Determining the Equation of a Quadratic Function by Using a Table

Problem #1

Mr. Pitt owns a peach orchard with 30 trees yielding an average of 400 peaches each. He wants to plant more peach trees in order to get the maximum yield from his orchard; however, for each additional tree planted, the average yield per tree will drop by 10 peaches.

Given that x represents the number of trees that Mr. Pitt adds to his orchard, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total yield of the orchard.

Number of peach trees to be added x	**Total number of peach trees**	**Average yield per peach tree**	**Total yield of orchard** y
0	30	400	$30 \times 400 = 12\,000$
1	$30 + 1$	$400 - 10 \times 1$	$(30 + 1) \times (400 - 10 \times 1) = 12\,090$
2	$30+2$	$400-10\times2$	$(30+2)\times(400-10\times2) = 12\,160$
3	$30+3$	$400-10\times3$	$(30+3)\times(400-10\times3) = 12\,210$
4	$30+4$	$400-10\times4$	$(30+4)\times(400-10\times4) = 12\,240$
5	$30+5$	$400-10\times5$	$(30+5)\times(400-10\times5) = 12\,250$
6	$30+6$	$400-10\times6$	$(30+6)\times(400-10\times6) = 12\,240$
7	$30+7$	$400-10\times7$	$(30+7)\times(400-10\times7) = 12\,210$
x	$30+x$	$400-10x$	$(30+x)\cdot(400-10x) = y$

Determine the equation for calculating the total yield of the orchard.

Equation: $y = (30+x)(400-10x) = 12000 - 300x + 400x - 10x^2$

$$y = -10x^2 + 100x + 12\,000$$

This equation is a ***second-degree equation*** because the greatest exponent of x is 2. It is also called a ***quadratic function*** or a ***quadratic equation***.

*Imp: tell them how they can do "checks" if they want to.

Problem #2

The XYZ publishing company prints 36 books a day, which are sold for $40 each. The company can produce 50 books per day. However, for each book printed over and above the number currently produced, the unit selling price decreases by $1.

Given that x represents the additional number of books sold, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total daily revenue from book sales.

Additional books x	**Total number of books**	**Selling price per book**	**Daily sales** y
0	36	$40	$36 \times \$40 = \$1\,440$
1	$36 + 1 = 37$	$\$40 - \$1 = \$39$	$37 \times \$39 = \$1443$
2	$36 + 2 = 38$	$\$40 - \$2 = \$38$	$38 \times \$38 = \$1444$
3	$36+3 = 39$	$40 - 3 = \$37$	$(39)(37) = 1443$
4	$36+4 = 40$	$40 - 4 = \$36$	$(40)(36) = 1440$
x	$36+x$	$40-x$	$(36+x)(40-x)$

$$y = (36+x)(40-x)$$
$$= -x^2 + 40x - 36x + 1440$$
$$y = -x^2 + 4x + 1440$$

Determine the equation for calculating the total daily revenue.

Equation: $y = -x^2 + 4x + 1440$

Did you know that...

...the term "quadratic" was chosen to designate this function because it comes from the Latin **"quadratus"** which means 'square'?

Procedure for finding the equation for problems involving a maximum:

1. Read the problem carefully.
2. Fill in the partially completed table of values with at least three other values for x.
3. Complete the last line of the table by choosing x as the variable in the first column and fill in the other columns as a function of x.
4. Do the multiplication indicated in the bottom right-hand corner of the table and then write the resulting expression as an equation of the form $y = ax^2 + bx + c$.

Problem #3

Mickey Mouse Electronics can sell 300 VCRs at a profit of \$60 per unit. Seeking to increase its profits, the company decides to take its VCRs off the market to increase the demand. It calculates that its sales will increase by 50 units for each week that VCRs are off the market, but that its profit will drop by \$5 per unit owing to storage costs.

Complete the following table, given that x represents the number of weeks the VCRs are off the market. Determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total profit.

Number of weeks off the market x	**Number of units sold**	**Profit on each unit**	**Total profit** y
0	300	\$60	\$60 × 300 = \$18 000
1	300 + (1 × 50) = 350	\$60 − (\$5 × 1) = \$55	\$55 × 350 = \$19 250
2	300 + (2·50) = 400	\$60 − (5×2) = \$50	\$50 × 400 = 20 000
3	300 + (3·50) = 450	\$60 − (5×3) = \$45	\$45 · 450 = 20 250
4	300 + (4·50) = 500	\$60 − (5·4) = \$40	\$40 · 500 = 20 000
5	300 + (5·50) = 550	\$60 − (5·5) = \$35	\$35 · 550 = 19 250
x	300 + (x·50)	\$60 − (5·x)	(300 + 50x)(60 − 5x)

$-250x^2 + 3000x - 1500x + 18\,000$

Equation: $y = -250x^2 + 1500x + 18\,000$

Problem #4

A survey conducted during an amateur theatre festival in Sherbrooke indicated that 100 people would attend the festival if the tickets were sold for \$4.50 apiece. For each \$.30 decrease in the price of a ticket, 20 more people would attend the festival.

Given that x represents the number of \$.30 decreases in price, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total revenue from ticket sales.

Number of extra people $x \times 20$	**Total number of people**	**Ticket price**	**Revenue y**
0	100	$\$4.50$	$\$4.50 \times 100 = \$450$
1×20	$100 + 20 = 120$	$\$4.50 - (\$0.30 \times 1)$	$\$4.20 \times 120 = \$504$
2×20	$100 + 40 = 140$	$\$4.50 - (0.30 \times 2)$	$\$3.90 \cdot 140 = \$546$
3×20	$100 + 60 = 160$	$\$4.50 - (0.30 \times 3)$	$\$3.60 \cdot 160 = \$576$
4×20	$100 + 80 = 180$	$\$4.50 - (0.30 \cdot 4)$	$\$3.30 \cdot 180 = \$594$
5×20	$100 + 100 = 200$	$\$4.50 - (0.30 \cdot 5)$	$\$3.00 \cdot 200 = \$600$
6×20	$100 + 120 = 220$	$\$4.50 - (0.30 \cdot 6)$	$\$2.70 \cdot 220 = \$594$
$x \times 20$	$100 + 20x$	$\$4.50 - 0.30x$	$(100+20x)(4.50-0.30x)$

$-6x^2 - 30x + 90x + 450$

Equation: $y = -6x^2 + 60x + 450$

All quadratic equations can be written in the form:

$y = ax^2 + bx + c,$

where $a, b, c \in \mathbb{R}$ and $a \neq 0$.

Problem # 5

A plum orchard with 30 trees yields an average of 500 plums per tree. If more plum trees were added, the yield would decrease by 15 plums per tree. Find the equation that describes this situation by completing the table below.

Number of plum trees added x	**Total number of plum trees**	**Yield per plum tree**	**Total yield of orchard** y
0	30	500	$30 \times 500 = 15\,000$
1	$30 + 1 = 31$	$500 - 15 \times 1 = 485$	$31 \times 485 = 15\,035$
2	$30 + 2 = 32$	$500 - 15 \cdot 2 = 470$	$32 \times 470 = 15\,040$
3	$30 + 3 = 33$	$500 - 15 \cdot 3 = 455$	$33 \cdot 455 = 15\,015$
x	$30 + x$	$500 - 15x$	$(30+x)(500-15x)$

$-15x^2 + 500x - 450x + 15000$

The equation is $y = -15x^2 + 50x + 15\,000$

Problem #6

At present, Jupiter Electronics could sell 200 computers at a profit of $300 per unit. The company prefers to wait in order to boost demand for its product. For each week the computers are off the market, the company will sell 40 additional computers, but its profit will decrease by $20 per unit. Find the corresponding equation by completing the following table.

Number of weeks off the market x	**Number of computers sold**	**Profit on each computer**	**Total profit** y
0	200	$300	$200 \times \$300 = \$60\,000$
1	$200 + 1 \times 40 = 240$	$\$300 - 1 \times \$20 = 280$	$240 \times \$280 = \$67\,200$
2	$200 + 2 \cdot 40 = 280$	$\$300 - 2 \cdot 20 = 260$	$280 \cdot \$260 = \$72\,800$
3	$200 + 3 \cdot 40 = 320$	$\$300 - 3 \cdot 20 = 240$	$320 \cdot \$240 = \$76\,800$
4	$200 + 4 \cdot 40 = 360$	$\$300 - 4 \cdot 20 = 220$	$360 \cdot \$220 = \$79\,200$
x	$200 + x \cdot 40$	$300 - x \cdot 20$	$(200 + 40x)(300 - 20x)$

The equation is $y = -800x^2 + 8000x + 60\,000$

$-800x^2 + 12000x$

$-4000x + 60000$

Problem #7

A machine reproduces 10 feature films per day; however, it could reproduce 16 films per day. Each reproduction sells for \$300. For each additional copy, the unit price will decrease by \$20. Find the corresponding equation by completing the following table.

Number of additional copies x	**Total number of copies**	**Selling price per copy**	**Total daily sales** y
0	10	\$300	$10 \times \$300 = \$3\,000$
1	$10 + 1 = 11$	$\$300 - 1 \times \$20 = \$280$	$11 \times \$280 = \$3\,080$
2	$10 + 2 = 12$	$\$300 - 2 \cdot 20 = \$260$	$12 \cdot \$260 = \$3\,120$
3	$10 + 3 = 13$	$300 - 3 \cdot 20 = \$240$	$13 \cdot \$240 = \$3\,120$
4	$10 + 4 = 14$	$300 - 4 \cdot 20 = \$220$	$14 \cdot 220 = 3080$
x	$10 + x$	$300 - x \cdot 20$	$(10+x)(300-20x)$

$-20x^2 + 300x - 200x + 3000$

The equation is $y = -20x^2 + 100x + 3000$

Problem # 8

A car dealer is trying to maximize his sales. A survey has shown that 80 people would buy the CRX model if it were to sell for $24 700; however, for each $475 price reduction, four more people would buy this vehicle.

Given that x represents the number of $475 price reductions, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the total revenue from the sale of these vehicles.

Number of $ 475 reductions	Price of vehicle $	Total number of buyers	Total revenue $
0	24 700	80	1 976 000
1	24 700 – (475 × 1) = 24 225	80+(4 x1) 84	2 034 900
2	24 700-(475·2)=23750	80+(4x2) 88	2 690 000
x	24 700 - 475x	80 + 4x	(24 700-475x)(80+4x)

Determine the equation for calculating the total revenue.

Equation: $y = -1900x^2 + 60\,800x + 1\,976\,000$

$y = (24\,700 - 475x)(80 + 4x)$

$y = 1976000 - 38000x + 98800x - 1900x^2$

$y = -1900x^2 + 60\,800x + 1976\,000$

Problem #9

Best Buy is selling a particular stereo system for $3 690. At this price, they sell an average of 9 such systems per week. Management at *Best Buy* has determined that for each $45 decrease in the price of the stereo, there would be two more stereos sold per week.

Given that x represents the number of $45 price reductions, complete the following table and determine the quadratic equation of the form $y = ax^2 + bx + c$ that represents the weekly revenue from the sale of these stereos.

Number of $45 reductions	Price of the Stereo	Number of buyers per week	Revenue per week ($)
0	3 690	9	33 210
1	$3690 - 1 \times 45$	$9 + 1 \times 2 = 11$	40 095
2	$3690 - 2 \times 45$	$9 + 2 \times 2$	46 800
x	$3690 - x \cdot 45$	$9 + x \times 2$	$(3690-45x)(9+2x)$

Equation: $y = -90x^2 + 6975x + 33\,210$

$y = (3690-45x)(9+2x)$

$= -90x^2 - 405x + 7380x + 33\,210$

$y = -90x^2 + 6975x + 33\,210$