Math 4111 Solving a System of Equations (Parabola and a Line)

To SOLVE a system of equations means to find the INTERSECTION POINTS

e.g. 1

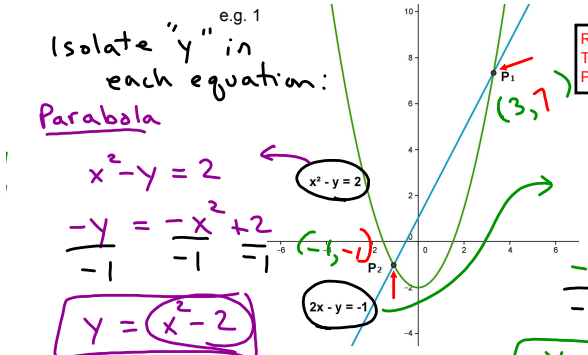
Isolate "y" in each equation:

Parabola

$x^2 - y = 2$

$\frac{-y}{-1} = \frac{-x^2}{-1} + \frac{2}{-1}$

$y = x^2 - 2$



RED ARROWS POINT TO INTERSECTION POINTS

Line

$2x - y = -1$

$\frac{-y}{-1} = \frac{-2x}{-1} + \frac{-1}{-1}$

$y = 2x + 1$

Method: To solve a system of equations, isolate "y" in each equation, then use comparison method to solve for x. Once you find each "x" you can "plug it into" either equation to find the "y".

Solve the system of equations shown above:

For intersection points: $y = y$

$x^2 - 2 = 2x + 1$

$x^2 - 2x - 2 - 1 = 0$

$1x^2 - 2x - 3 = 0$

$a = 1 \quad b = -2 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$= \frac{2 \pm \sqrt{16}}{2}$$

$$= \frac{2 \pm 4}{2}$$

$\oplus \rightarrow \frac{2+4}{2} = \frac{6}{2} = 3$

$\ominus \rightarrow \frac{2-4}{2} = \frac{-2}{2} = -1$

Find "y" that goes with each "x"

(3, 7) (-1, -1)

$y = 2x + 1$

$y = 2(3) + 1$

$= 6 + 1 = 7$

$y = 2x + 1$

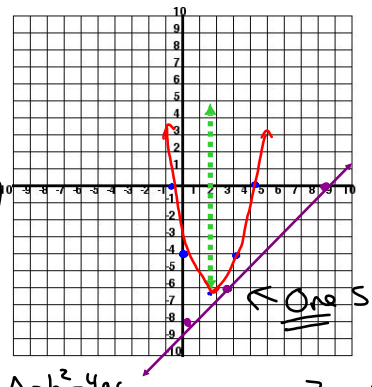
$y = 2(-1) + 1$

$= -2 + 1 = -1$

Answer: (3,7) and (-1,-1)

e.g. 2 Solve the following system of equations:

1. $y = x^2 - 3x - 4$
2. $y = x - 8$



① Let's graph

Parabola: $y = x^2 - 3x - 4$

y-int: $(0, c)$ $\boxed{(0, -4)}$

Vertex

$\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$

$\left(\frac{3}{2(1)}, \frac{-25}{4(1)}\right)$

$\left(\frac{3}{2}, \frac{-25}{4}\right)$

$(1.5, -6\frac{1}{4})$

$\Delta = b^2 - 4ac$

$= (-3)^2 - 4(1)(-4)$

$= 9 + 16$

$= 25$

← One Solution

Zeros

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x_1 = 4 \quad x_2 = -1$

Line: $y = x - 8$

$b = -8 \quad (0, -8)$

Let $y = 0$

$0 = x - 8 \quad (8, 0)$

$8 = x$ x-int

Parabola: $y = x^2 - 3x - 4$

Line: $y = x - 8$

Let $y = y$

$x^2 - 3x - 4 = x - 8$

$x^2 - 3x - 4 - x + 8 = 0$

$1x^2 - 4x + 4 = 0$

$a = 1 \quad b = -4 \quad c = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$

$= \frac{4 \pm \sqrt{16 - 16}}{2}$

$= \frac{4 \pm 0}{2} \rightarrow \frac{4}{2} = 2 = x$ (intersection x)

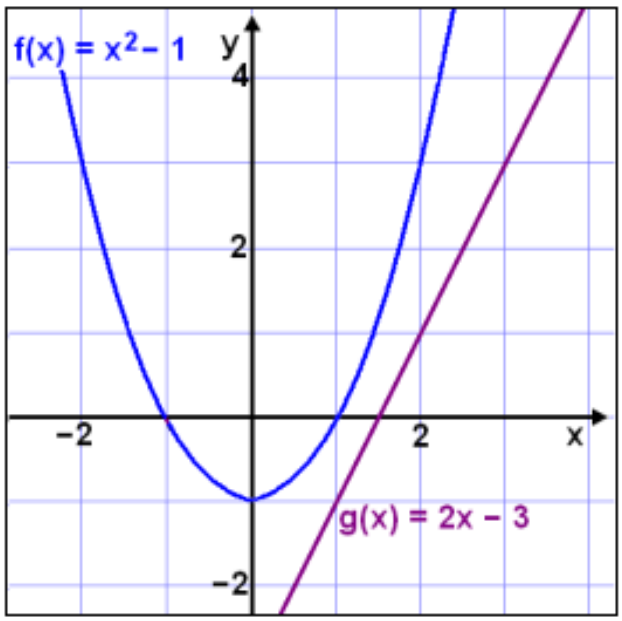
Plug $x = 2$ into one of the equations:

$y = x - 8$

$y = 2 - 8 = -6$

Answer: $(2, -6)$

e.g. 3 Solve the following system of equations:



$y = y$

$x^2 - 1 = 2x - 3$

$x^2 - 1 - 2x + 3 = 0$

$x^2 - 2x + 2 = 0$

$a = 1 \quad b = -2 \quad c = 2$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(1)(2)$

$= 4 - 8 = -4$

Answer: No Solution

3 possible scenarios: