MTH-4106-1 REVIEW: FACTORING AND ALGEBRAIC FRACTIONS

Question 1.

Factor the following polynomials:

a) $4ab^2 - 16b^4$ $= \boxed{4b^2(a - 4b^2)}$

b) $10x^6y^8 - 15x^7y^7 + 20x^5y^8 - 5x^5y^6 - 5x^4y^5 + 5x^5y^5$

$\boxed{5x^4y^5(2x^2y^3 - 3x^3y^2 + 4xy^3 - xy - 1 + x)}$

c) $a^2 - 5a - 6$

$\boxed{(a-6)(a+1)}$

d) $2x^2 - 5xy + 3y^2$ $p = 6$ $s = -5$

$(2x^2 - 2xy)(-3xy + 3y^2)$ $-2, -3$

$2x(x-y) - 3y(x-y)$

$\boxed{(2x-3y)(x-y)}$

e) $256 - 1.69x^2$

$\boxed{(16 - 1.3x)(16 + 1.3x)}$

l) $25yx^4 - 625x^6$

$\boxed{25x^4(y - 25x^2)}$

m) $a^2 - a - 72$

$\boxed{(a-9)(a+8)}$

n) $5y^2 - 26xy + 5x^2$

$p = 25$

$s = -26$

$-25, -1$

$(5y^2 - 25xy)(-1xy + 5x^2)$

$5y(y-5x) - x(y-5x)$

$\boxed{(5y-x)(y-5x)}$

o) $\frac{36y^2}{25} - 64z^2$

$\boxed{\left(\frac{6y}{5} - 8z\right)\left(\frac{6y}{5} + 8z\right)}$

p) $(8p^4r^2 - 48p)(-25p^3r^4 + 150r^2)$

$8p(p^3r^2 - 6) - 25r^2(p^3r^2 - 6)$

$\boxed{(8p - 25r^2)(p^3r^2 - 6)}$

q) $y^2 + y - 12$

$\boxed{(y+4)(y-3)}$

x) $4t^2 - 16u^4$

$4(t^2 - 4u^4)$

$\boxed{4(t-2u^2)(t+2u^2)}$

y) $-x^2 - xy + 6y^2$

$(-x^2 - 3xy) + (2xy + 6y^2)$

$p = -6$

$-x(x+3y) + 2y(x+3y)$

$s = -1$

$\boxed{(-x+2y)(x+3y)}$

$-3, +2$

z) $-2a^2 + 5ab - 2b^2$

$(-2a^2 + 4ab) + (1ab - 2b^2)$

$p = {}^{+}4$

$-2a(a-2b) + b(a-2b)$

$s = {}^{+}5$

$\boxed{(-2a+b)(a-2b)}$

$^{+}4, ^{+}1$

Question 2

Factor the following polynomials completely.
Show all the steps in the solutions.

a) $18m^5n^4 - 78m^4n^5 + 24m^3n^6$

$6m^3n^4\underbrace{(3m^2 - 13mn + 4n^2)}$

$p = +12$

$(3m^2 - 12mn)(-1mn + 4n^2)$

$s = -13$

$3m(m-4n) - n(m-4n)$

$-12, -1$

$(3m-n)(m-4n)$

answer : $\boxed{6m^3n^4(3m-n)(m-4n)}$

e) $20x^6y^3 - 64x^5y^4 + 12x^4y^5$

$$4x^4y^3\underbrace{(5x^2 - 16xy + 3y^2)}$$

$p = {}^{+}15$

$s = -16$

$-15, -1$

$$(5x^2 - 15xy)(-1xy + 3y^2)$$

$$5x(x-3y) - y(x-3y)$$

$$(5x-y)(x-3y)$$

answer: $\boxed{4x^4y^3(5x-y)(x-3y)}$

f) $-7y^3z + 50y^2z^2 - 7yz^3$

$$yz\underbrace{(-7y^2 + 50yz - 7z^2)}$$

$p = {}^{+}49$

$s = {}^{+}50$

$+49, +1$

$$(-7y^2 + 49yz) + (1yz - 7z^2)$$

$$-7y(y-7z) + z(y-7z)$$

$$(-7y+z)(y-7z)$$

answer: $\boxed{yz(-7y+z)(y-7z)}$

g) $16x^2 - 64a^4$

$$16(x^2 - 4a^4)$$

$$\boxed{16(x-2a^2)(x+2a^2)}$$

Question 3

Reduce the following algebraic fractions to their lowest terms.
Show all the steps in the solutions.

a) $$\frac{16m^4 - n^6}{3m^2n^4 - 12m^4n} = \frac{(4m^2 - n^3)(4m^2 + n^3)}{3m^2n(n^3 - 4m^2)}$$

$$= \frac{-1\,\cancel{(-4m^2 + n^3)}(4m^2 + n^3)}{3m^2n\,\cancel{(n^3 - 4m^2)}}$$

$$= \boxed{\frac{-(4m^2 + n^3)}{3m^2n}}$$

b) $$\frac{x^2 - 8x + 7}{147 - 3x^2} = \frac{(x-7)(x-1)}{3(49 - x^2)} = \frac{(x-7)(x-1)}{3(7-x)(7+x)}$$

$$= \frac{-1\,\cancel{(-x+7)}(x-1)}{3\,\cancel{(7-x)}(7+x)}$$

$$= \boxed{\frac{-(x-1)}{3(7+x)}}$$

c) $$\frac{25x^4 - 4y^2}{4xy^3 - 10x^3y^2} = \frac{(5x^2 - 2y)(5x^2 + 2y)}{2xy^2(2y - 5x^2)}$$

$$= \frac{-1\,\cancel{(-5x^2 + 2y)}(5x^2 + 2y)}{2xy^2\,\cancel{(2y - 5x^2)}} = \boxed{\frac{-(5x^2 + 2y)}{2xy^2}}$$

b) $\frac{5x^2}{x^3} - \frac{4x-x^2}{(x-4)} = \frac{5}{x} - \frac{x(4-x)}{(x-4)}$

$= \frac{5}{x} - \frac{-x(\cancel{-4+x})}{\cancel{(x-4)}}$

$= \frac{5}{x} + \frac{x}{1}$ c.d = x

$= \frac{5}{x} + \frac{x^2}{x}$

$= \boxed{\frac{5+x^2}{x}}$

c) $\frac{9-x^2}{x} \times \frac{3x-9}{-9x-3x^2}$

$= \frac{(3-x)(3+x)}{x} \cdot \frac{3(x-3)}{-3x(3+x)} = \frac{(3-x)\cancel{(3+x)}}{x} \cdot \frac{\cancel{3}(x-3)}{-\cancel{3}x\cancel{(3+x)}}$

$= \boxed{\frac{-(3-x)(x-3)}{x^2}}$

OR $\frac{(x-3)(x-3)}{x^2} = \frac{(x-3)^2}{x^2}$

d) $\frac{1}{4-y} + \frac{y-3}{y(y-7)}$ c.d = $y(4-y)(y-7)$

$\frac{y(y-7)}{c.d.} + \frac{(y-3)(4-y)}{c.d.} = \frac{y^2-7y}{c.d.} + \frac{-y^2+7y-12}{c.d.}$

$= \frac{y^2-7y-y^2+7y-12}{c.d.}$

$= \boxed{\frac{-12}{y(4-y)(y-7)}}$

h) $\frac{1}{5-w} + \frac{w-4}{w(w-9)}$ c.d. $= w(5-w)(w-9)$

$$\frac{w(w-9)}{c.d.} + \frac{(w-4)(5-w)}{c.d.}$$

$$\frac{w^2-9w}{c.d.} + \frac{-w^2+9w-20}{c.d.} = \frac{w^2-9w-w^2+9w-20}{c.d.}$$

$$= \boxed{\frac{-20}{w(5-w)(w-9)}}$$

i) $\frac{4-y^2}{y-2} \div \frac{y^2-2y-8}{y-4}$

$$\frac{(2-y)(2+y)}{(y-2)} \times \frac{(y-4)}{(y-4)(y+2)}$$

$$\frac{-1\cancel{(-2+y)}\cancel{(2+y)}}{\cancel{(y-2)}} \cdot \frac{\cancel{(y-4)}}{\cancel{(y-4)}\cancel{(y+2)}} = \boxed{-1}$$

j) $\frac{12m^4}{m^5} - \frac{6m-m^2}{m-6} = \frac{12}{m} - \frac{m(6-m)}{(m-6)}$

$$= \frac{12}{m} - \frac{m(-1)\cancel{(-6+m)}}{\cancel{(m-6)}}$$

$$= \frac{12}{m} - \frac{-m}{1} = \frac{12}{m} + \frac{m}{1} \qquad c.d = m$$

$$= \frac{12}{m} + \frac{m^2}{m} = \boxed{\frac{12+m^2}{m}}$$

n) $\frac{6}{x+3} - \frac{x+3}{x}$ c.d. $= x(x+3)$

$$\frac{6x}{c.d.} - \frac{(x+3)(x+3)}{c.d.}$$

$$\frac{6x}{c.d} - \frac{(x^2+6x+9)}{c.d.} = \frac{6x-x^2-6x-9}{c.d.}$$

$$= \boxed{\frac{-x^2-9}{x(x+3)} \text{ OR } \frac{-(x^2+9)}{x(x+3)}}$$

o) ① $\frac{25y^2-10y+1}{y-4} \times \frac{y^2-11y+28}{②\,5y^2-36y+7}$

$$\frac{(5y-1)(5y-1)}{y-4} \times \frac{(y-4)(y-7)}{(5y-1)(y-7)}$$

$$\frac{(5y-1)\cancel{(5y-1)}}{\cancel{(y-4)}} \times \frac{\cancel{(y-4)}\cancel{(y-7)}}{\cancel{(5y-1)}\cancel{(y-7)}}$$

$$\boxed{5y-1}$$

① $25y^2-10y+1$ $p=25$ $s=-10$

$(25y^2-5y)(-5y+1)$ $-5,-5$

$5y(5y-1)-1(5y-1)$

$(5y-1)(5y-1)$

② $5y^2-36y+7$ $p=35$ $s=-36$

$(5y^2-35y)(-1y+7)$

$5y(y-7)-1(y-7)$ $-35,-1$

$(5y-1)(y-7)$

p) $\frac{x^2}{5x^2+x} + \frac{4-x}{5x} = \frac{x^2}{x(5x+1)} + \frac{4-x}{5x}$ c.d $=(5)(x)(5x+1)$

$$= \frac{5x^2}{c.d.} + \frac{(4-x)(5x+1)}{c.d.}$$

$$= \frac{5x^2}{c.d.} + \frac{-5x^2+19x+4}{c.d.}$$

$$= \boxed{\frac{19x+4}{5x(5x+1)}}$$

Question 5

In each of the following problems the two algebraic expressions are equivalent. In each problem, demonstrate the equivalence by transforming the expression on the left side. Show all the steps in the solutions.

a) $$\frac{-b^2-b+30}{b^2+6b} + \frac{b}{b+5} = \frac{25}{b^2+5b}$$

$-b^2-b+30$ $\quad p=-30$

$s=-1$

$-6, +5$

$(b^2-6b)+(5b+30)$

$-b(b+6)+5(b+6)$

$(-b+5)(b+6)$

$$\frac{(-b+5)(b+6)}{b(b+6)} + \frac{b}{(b+5)}$$

$$\frac{(-b+5)\cancel{(b+6)}}{b\cancel{(b+6)}} + \frac{b}{(b+5)} \qquad \text{c.d.} = b(b+5)$$

$$\frac{(-b+5)(b+5)}{\text{c.d.}} + \frac{b^2}{\text{c.d.}}$$

$$= \frac{-b^2+25}{\text{c.d.}} + \frac{b^2}{\text{c.d.}}$$

$$= \frac{-b^2+25+b^2}{\text{c.d.}}$$

$$\boxed{= \frac{25}{b(b+5)} = \frac{25}{b^2+5b}}$$

c) $\dfrac{-x^2-x+42}{x^2+7x}+\dfrac{x}{x+6}=\dfrac{36}{x^2+6x}$

$-x^2-x+42$

$p=-42$

$s=-1$

$-7,+6$

$(-x^2-7x)+(6x+42)$

$-x(x+7)+6(x+7)$

$(-x+6)(x+7)$

$$\frac{(-x+6)(x+7)}{x(x+7)}+\frac{x}{x+6}$$

$$\frac{(-x+6)\cancel{(x+7)}}{x\cancel{(x+7)}}+\frac{x}{(x+6)} \qquad \text{c.d.}=x(x+6)$$

$$\frac{(-x+6)(x+6)}{\text{c.d.}}+\frac{x^2}{\text{c.d.}}$$

$$\frac{-x^2+36}{\text{c.d.}}+\frac{x^2}{\text{c.d.}}$$

$$\boxed{\frac{36}{x(x+6)}=\frac{36}{x^2+6x}}$$

Question 6

In each of the following problems the two algebraic expressions are again equivalent. This time, demonstrate that equivalence by transforming both algebraic expressions (i.e. on the left **and** right sides). Show all the steps in the solutions.

a) $$\frac{3(x+2)}{x^2-2x-8} - \frac{2}{x-3} = \frac{1}{3-x} + \frac{2x-5}{x^2-7x+12}$$

$$\frac{3(x+2)}{(x-4)(x+2)} - \frac{2}{(x-3)}$$

$$\frac{3\cancel{(x+2)}}{(x-4)\cancel{(x+2)}} - \frac{2}{(x-3)}$$

$$\frac{3}{(x-4)} - \frac{2}{(x-3)}$$

c.d. $= (x-4)(x-3)$

$$\frac{3(x-3)}{\text{c.d.}} - \frac{2(x-4)}{\text{c.d.}}$$

$$\frac{3x-9}{\text{c.d.}} - \frac{(2x-8)}{\text{c.d.}}$$

$$\frac{3x-9-2x+8}{\text{c.d.}}$$

$$\boxed{\frac{(x-1)}{(x-4)(x-3)}} = \boxed{\frac{(x-1)}{(x-4)(x-3)}}$$

$$\frac{1}{(3-x)} + \frac{2x-5}{(x-3)(x-4)}$$

$$\frac{-1}{(x-3)} + \frac{(2x-5)}{(x-3)(x-4)}$$

c.d. $= (x-3)(x-4)$

$$\frac{-1(x-4)}{\text{c.d.}} + \frac{(2x-5)}{\text{c.d.}}$$

$$\frac{-x+4}{\text{c.d.}} + \frac{2x-5}{\text{c.d.}}$$

$$\frac{-x+4+2x-5}{\text{c.d.}}$$

c) $\dfrac{4(x+6)}{x^2+2x-24} - \dfrac{5}{x-5} = \dfrac{1}{5-x} - \dfrac{4}{x^2-9x+20}$

$$\frac{4(x+6)}{(x+6)(x-4)} - \frac{5}{(x-5)}$$

$$\frac{4\cancel{(x+6)}}{\cancel{(x+6)}(x-4)} - \frac{5}{(x-5)}$$

$$\frac{4}{(x-4)} - \frac{5}{(x-5)}$$

$$\text{c.d.} = (x-4)(x-5)$$

$$\frac{4(x-5)}{\text{c.d.}} - \frac{5(x-4)}{\text{c.d.}}$$

$$\frac{4x-20}{\text{c.d.}} - \frac{(5x-20)}{\text{c.d.}}$$

$$\frac{4x-20-5x+20}{\text{c.d.}}$$

$$\boxed{\frac{-x}{(x-4)(x-5)}}$$

$$\frac{1}{(5-x)} - \frac{4}{(x-4)(x-5)}$$

$$\frac{-1}{(x-5)} - \frac{4}{(x-4)(x-5)}$$

$$\text{c.d.} = (x-4)(x-5)$$

$$\frac{-1(x-4)}{\text{c.d.}} - \frac{4}{\text{c.d.}}$$

$$\frac{-x+4}{\text{c.d.}} - \frac{4}{\text{c.d.}}$$

$$\frac{-x+4-4}{\text{c.d.}}$$

$$\boxed{\frac{-x}{(x-4)(x-5)}}$$

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b) $10x^6y^8 - 15x^7y^7 + 20x^5y^8 - 5x^5y^6 - 5x^4y^5 + 5x^5y^5$

$\boxed{5x^4y^5(2x^2y^3 - 3x^3y^2 + 4xy^3 - xy - 1 + x)}$

c) $a^2 - 5a - 6$

$\boxed{(a-6)(a+1)}$

d) $2x^2 - 5xy + 3y^2$ $p = 6$ $s = -5$

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y) $-x^2 - xy + 6y^2$

$(-x^2 - 3xy) + (2xy + 6y^2)$

$p = -6$

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$s = -1$

$\boxed{(-x+2y)(x+3y)}$

$-3, +2$

z) $-2a^2 + 5ab - 2b^2$

$(-2a^2 + 4ab) + (1ab - 2b^2)$

$p = {}^{+}4$

$-2a(a-2b) + b(a-2b)$

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Show all the steps in the solutions.

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answer : $\boxed{6m^3n^4(3m-n)(m-4n)}$

e) $20x^6y^3 - 64x^5y^4 + 12x^4y^5$

$$4x^4y^3\underbrace{(5x^2 - 16xy + 3y^2)}$$

$p = {}^{+}15$

$s = -16$

$-15, -1$

$$(5x^2 - 15xy)(-1xy + 3y^2)$$

$$5x(x - 3y) - y(x - 3y)$$

$$(5x - y)(x - 3y)$$

answer: $\boxed{4x^4y^3(5x - y)(x - 3y)}$

f) $-7y^3z + 50y^2z^2 - 7yz^3$

$$yz\underbrace{(-7y^2 + 50yz - 7z^2)}$$

$p = {}^{+}49$

$s = {}^{+}50$

${}^{+}49, {}^{+}1$

$$(-7y^2 + 49yz) + (1yz - 7z^2)$$

$$-7y(y - 7z) + z(y - 7z)$$

$$(-7y + z)(y - 7z)$$

answer: $\boxed{yz(-7y + z)(y - 7z)}$

g) $16x^2 - 64a^4$

$$16(x^2 - 4a^4)$$

$$\boxed{16(x - 2a^2)(x + 2a^2)}$$

Question 3

Reduce the following algebraic fractions to their lowest terms.
Show all the steps in the solutions.

a) $\dfrac{16m^4 - n^6}{3m^2n^4 - 12m^4n} = \dfrac{(4m^2 - n^3)(4m^2 + n^3)}{3m^2n(n^3 - 4m^2)}$

$= \dfrac{-1\cancel{(-4m^2 + n^3)}(4m^2 + n^3)}{3m^2n\cancel{(n^3 - 4m^2)}}$

$= \boxed{\dfrac{-(4m^2 + n^3)}{3m^2n}}$

b) $\dfrac{x^2 - 8x + 7}{147 - 3x^2} = \dfrac{(x-7)(x-1)}{3(49 - x^2)} = \dfrac{(x-7)(x-1)}{3(7-x)(7+x)}$

$= \dfrac{-1\cancel{(-x+7)}(x-1)}{3\cancel{(7-x)}(7+x)}$

$= \boxed{\dfrac{-(x-1)}{3(7+x)}}$

c) $\dfrac{25x^4 - 4y^2}{4xy^3 - 10x^3y^2} = \dfrac{(5x^2 - 2y)(5x^2 + 2y)}{2xy^2(2y - 5x^2)}$

$= \dfrac{-1\cancel{(-5x^2 + 2y)}(5x^2 + 2y)}{2xy^2\cancel{(2y - 5x^2)}} = \boxed{\dfrac{-(5x^2 + 2y)}{2xy^2}}$

b) $\dfrac{5x^2}{x^3} - \dfrac{4x-x^2}{(x-4)} = \dfrac{5}{x} - \dfrac{x(4-x)}{(x-4)}$

$$= \frac{5}{x} - \frac{-x(\cancel{-4+x})}{\cancel{(x-4)}}$$

$$= \frac{5}{x} + \frac{x}{1} \qquad c.d = x$$

$$= \frac{5}{x} + \frac{x^2}{x}$$

$$= \boxed{\frac{5+x^2}{x}}$$

c) $\dfrac{9-x^2}{x} \times \dfrac{3x-9}{-9x-3x^2}$

$$= \frac{(3-x)(3+x)}{x} \cdot \frac{3(x-3)}{-3x(3+x)} = \frac{(3-x)\cancel{(3+x)}}{x} \cdot \frac{\cancel{3}(x-3)}{-\cancel{3}x\cancel{(3+x)}}$$

$$= \boxed{\frac{-(3-x)(x-3)}{x^2}}$$

OR $\dfrac{(x-3)(x-3)}{x^2} = \dfrac{(x-3)^2}{x^2}$

d) $\dfrac{1}{4-y} + \dfrac{y-3}{y(y-7)} \qquad c.d = y(4-y)(y-7)$

$$\frac{y(y-7)}{c.d.} + \frac{(y-3)(4-y)}{c.d.} = \frac{y^2-7y}{c.d.} + \frac{-y^2+7y-12}{c.d.}$$

$$= \frac{y^2-7y-y^2+7y-12}{c.d.}$$

$$= \boxed{\frac{-12}{y(4-y)(y-7)}}$$

h) $\dfrac{1}{5-w} + \dfrac{w-4}{w(w-9)}$ $\quad$ c.d. $= w(5-w)(w-9)$

$$\frac{w(w-9)}{c.d.} + \frac{(w-4)(5-w)}{c.d.}$$

$$\frac{w^2-9w}{c.d.} + \frac{-w^2+9w-20}{c.d.} = \frac{w^2-9w-w^2+9w-20}{c.d.}$$

$$= \boxed{\frac{-20}{w(5-w)(w-9)}}$$

i) $\dfrac{4-y^2}{y-2} \div \dfrac{y^2-2y-8}{y-4}$

$$\frac{(2-y)(2+y)}{(y-2)} \times \frac{(y-4)}{(y-4)(y+2)}$$

$$-1\frac{\cancel{(-2+y)}\cancel{(2+y)}}{\cancel{(y-2)}} \cdot \frac{\cancel{(y-4)}}{\cancel{(y-4)}\cancel{(y+2)}} = \boxed{-1}$$

j) $\dfrac{12m^4}{m^5} - \dfrac{6m-m^2}{m-6} = \dfrac{12}{m} - \dfrac{m(6-m)}{(m-6)}$

$$= \frac{12}{m} - \frac{m(-1)\cancel{(-6+m)}}{\cancel{(m-6)}}$$

$$= \frac{12}{m} - \frac{-m}{1} = \frac{12}{m} + \frac{m}{1} \qquad c.d = m$$

$$= \frac{12}{m} + \frac{m^2}{m} = \boxed{\frac{12+m^2}{m}}$$

n) $\dfrac{6}{x+3} - \dfrac{x+3}{x}$ c.d. = $x(x+3)$

$$\frac{6x}{c.d.} - \frac{(x+3)(x+3)}{c.d.}$$

$$\frac{6x}{c.d} - \frac{(x^2+6x+9)}{c.d.} = \frac{6x - x^2 - 6x - 9}{c.d.}$$

$$= \boxed{\frac{-x^2-9}{x(x+3)} \text{ OR } \frac{-(x^2+9)}{x(x+3)}}$$

o) ① $\dfrac{25y^2-10y+1}{y-4} \times$ ② $\dfrac{y^2-11y+28}{5y^2-36y+7}$

① $25y^2-10y+1$ p = 25 s = −10 −5, −5

$(25y^2-5y)(-5y+1)$

$5y(5y-1)-1(5y-1)$

$(5y-1)(5y-1)$

② $5y^2-36y+7$ p = 35 s = −36 −35, −1

$(5y^2-35y)(-1y+7)$

$5y(y-7)-1(y-7)$

$(5y-1)(y-7)$

$$\frac{(5y-1)(5y-1)}{y-4} \times \frac{(y-4)(y-7)}{(5y-1)(y-7)}$$

$$\frac{(5y-1)\cancel{(5y-1)}}{\cancel{(y-4)}} \times \frac{\cancel{(y-4)}\cancel{(y-7)}}{\cancel{(5y-1)}\cancel{(y-7)}}$$

$$\boxed{5y-1}$$

p) $\dfrac{x^2}{5x^2+x} + \dfrac{4-x}{5x} = \dfrac{x^2}{x(5x+1)} + \dfrac{4-x}{5x}$ c.d = $(5)(x)(5x+1)$

$$= \frac{5x^2}{c.d.} + \frac{(4-x)(5x+1)}{c.d.}$$

$$= \frac{5x^2}{c.d.} + \frac{-5x^2+19x+4}{c.d.}$$

$$= \boxed{\frac{19x+4}{5x(5x+1)}}$$

Question 5

In each of the following problems the two algebraic expressions are equivalent. In each problem, demonstrate the equivalence by transforming the expression on the left side. Show all the steps in the solutions.

a) $$\frac{-b^2-b+30}{b^2+6b} + \frac{b}{b+5} = \frac{25}{b^2+5b}$$

$-b^2-b+30$ $\quad p = -30$

$s = -1$

$-6, +5$

$(b^2-6b)+(5b+30)$

$-b(b+6)+5(b+6)$

$(-b+5)(b+6)$

$$\frac{(-b+5)(b+6)}{b(b+6)} + \frac{b}{(b+5)}$$

$$\frac{(-b+5)\cancel{(b+6)}}{b\cancel{(b+6)}} + \frac{b}{(b+5)} \qquad \text{c.d.} = b(b+5)$$

$$\frac{(-b+5)(b+5)}{\text{c.d.}} + \frac{b^2}{\text{c.d.}}$$

$$= \frac{-b^2+25}{\text{c.d.}} + \frac{b^2}{\text{c.d.}}$$

$$= \frac{-b^2+25+b^2}{\text{c.d.}}$$

$$\boxed{= \frac{25}{b(b+5)} = \frac{25}{b^2+5b}}$$

c) $\frac{-x^2-x+42}{x^2+7x} + \frac{x}{x+6} = \frac{36}{x^2+6x}$

$-x^2-x+42$

$p = -42$

$s = -1$

$-7, +6$

$(-x^2-7x)+(6x+42)$

$-x(x+7)+6(x+7)$

$(-x+6)(x+7)$

$$\frac{(-x+6)(x+7)}{x(x+7)} + \frac{x}{x+6}$$

$$\frac{(-x+6)\cancel{(x+7)}}{x\cancel{(x+7)}} + \frac{x}{(x+6)} \quad \text{c.d.} = x(x+6)$$

$$\frac{(-x+6)(x+6)}{\text{c.d.}} + \frac{x^2}{\text{c.d.}}$$

$$\frac{-x^2+36}{\text{c.d.}} + \frac{x^2}{\text{c.d.}}$$

$$\boxed{\frac{36}{x(x+6)} = \frac{36}{x^2+6x}}$$

Question 6

In each of the following problems the two algebraic expressions are again equivalent. This time, demonstrate that equivalence by transforming both algebraic expressions (i.e. on the left **and** right sides). Show all the steps in the solutions.

a) $$\frac{3(x+2)}{x^2-2x-8} - \frac{2}{x-3} = \frac{1}{3-x} + \frac{2x-5}{x^2-7x+12}$$

$$\frac{3(x+2)}{(x-4)(x+2)} - \frac{2}{(x-3)}$$

$$\frac{3\cancel{(x+2)}}{(x-4)\cancel{(x+2)}} - \frac{2}{(x-3)}$$

$$\frac{3}{(x-4)} - \frac{2}{(x-3)}$$

c.d. = $(x-4)(x-3)$

$$\frac{3(x-3)}{\text{c.d.}} - \frac{2(x-4)}{\text{c.d.}}$$

$$\frac{3x-9}{\text{c.d.}} - \frac{(2x-8)}{\text{c.d.}}$$

$$\frac{3x-9-2x+8}{\text{c.d.}}$$

$$\boxed{\frac{(x-1)}{(x-4)(x-3)}}$$

$$\frac{1}{(3-x)} + \frac{2x-5}{(x-3)(x-4)}$$

$$\frac{-1}{(x-3)} + \frac{(2x-5)}{(x-3)(x-4)}$$

c.d. = $(x-3)(x-4)$

$$\frac{-1(x-4)}{\text{c.d.}} + \frac{(2x-5)}{\text{c.d.}}$$

$$\frac{-x+4}{\text{c.d.}} + \frac{2x-5}{\text{c.d.}}$$

$$\frac{-x+4+2x-5}{\text{c.d.}}$$

$$\boxed{\frac{(x-1)}{(x-4)(x-3)}}$$

$$\boxed{\frac{(x-1)}{(x-4)(x-3)}} = \boxed{\frac{(x-1)}{(x-4)(x-3)}}$$

c) $\dfrac{4(x+6)}{x^2+2x-24} - \dfrac{5}{x-5} = \dfrac{1}{5-x} - \dfrac{4}{x^2-9x+20}$

$$\frac{4(x+6)}{(x+6)(x-4)} - \frac{5}{(x-5)}$$

$$\frac{4\cancel{(x+6)}}{\cancel{(x+6)}(x-4)} - \frac{5}{(x-5)}$$

$$\frac{4}{(x-4)} - \frac{5}{(x-5)}$$

$$\text{c.d.} = (x-4)(x-5)$$

$$\frac{4(x-5)}{\text{c.d.}} - \frac{5(x-4)}{\text{c.d.}}$$

$$\frac{4x-20}{\text{c.d.}} - \frac{(5x-20)}{\text{c.d.}}$$

$$\frac{4x-20-5x+20}{\text{c.d.}}$$

$$\boxed{\frac{-x}{(x-4)(x-5)}}$$

$$\frac{1}{(5-x)} - \frac{4}{(x-4)(x-5)}$$

$$\frac{-1}{(x-5)} - \frac{4}{(x-4)(x-5)}$$

$$\text{c.d.} = (x-4)(x-5)$$

$$\frac{-1(x-4)}{\text{c.d.}} - \frac{4}{\text{c.d.}}$$

$$\frac{-x+4}{\text{c.d.}} - \frac{4}{\text{c.d.}}$$

$$\frac{-x+4-4}{\text{c.d.}}$$

$$\boxed{\frac{-x}{(x-4)(x-5)}}$$

$$=$$