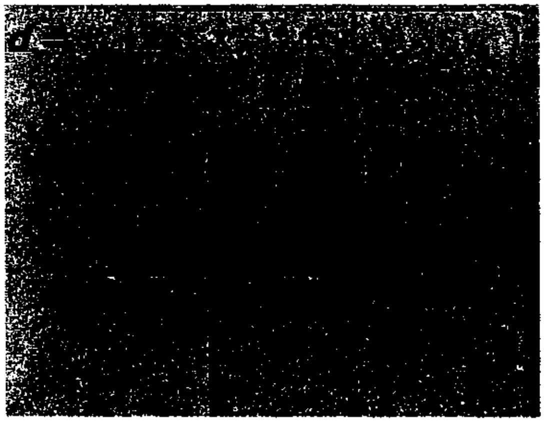
Answers

Mth-4107 **The Distance Formula**



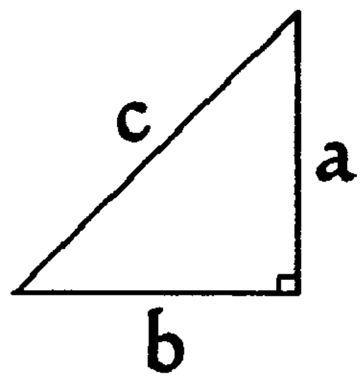
Important to note for distance formula:

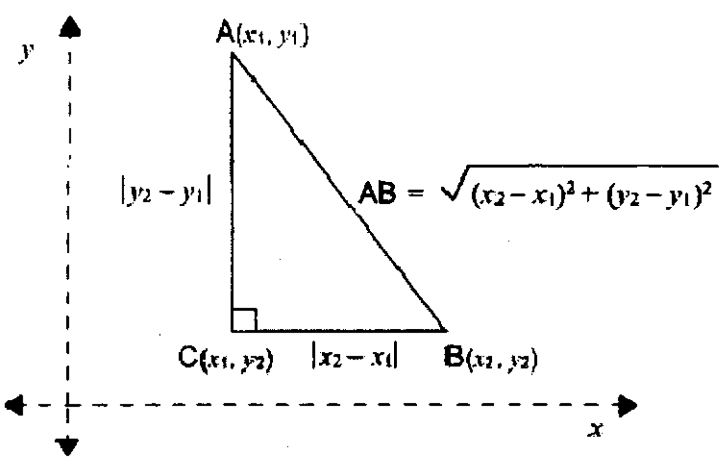
It doesn't matter which point is $(x_1 y_1)$ and which is $(x_2 y_2)$. The final answer will be the same!

Derivation of the Distance Formula:

Remember the Pythagorean Theorem?

$$a^2 + b^2 = c^2$$





Examples:

This problem is done for you, as an example:

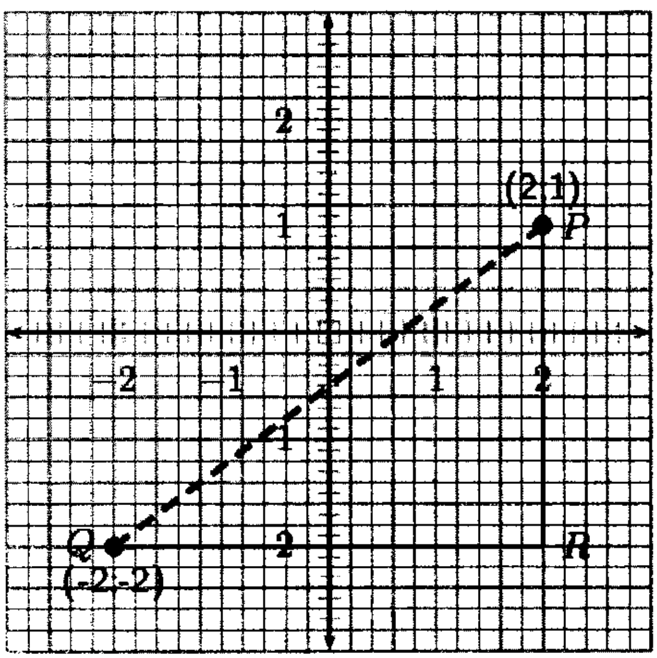
Let $(-3, 4) = (x_1, y_1)$ and $(5, 2) = (x_2, y_2)$. Then

$$d = \sqrt{(5-(-3))^2+(2-4)^2}$$
$$d = \sqrt{(8)^2+(2)^2}$$
$$d = \sqrt{64+4}$$
$$d = \sqrt{68}$$
$$d = \sqrt{(4)(17)}$$
$$d = 2\sqrt{17} = 2 \times 4.123 = 8.25u$$

Now, let's try some on our own!

$\underset{x_1\ \ y_1}{(-2,-2)}$ $\underset{x_2\ \ y_2}{(2,1)}$

1. Calculate the length of segment PQ.



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2+2)^2+(1+2)^2}$$
$$= \sqrt{4^2+3^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$= 5$units or $5u$

2. The structures on a family's property are represented in the following Cartesian plane. Each structure corresponds to a given point.

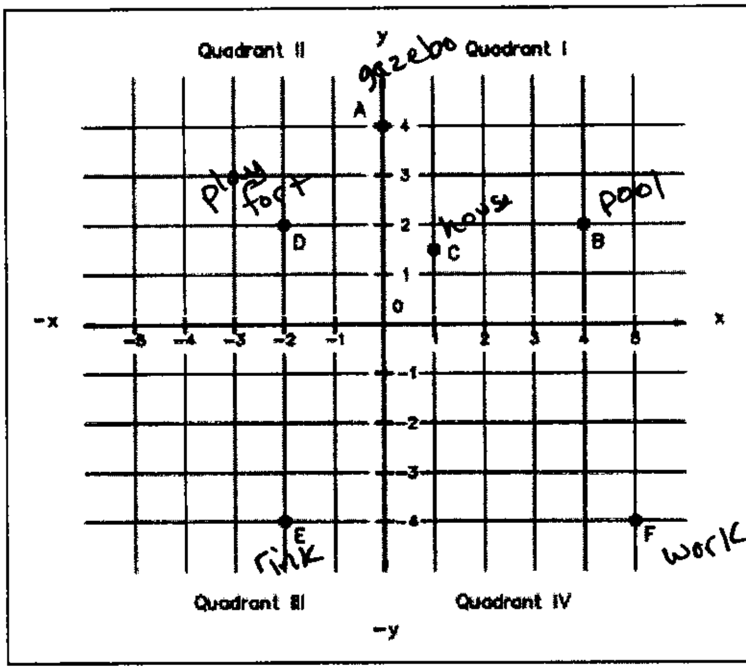


Figure 1 The Cartesian System

Scale: 1 unit ≙ 5m

a) Determine the coordinates of:

A: The gazebo: (0, 4)

B: The pool: (4, 2)

C: The main house: (1, 1.5)

D: The kids' play fort: (−2, 2)

E: The hockey rink: (−2, −4)

F: The workshop: (5, −4)

b) Calculate the distance (in meters) between: *Round to the nearest hundredth

i) the pool and the rink (4, 2) and (−2, −4)

x_1 y_1 x_2 y_2

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-2-4)^2+(-4-2)^2}$$

$$= \sqrt{(-6)^2+(-6)^2} = \sqrt{36+36} = \sqrt{72}$$

$$= 8.485u \times 5m$$

$$= \boxed{42.43m}$$

$x_1\ y_1$ $(-2,2)$ $x_2\ y_2$ $(5,-4)$

ii) the kids' play fort and the workshop

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5+2)^2+(-4-2)^2}$$
$$= \sqrt{(7)^2+(-6)^2} = \sqrt{49+36} = \sqrt{85} = 9.22 \times 5m$$
$$= \boxed{46.10m}$$

iii) the hockey rink and the workshop

$(-2,-4)$ $x_1\ y_1$ $(5,-4)$ $x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5+2)^2+(-4+4)^2} = \sqrt{(7)^2+(0)^2} = \sqrt{49} = 7u \times 5m = \boxed{35.00m}$$

iv) the kids' play fort and the rink

$(-2,2)$ $x_1\ y_1$ $(-2,-4)$ $x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2+2)^2+(-4-2)^2}$$
$$= \sqrt{0^2+(-6)^2} = \sqrt{36} = 6u \times 5m = \boxed{30.00m}$$

v) the pool and the kids' play fort

$(4,2)$ $x_1\ y_1$ $(-2,2)$ $x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-4)^2+(2-2)^2} = \sqrt{(-6)^2+0^2} = \sqrt{36} = 6u \times 5m = \boxed{30.00m}$$

vi) the gazebo and the main house

$(0,4)$ $x_1\ y_1$ $(1,1.5)$ $x_2\ y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(1-0)^2+(1.5-4)^2}$$
$$= \sqrt{(1)^2+(-2.5)^2} = \sqrt{1+6.25}$$
$$= \sqrt{7.25}$$
$$= 2.69u \times 5m$$
$$= \boxed{13.46m}$$

3. **Calculate the distance between each of the following pairs of points. Since there is no scale conversion given here, the unit, u will be used.** * Round to nearest tength.

a) $\overset{x_1\ y_1}{(3,4)}$ and $\overset{x_2\ y_2}{(6,8)}$

a) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(6-3)^2+(8-4)^2}$

$= \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25}$

a) $= \boxed{5.0u}$

b) $\overset{x_1\ y_1}{(9,-3)}$ and $\overset{x_2\ y_2}{(-3,2)}$

b) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3-9)^2+(2+3)^2}$

$= \sqrt{(-12)^2+(5)^2} = \sqrt{144+25}$

$= \sqrt{169}$ b) $= \boxed{13.0u}$

c) $\overset{x_1\ y_1}{(-2.5,-7.3)}$ and $\overset{x_2\ y_2}{(5.2,0)}$

c) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(5.2+2.5)^2+(0+7.3)^2}$

$= \sqrt{(7.7)^2+(7.3)^2} = \sqrt{59.29+53.29}$

$= \sqrt{112.58}$ c) $= \boxed{10.6u}$

d) $\overset{x_1\ y_1}{(-7,3)}$ and $\overset{x_2\ y_2}{(-7,12)}$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-7+7)^2+(12-3)^2}$

$= \sqrt{(0)^2+9^2} = \sqrt{81}$

d) $= \boxed{9.0u}$

e) $\underset{x_1\ y_1}{(2,4.3)}$ and $\underset{x_2\ y_2}{(-5,4.3)}$

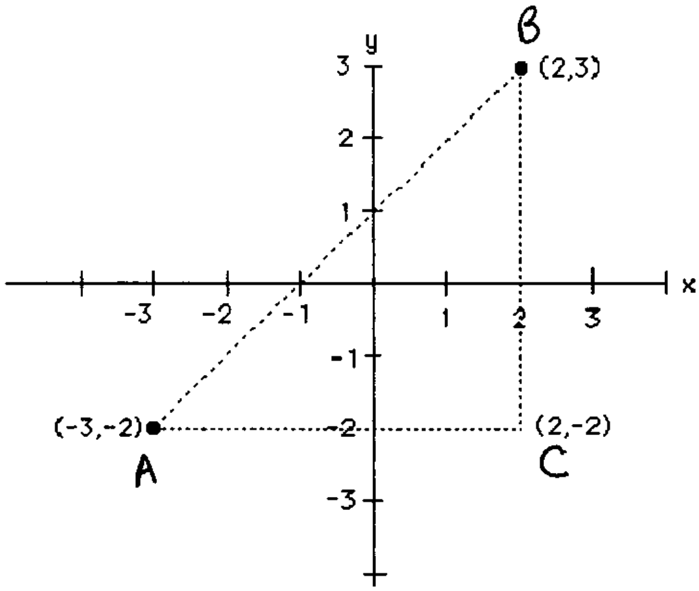
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-5-2)^2+(4.3-4.3)^2}$

$= \sqrt{(-7)^2+0^2} = \sqrt{49}$

e) $= \boxed{7.0u}$

4. **Calculate the perimeter of the following triangle. Use a scale of 1 unit $\hat{=}$ 1 cm.**

* Round to nearest hundredth.



$(-3,-2)$; $(2,3)$
$x_1\ y_1$ $\quad x_2\ y_2$

$\overline{AB}$: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2+3)^2+(3+2)^2}$

$= \sqrt{(5)^2+(5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50} = \boxed{7.07u} = \overline{AB}$

$\overline{BC}$ $\quad (2,3)$; $(2,-2)$
$x_1\ y_1$ $\quad x_2\ y_2$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-2)^2+(-2-3)^2}$

$= \sqrt{(0)^2+(-5)^2} = \sqrt{25} = \boxed{5u} = \overline{BC}$

NB $\overline{BC}$ and $\overline{AC}$ could be determined without the distance formula → for the ☺ people who "think outside the box".

$\overline{AC}$ $\quad (-3,-2)$; $(2,-2)$
$x_1\ y_1$ $\quad x_2\ y_2$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2+3)^2+(-2+2)^2}$

$= \sqrt{5^2+0^2}$

$= \sqrt{25} = \boxed{5u} = \overline{AC}$

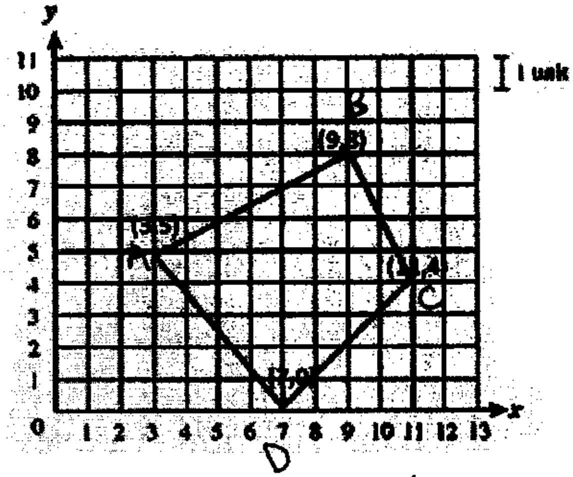
Perimeter =
$\overline{AB} + \overline{BC} + \overline{AC}$

$= 7.07u + 5u + 5u$

$= \boxed{17.07u} = 17.07cm$ (since $1u \hat{=} 1cm$)

5. **Calculate the perimeter of the following polygon. Use a scale of 1 unit $\hat{=}$ 1 cm.**

* Round answer to nearest tenth!



$\overline{AB}$: (3,5) and (9,8)
$x_1 \; y_1$ $\quad x_2 \; y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(9-3)^2+(8-5)^2}$$
$$= \sqrt{(6)^2+(3)^2}$$
$$= \sqrt{36+9} = \sqrt{45}$$

$\overline{AB}$: $\boxed{= 6.71u}$

$\overline{BC}$: (9,8) and (11,4)
$x_1 \; y_1$ $\quad x_2 \; y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(11-9)^2+(4-8)^2}$$
$$= \sqrt{(2)^2+(-4)^2}$$
$$= \sqrt{4+16}$$
$$\overline{BC} = \sqrt{20} \boxed{= 4.47u}$$

$\overline{CD}$: (11,4) (7,0)
$x_1 \; y_1$ $\quad x_2 \; y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(7-11)^2+(0-4)^2}$$
$$= \sqrt{(-4)^2+(-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = \boxed{5.66u}$$

$\overline{CD}$:

$\overline{AD}$: (3,5) and (7,0)
$x_1 \; y_1$ $\quad x_2 \; y_2$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(7-3)^2+(0-5)^2}$$
$$= \sqrt{(4)^2+(-5)^2}$$
$$= \sqrt{16+25} = \sqrt{41}$$
$$= 6.40$$

Perimeter =

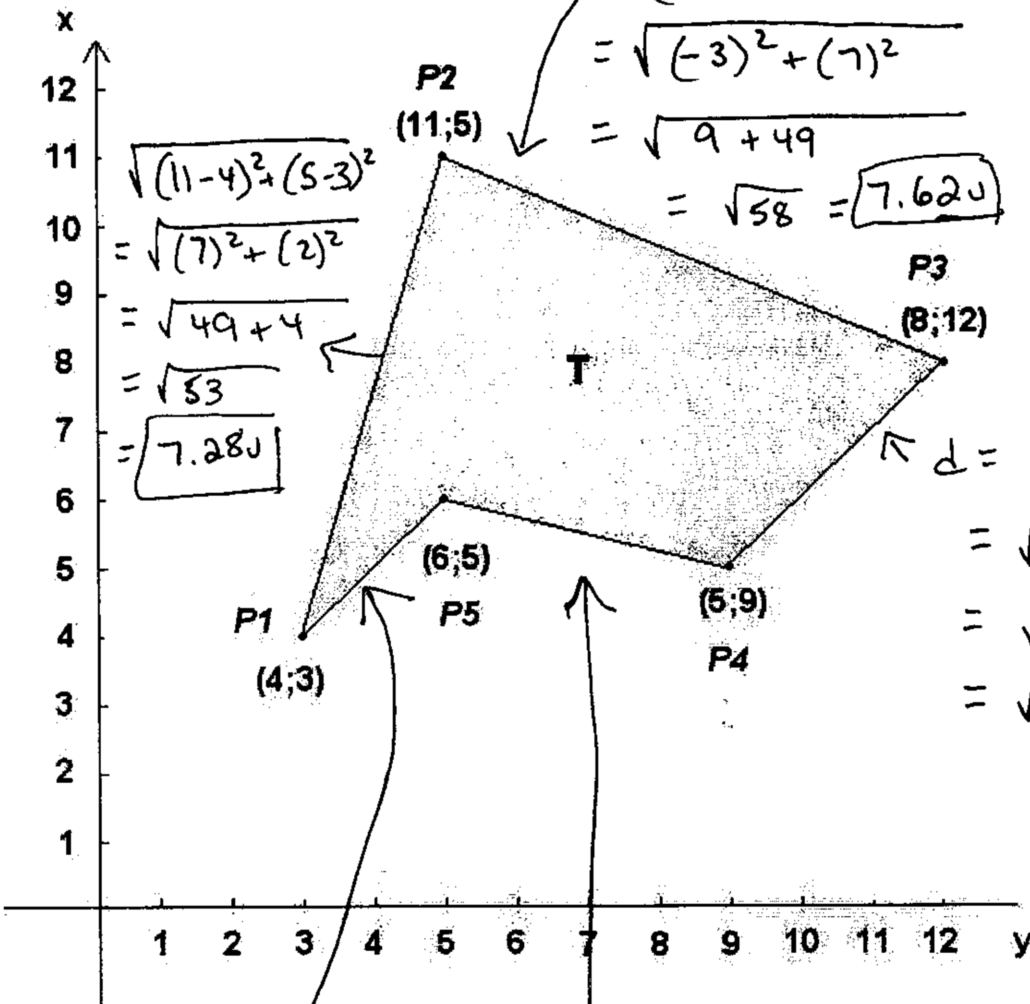
$\overline{AB} + \overline{BC} + \overline{CD} + \overline{AD}$

$= 6.71u + 4.47u + 5.66u + 6.40u$

$= 23.24u = \boxed{\boxed{23.2\text{cm}}}$

* Round to nearest hundredth.

6. **The cartesian plane below depicts the path and distances involved in a motorbike race. The bikers start at P1 and continue on through P2, P3, P4, P5, and then back to P1. Given that each unit corresponds to 7km, what is the total distance of the race?**



$$\sqrt{(11-4)^2+(5-3)^2} = \sqrt{(7)^2+(2)^2} = \sqrt{49+4} = \sqrt{53} = \boxed{7.28u}$$

$$\sqrt{(8-11)^2+(12-5)^2} = \sqrt{(-3)^2+(7)^2} = \sqrt{9+49} = \sqrt{58} = \boxed{7.62u}$$

$$d = \sqrt{(5-8)^2+(9-12)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = \boxed{4.24u}$$

$$d = \sqrt{(5-6)^2+(9-5)^2} = \sqrt{(-1)^2+(4)^2} = \sqrt{1+16} = \sqrt{17} = \boxed{4.12u}$$

$$d = \sqrt{(6-4)^2+(5-3)^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = \boxed{2.83u}$$

Total Distance $= 7.28u + 7.62u + 4.24u + 4.12u + 2.83u$

$= 26.09u \times 7Km = \boxed{182.63 \text{ Km}}$

Ch. 7.

The following expressions represent the distance between two points.

1) $\sqrt{(4+2)^2+(3+1)^2}$

2) $|-3+8|$

3) $\sqrt{(-1-4)^2+(-2-3)^2}$

4) $\sqrt{(4+6)^2+(3+2)^2}$

5) $\sqrt{(-6+1)^2+(-2+2)^2}$

Points $A(-1,-2)$, $B(4,3)$, and $C(-6,-2)$ were used to define the segments below. Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AB}$ 3

b) $\overline{BC}$ 4

c) $\overline{AC}$ 2, 5

$\overline{AB}$: $\sqrt{(3+2)^2+(4+1)^2}$

$\overline{BC}$: $\sqrt{(-2-3)^2+(-6-4)^2}$

$= \sqrt{(-5)^2+(-10)^2}$

$\overline{AC}$: $\sqrt{(-2+2)^2+(-6+1)^2}$

$= \sqrt{0^2+(-5)^2}$

8.

The following expressions represent the distance between two points.

1) $\sqrt{(4+4)^2+(-6+6)^2}$

2) $|-6-6|$

3) $\sqrt{(-4-3)^2+(-6-0)^2}$

4) $|-4-4|$

5) $\sqrt{(3-4)^2+(0+6)^2}$

Points $A(-4,-6)$, $B(3,0)$, and $C(4,-6)$ were used to define the segments below. Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AB}$ 3

b) $\overline{BC}$ 5

c) $\overline{AC}$ 4, 1

$\overline{AB} = \sqrt{(0+6)^2+(3+4^2}$

$= \sqrt{(6)^2+(7)^2}$

$\overline{BC} = \sqrt{(-6-0)^2+(4-3)^2}$

$= \sqrt{(-6)^2+(1)^2}$

$AC = \sqrt{(-6+6)^2+(4+4)^2}$

$= \sqrt{0^2+(8)^2}$

9.

The following expressions represent the distance between two points.

1) $\sqrt{(-3-2)^2+(6+1)^2}$

2) $|-2-2|$

3) $\sqrt{(-2+3)^2+(-1-6)^2}$

4) $|-1-1|$

5) $\sqrt{(2+2)^2+(-1+1)^2}$

Points $A(-2,-1)$, $B(-3,6)$, and $C(2,-1)$ were used to define the segments below. Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AB}$ 3

b) $\overline{BC}$ 1

c) $\overline{AC}$ 2, 5

$\overline{AB}$: $\sqrt{(6+1)^2+(-3+2)^2}$

$= \sqrt{(7)^2+(-1)^2}$

$\overline{BC}$: $\sqrt{(-1-6)^2+(2+3)^2}$

$= \sqrt{(-7)^2+(5)^2}$

$\overline{AC}$: $\sqrt{(-1+1)^2+(2+2)^2}$

$= \sqrt{0^2+(4)^2}$

$= \sqrt{16} = 4$