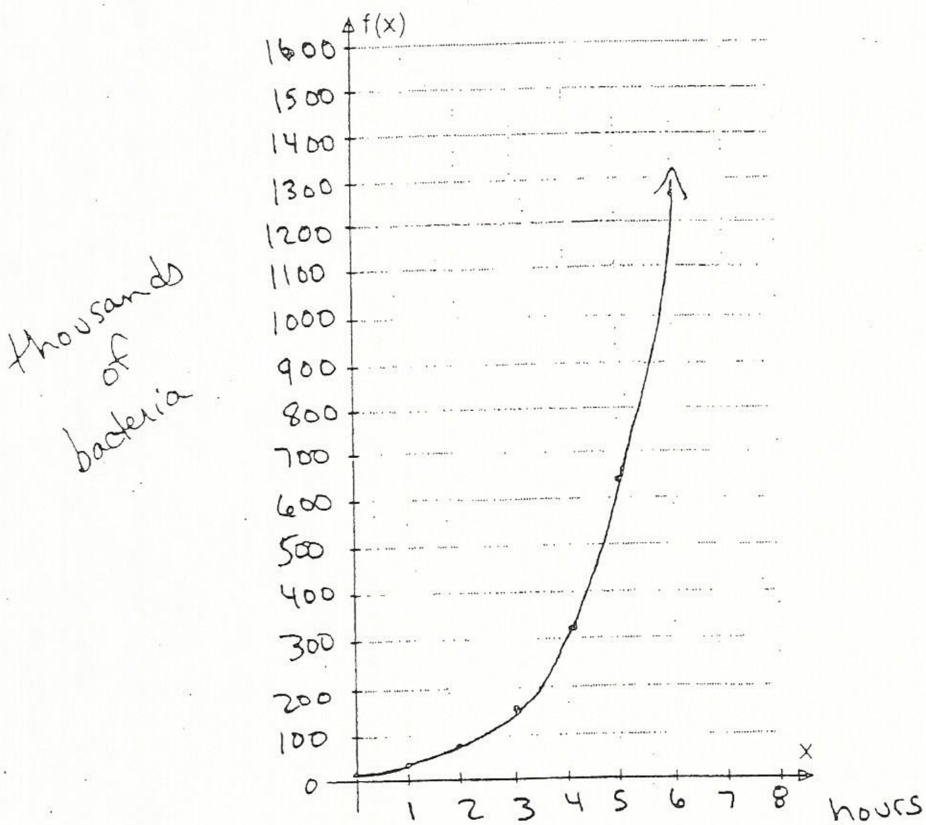
Company A experienced the longest growth.

In a laboratory, a biologist is studying a new type of bacterium. For his study, he places 20 thousand bacteria on a plate and notes that their number doubles every hour.

a) Complete the following table of values:

x (hours)	0	1	2	3	4	5	6
f(x) (thousands of bacteria)	20	40	80	160	320	640	1280

b) Graph this functional situation.



c) Is the function decreasing or increasing? Answer: increasing

Explain your answer by giving two ordered pairs. (1, 40) (4, 320)

4 > 1

320 > 40

d) What is the range of this function?

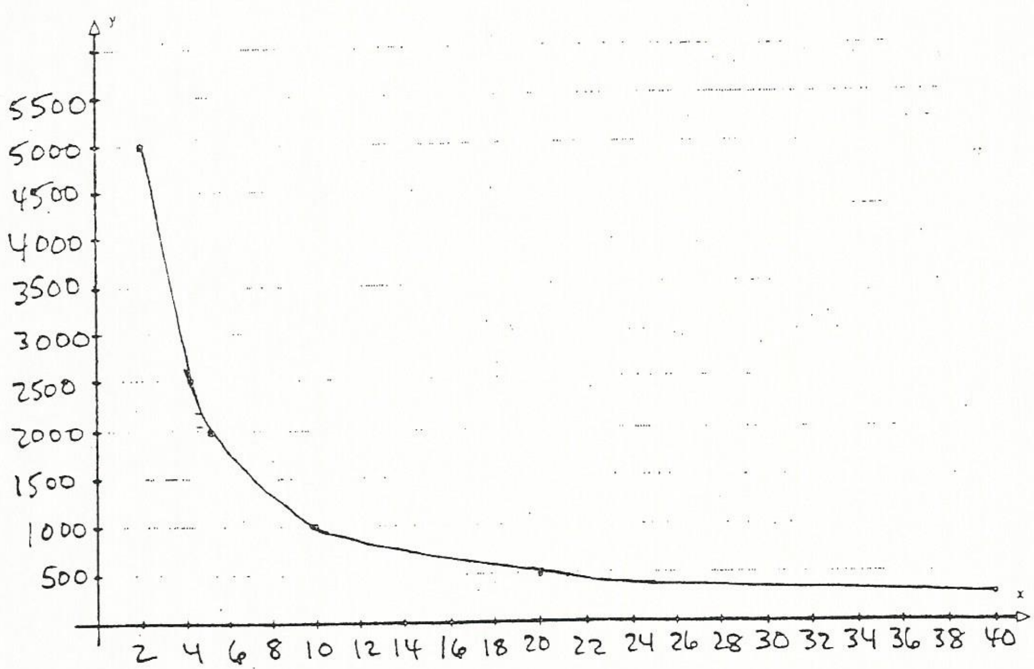
Answer: [20,000, ∞

8) An elementary school is holding a comedy festival in order to raise $10 000 for new playground equipment. The organizing committee is not sure how much to sell the tickets for. They want to ask at least $2 per ticket, but no more than $40 per ticket. The committee considers different prices for a ticket and, for each price, the number of tickets they expect to sell.

a) Complete the following table of values:

x (price per ticket)	2	4	5	10	20	25	40
y (number of tickets)	5000	2500	2000	1000	500	400	250

b) Graph this functional situation.



c) What is the domain of this function?

Answer: [2, 40]

d) Is this function decreasing or increasing?

Answer: decreasing

Explain your answer by giving two ordered pairs: (2, 5000), (4, 2500)

2 < 4

5000 > 2500

MTH-4109-1 Sets, Relations and Functions Quiz Name:

Date:

1. An actor preparing for a role loses $\frac{1}{2}$ lb every day. Determine the dependent variable for this functional situation.

Answer: weight loss

2. A cell biologist observes the growth of a bacterial culture and notes that the number of bacteria doubles every 12 hours. Determine the dependent variable for this functional situation.

Answer: bacterial population or # bacteria