Converting Exponents to Radicals & Vice Versa

$$2^{\frac{3}{5}} = \sqrt[5]{8}$$

root index of radical

radicand

base with exponent

Radical (has "$\sqrt{\ }$" sign)

FOR RADICALS: if the root index is not given it's understood to be "2".

e.g. $\sqrt[3]{8} = 2$

$\sqrt{4} = 2$

root index understood to be "2"

$\sqrt[4]{81} = 3$

Converting Exponent → Radical

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

e.g. $5^{\frac{3}{2}} = \sqrt[2]{5^3} = \sqrt{5^3}$

e.g. $2^{\frac{7}{3}} = \sqrt[3]{2^7}$

e.g. $4^{\frac{1}{2}} \longrightarrow \left(2^2\right)^{\frac{1}{2}} = 2$

$\searrow \sqrt{4} = 2$

e.g. $9^{\frac{1}{2}} \longrightarrow \left(3^2\right)^{\frac{1}{2}} = 3$

$\searrow \sqrt{9} = 3$

e.g. $8^{\frac{1}{3}} \rightarrow \left(2^3\right)^{\frac{1}{3}} = 2$

$\sqrt[3]{8} = 2$

e.g. $-25^{\frac{2}{3}} = -\sqrt[3]{25^2}$

$= -\sqrt[3]{(5^2)^2}$

$= -\sqrt[3]{5^4}$

Convert these exponents to radicals:

a) $2^{\frac{4}{3}} = \sqrt[3]{2^4}$ or $\sqrt[3]{16}$

b) $x^{\frac{3}{7}} = \sqrt[7]{x^3}$

c) $-3^{\frac{2}{3}} = -\sqrt[3]{3^2}$ OR $-\sqrt[3]{9}$

d) $a^{\frac{1}{2}} = \sqrt{a}$

e) $5^{\frac{3}{4}} = \sqrt[4]{5^3}$ or $\sqrt[4]{125}$

f) $-2^{\frac{7}{2}} = -\sqrt{2^7}$

Now, we'll convert radical → exponent...

e.g. $\sqrt[5]{a^3} = a^{\frac{3}{5}}$

e.g. $-\sqrt{32} = -32^{\frac{1}{2}} = -\left(2^5\right)^{\frac{1}{2}} = -2^{\frac{5}{2}}$

OR

$\hookrightarrow -\sqrt{2^5} = -2^{\frac{5}{2}}$

e.g. $\sqrt[5]{8^2} = \sqrt[5]{(2^3)^2}$

$= \sqrt[5]{2^6} = 2^{\frac{6}{5}}$

e.g. $\sqrt[6]{\frac{1}{16}} = \sqrt[6]{2^{-4}}$

$= 2^{-\frac{4}{6}}$

$= 2^{-\frac{2}{3}} = \frac{1}{2^{\frac{2}{3}}}$

e.g. $\sqrt[3]{8^4}$

$\sqrt[3]{(2^3)^4} = \sqrt[3]{2^{12}}$

$= 2^{\frac{12}{3}} = 2^4$

e.g. $-\sqrt[5]{243} = -\sqrt[5]{3^5}$

$= -3^{\frac{5}{5}} = -3$

e.g. $-\sqrt{64} = -\sqrt{2^6} = -2^{\frac{6}{2}}$

$= -2^3$

e.g. $\sqrt[4]{\frac{1}{27}}$

$= \sqrt[4]{3^{-3}} = 3^{\frac{-3}{4}} = \frac{1}{3^{\frac{3}{4}}}$

e.g. $a^3 \cdot \sqrt[5]{a^2}$

$a^3 \cdot a^{\frac{2}{5}}$

$a^{\frac{17}{5}}$

$\frac{3 \cdot 5}{1 \cdot 5} + \frac{2}{5}$

$\frac{15}{5} + \frac{2}{5} = \frac{17}{5}$

e.g. $4\sqrt[5]{\frac{1}{8}}$

$2^2 \cdot \sqrt[5]{2^{-3}} = 2^2 \cdot 2^{-\frac{3}{5}}$

$= \boxed{2^{\frac{7}{5}}}$

$\frac{2 \cdot 5}{1 \cdot 5} - \frac{3}{5}$

$\frac{10}{5} - \frac{3}{5} = \frac{7}{5}$

e.g. $\frac{1}{7^3}\sqrt[4]{49^{-5}}$

$7^{-3}\sqrt[4]{(7^2)^{-5}}$

$7^{-3}\sqrt[4]{7^{-10}}$

$7^{-3} \cdot 7^{-\frac{10}{4}}$

$7^{-3} \cdot 7^{-\frac{5}{2}}$

$7^{-\frac{11}{2}} = \boxed{\frac{1}{7^{\frac{11}{2}}}}$

$\frac{-3 \cdot 2}{1 \cdot 2} - \frac{5}{2}$

$\frac{-6}{2} - \frac{5}{2} = \frac{-11}{2}$

Do: ① Handout from
Chap 3 in book

② Review Booklet
Question 13 only

③ Quiz pkg: last 2 pages