double sided !

MTH – 4107 – 1
Straight Lines II

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DATE: _____

Review Booklet

Question 1

a) Determine the equation of the line that passes through point $\left(\frac{1}{3},-2\right)$ and is parallel to the line whose equation is $2x-6y=7$.

Clearly show all your work. (10 marks)

l_1

$2x-6y=7$

$\frac{-6y}{-6} = \frac{-2x}{-6} + \frac{7}{-6}$

$y = \frac{1}{3}x - \frac{7}{6}$

$m = \frac{1}{3}$

l_2

$m = \frac{1}{3}$

$y = mx + b$

$-2 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + b$

$-2 = \frac{1}{9} + b$

$-2\frac{1}{9} = b$

$y = \frac{1}{3}x - 2\frac{1}{9}$

b) Determine the equation of the line that passes through point $(2,2)$ and is parallel to the line whose equation is $x-\frac{2}{3}y=-3$.

Clearly show all your work. (10 marks)

l_1

$x-\frac{2}{3}y=-3$

$\left(\frac{-3}{2}\right)-\frac{2}{3}y=(-x-3)\frac{-3}{2}$

$y=\frac{3}{2}x+\frac{9}{2}$

$m=\frac{3}{2}$

l_2

$m=\frac{3}{2}$

$y=mx+b$

$2=\frac{3}{2}(2)+b$

$2=\frac{6}{2}+b$

$2-3=b$

$-1=b$

$y=\frac{3}{2}x-1$

c) Determine the equation of the line that passes through point $\left(\frac{1}{4},-1\right)$ and is parallel to the line whose equation is $2x-8y=9$.

Clearly show all your work. (10 marks)

l_1

$2x-8y=9$

$\frac{-8y}{-8} = \frac{-2x}{-8} + \frac{9}{-8}$

$y = \frac{1}{4}x - \frac{9}{8}$

$m = \frac{1}{4}$

l_2

$m = \frac{1}{4}$

$y = mx + b$

$-1 = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + b$

$-1 = \frac{1}{16} + b$

$-1\frac{1}{16} = b$

$y = \frac{1}{4}x - 1\frac{1}{16}$

d) Determine the equation of the line that passes through point $(5,-2)$ and is parallel to the line whose equation is $x-\frac{5}{4}y=-1$.

Clearly show all your work. (10 marks)

l_1

$x-\frac{5}{4}y=-1$

$\left(-\frac{4}{5}\right)-\frac{5}{4}y=\left(-x-1\right)-\frac{4}{5}$

$y=\frac{4}{5}x+\frac{4}{5}$

$m=\frac{4}{5}$

l_2

$m=\frac{4}{5}$

$y=mx+b$

$-2=\left(\frac{4}{5}\right)(5)+b$

$-2=\frac{20}{5}+b$

$-2=4+b$

$-2-4=b$

$-6=b$

$y=\frac{4}{5}x-6$

Question 2

a) Determine the equation of the line that passes through point $\left(5,-\frac{1}{6}\right)$ and is perpendicular to the line whose equation is $6x-5y+4=0$.

Clearly show all your work. (10 marks)

l_1

$6x-5y+4=0$

$\frac{-5y}{-5} = \frac{-6x}{-5} - \frac{4}{-5}$

$y = \frac{6}{5}x + \frac{4}{5}$

$m = \frac{6}{5}$

l_2

$m = -\frac{5}{6}$

$y = mx+b$

$-\frac{1}{6} = \left(-\frac{5}{6}\right)(5) + b$

$-\frac{1}{6} = -\frac{25}{6} + b$

$-\frac{1}{6} + \frac{25}{6} = b$

$\frac{24}{6} = b$

$4 = b$

$y = -\frac{5}{6}x + 4$

b) Determine the equation of the line that passes through point $(2,-4)$ and is perpendicular to the line whose equation is $2x-\frac{3y}{4}-2=0$.

Clearly show all your work. (10 marks)

l_1

$2x-\frac{3y}{4}-2=0$

$\left(-\frac{4}{3}\right)-\frac{3y}{4}=(-2x+2)\left(\frac{-4}{3}\right)$

$y=\frac{8}{3}x-\frac{8}{3}$

$m=\frac{8}{3}$

l_2

$m=-\frac{3}{8}$

$y=mx+b$

$-4=\left(\frac{-3}{8}\right)(2)+b$

$-4=\frac{-6}{8}+b$

$-4=-\frac{3}{4}+b$

$-4+\frac{3}{4}=b$

$-3\frac{1}{4}=b$

$y=-\frac{3}{8}x-3\frac{1}{4}$

c) Determine the equation of the line that passes through point $\left(3,-\frac{1}{8}\right)$ and is perpendicular to the line whose equation is $8x-3y+6=0$.

Clearly show all your work. (10 marks)

l_1

$8x-3y+6=0$

$\frac{-3y}{-3} = \frac{-8x}{-3} \frac{-6}{-3}$

$y = \frac{8}{3}x + 2$

$m = \frac{8}{3}$

l_2

$m = -\frac{3}{8}$

$y = mx + b$

$-\frac{1}{8} = \left(-\frac{3}{8}\right)(3) + b$

$-\frac{1}{8} = -\frac{9}{8} + b$

$-\frac{1}{8} + \frac{9}{8} = b$

$\frac{8}{8} = b$

$1 = b$

$y = -\frac{3}{8}x + 1$

d) Determine the equation of the line that passes through point $(2,-3)$ and is perpendicular to the line whose equation is $2x-\frac{4y}{5}-3=0$.

Clearly show all your work. (10 marks)

l_1

$2x-\frac{4y}{5}-3=0$

$\left(-\frac{5}{4}\right)-\frac{4}{5}y=-2x+3\left(-\frac{5}{4}\right)$

$y=\frac{10}{4}x-\frac{15}{4}$

$y=\frac{5}{2}x-\frac{15}{4}$

$m=\frac{5}{2}$

l_2

$m=-\frac{2}{5}$

$y=mx+b$

$-3=\left(-\frac{2}{5}\right)(2)+b$

$-3=-\frac{4}{5}+b$

$-3+\frac{4}{5}=b$

$-2\frac{1}{5}=b$

$y=-\frac{2}{5}x-2\frac{1}{5}$

Question 3

a) Determine the equation of the line that passes through point $\left(-\frac{3}{4}, 2\right)$ and is parallel to the line whose equation is $-2x-5=0$.

Clearly show all your work. (10 marks)

l_1

$-2x-5=0$

$\frac{-2x}{-2} = \frac{5}{-2}$

$x = \frac{-5}{2}$

Vertical line through $x = \frac{-5}{2}$

l_2

ANS: $x = \frac{-3}{4}$

Vertical line through $x = \frac{-3}{4}$

b) Determine the equation of the line that passes through point $\left(-1, \frac{1}{4}\right)$ and is parallel to the line whose equation is $-\frac{3}{2}y + 6 = 0$.

Clearly show all your work. (10 marks)

l_1

$-\frac{3}{2}y + 6 = 0$

$\left(\frac{-2}{3}\right) -\frac{3}{2}y = -6\left(\frac{-2}{3}\right)$

$y = \frac{12}{3} = 4$

horizontal line through $y = 4$

l_2

ANS: $y = \frac{1}{4}$

(horizontal line through $y = \frac{1}{4}$)

c) Determine the equation of the line that passes through point $\left(-\frac{3}{7},1\right)$ and is parallel to the line whose equation is $-6x-14=0$.

Clearly show all your work. (10 marks)

l_1

$-6x-14=0$

$\frac{-6x}{-6} = \frac{14}{-6}$

$x = \frac{-14}{6} = \frac{-7}{3}$

Vertical line

l_2

Vertical line

ANS: $x = \frac{-3}{7}$

d) Determine the equation of the line that passes through point $\left(-\frac{2}{5}, \frac{3}{7}\right)$ and is parallel to the line whose equation is $-\frac{9}{4}y + 9 = 0$.

Clearly show all your work. (10 marks)

l_1

$-\frac{9}{4}y + 9 = 0$

$\left(-\frac{4}{9}\right) - \frac{9}{4}y = -9\left(\frac{-4}{9}\right)$

$y = \frac{36}{9} = 4$

horizontal line

l_2

ANS: $y = \frac{3}{7}$

horizontal line

e) Determine the equation of the line that passes through point $\left(-\frac{1}{2},6\right)$ and is parallel to the line whose equation is $-3x-10=0$.

Clearly show all your work. (10 marks)

l_1

$-3x-10=0$

$\frac{-3x}{-3} = \frac{10}{-3}$

$x = \frac{-10}{3}$

vertical line

l_2

ans : $\boxed{x = -\frac{1}{2}}$

vertical line

f) Determine the equation of the line that passes through point $\left(-\frac{13}{7}, \frac{5}{6}\right)$ and is parallel to the line whose equation is $-\frac{12}{5}y + 12 = 0$.

Clearly show all your work. (10 marks)

l_1

l_2

$-\frac{12}{5}y + 12 = 0$

ANS : $\boxed{y = \frac{5}{6}}$

$\left(-\frac{5}{12}\right)\frac{-12}{5}y = -12\left(-\frac{5}{12}\right)$

$y = 5$

Question 4

I. Given the following five equations:

l_1: $\frac{-5}{4}x = 5$

l_2: $2y - 3x = 10$

$\frac{2y}{2} = \frac{3x}{2} + \frac{10}{2}$ $\quad m = \frac{3}{2}$

l_3: $8x + 12y + 15 = 0$

$\frac{12y}{12} = \frac{-8x}{12} - \frac{15}{12}$ $\quad m = -\frac{2}{3}$

l_4: $2x + 3y - 4 = 0$

$\frac{3y}{3} = -\frac{2x}{3} + \frac{4}{3}$ $\quad m = -\frac{2}{3}$

l_5: $2y = 10$

a) Determine what line is concurrent with l_1 in point (-4,4). Clearly show all your work and justify it.

ans: l_4 : $2x + 3y - 4 = 0$

$2(-4) + 3(4) - 4 = 0$

$-8 + 12 - 4 = 0$

b) Determine what line is concurrent with l_5 in its *y*-intercept. Clearly show all your work and justify it.

l_5: $\frac{2y}{2} = \frac{10}{2}$ $\quad y = 5$

$y\text{-int} = 5$

ans: l_2 : $2y - 3x = 10$

$\frac{2y}{2} = \frac{3x}{2} + \frac{10}{2}$ (y-int.)

$y = \frac{3}{2}x + 5$

c) Find a line that is perpendicular to l_4. Clearly show all your work and justify it.

l_4: $m = -\frac{2}{3}$

$l_2 \perp l_4$

ans: l_2 is perpendicular to l_4 since l_2 has a slope $= \frac{3}{2}$, which is the negative reciprocal of l_4's slope $(-\frac{2}{3})$.

d) Find a line that is parallel to l_3. Clearly show all your work and justify it.

l_3 : $m = -\frac{2}{3}$ (see above)

$\therefore$ l_4 is parallel to l_3 since they have the same slope.

$l_4 \parallel l_3$

II. Given the following five equations:

l_1: $\frac{1x}{1.25} = \frac{1.25y}{1.25}$ $\therefore$ $y = 0.8x$ (or $y = \frac{4}{5}x$)

l_2: $\frac{-7}{5}x = 7$

l_3: $2y = 4$

l_4: $5x + 4y + 5 = 0$

$\frac{4y}{4} = -\frac{5x}{4} - \frac{5}{4}$ $m = -\frac{5}{4}$

l_5: $5y - 4x = 10$

$\frac{5y}{5} = \frac{4x}{5} + \frac{10}{5}$ $m = \frac{4}{5}$

a) Find a line that is parallel to l_1. Clearly show all your work and justify it.

$l_1 \parallel l_5$ both have $m = \frac{4}{5}$ (see above calculations)

b) Determine what line is concurrent with l_2 in point (-5,5). Clearly show all your work and justify it.

ans: l_4 →

$5x + 4y + 5 = 0$

$5(-5) + 4(5) + 5 = 0$

$-25 + 20 + 5 = 0$

$0 = 0$ ✓

c) Find a line that is perpendicular to l_1. Clearly show all your work and justify it.

$l_4 \perp l_1$ since $m_{l_4} \cdot m_{l_1} = -1$

(See above calculations)

d) Determine what line is concurrent with l_3 in its y-intercept. Clearly show all your work and justify it.

ans: l_5

l_3: $\frac{2y}{2} = \frac{4}{2}$

$y = 2$

y-int = 2

$5y - 4x = 10$

$\frac{5y}{5} = \frac{4x}{5} + \frac{10}{5}$

$y = \frac{4}{5}x + 2$

↑

y-int = 2

III. Given the following five equations:

l_1: $x-3y=7$

$\frac{-3y}{-3} = \frac{-x}{-3} + \frac{7}{-3}$ $\quad y = \frac{1}{3}x - \frac{7}{3}$ $\quad m = \frac{1}{3}$

l_2: $-\frac{5}{6}x = 5$

undefined slope

l_3: $\frac{3y}{3} = \frac{7}{3}$ $\quad y = \frac{7}{3}$ $\quad \therefore$ y-int $= \frac{7}{3}$ $\quad m = 0$

l_4: $6x+2y+5=0$

$\frac{2y}{2} = \frac{-6x}{2} - \frac{5}{2}$ $\quad y = -3x - \frac{5}{2}$ $\quad m = -3$

l_5: $3y-9x=7$

$\frac{3y}{3} = \frac{9x}{3} + \frac{7}{3}$ $\quad m = 3$

$y = 3x + \frac{7}{3}$

a) Determine whether ~~l_1 is perpendicular~~ to l_4. Clearly show all your work and justify it.

yes, $l_1 \perp l_4$ since $m_{l_1} \cdot m_{l_4} = -1$

(See above for slope calculations)

($m_{l_1} = \frac{1}{3}$, $m_{l_4} = -3$)

b) Determine what line is concurrent with l_2 in point $\left(-6, 15\frac{1}{2}\right)$. Clearly show all your work and ~~justify~~ it.

ans: l_4

$6x+2y+5=0$

$6(-6)+2(15\frac{1}{2})+5=0$

$-36+31+5=0$

$0=0$ ✓

c) Determine what line is concurrent with l_3 in its y-intercept. Clearly show all your work and justify it.

ans: l_5

$3y-9x=7$

$\frac{3y}{3} = \frac{9x}{3} + \frac{7}{3}$

$y = 3x + \frac{7}{3}$ ← y-int.

d) Find a line that is parallel to l_5. Clearly show all your work and justify it.

l_5: $m=3$, (See above calculations) There is no line parallel to l_5. No other line has $m=3$.

e) Determine whether l_2 and l_3 are perpendicular lines. Clearly show all your work and justify it.

yes $l_2 \perp l_3$ → since l_2 is a vertical line and l_3 is a horizontal line, they are perpendicular: ⊥

Question 5

I. The following expressions represent the distance between two points.

1) $\sqrt{(-1+4)^2+(-7-2)^2}$

2) $|-1-1|$

3) $\sqrt{(1+1)^2+(-7+7)^2}$

4) $\sqrt{(-4-1)^2+(2+7)^2}$

5) $|-7-7|$

Points A(-1,-7), B(-4,2), and C(1,-7) were used to define the segments below.

Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AC}$ 2, 3

b) $\overline{BA}$ 1

c) $\overline{BC}$ 4

$\overline{AC} = \sqrt{(-7+7)^2+(1+1)^2}$

$= \sqrt{0^2+2^2} = \sqrt{4} = 2$

$\overline{BA} = \sqrt{(2+7)^2+(-4+1)^2}$

$= \sqrt{9^2+(-3)^2}$

$= \sqrt{81+9}$

$\overline{BC} = \sqrt{(-7-2)^2+(1+4)^2}$

$= \sqrt{(-9)^2+(5)^2}$

II. The following expressions represent the distance between two points.

1) $\sqrt{(5-4)^2+(0+8)^2}$

2) $\sqrt{(5+8)^2+(0+3)^2}$

3) $\sqrt{(-3-5)^2+(-8-0)^2}$

4) $\sqrt{(4+3)^2+(-8+8)^2}$

5) $|-4+11|$

Points A(-3,-8), B(5,0), and C(4,-8) were used to define the segments below.

Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AC}$ 5, 4

b) $\overline{BA}$ 3

c) $\overline{BC}$ 1

$\overline{AC} = \sqrt{(-8+8)^2+(4+3)^2}$

$= \sqrt{0^2+(7)^2} = \sqrt{49} = 7$

$\overline{BA} = \sqrt{(0+8)^2+(5+3)^2}$

$= \sqrt{8^2+8^2}$

$\overline{BC} = \sqrt{(-8-0)^2+(4-5)^2}$

$= \sqrt{(-8)^2+(-1)^2}$

III. The following expressions represent the distance between two points.

1) $\sqrt{(-2+1)^2+(5+9)^2}$

2) $|-1-1|$

3) $\sqrt{(5+9)^2+(-2-0)^2}$

4) $|3-2|$

5) $\sqrt{(-1-0)^2+(-9+9)^2}$

Points A(-1,-9), B(-2,5), and C(0,-9) were used to define the segments below.

Determine which expression(s) correspond(s) to each segment. Write the number for the expression in the space provided.

a) $\overline{AC}$ 4, 5

b) $\overline{BA}$ 1

c) $\overline{BC}$ 3

$\overline{AC}$: $\sqrt{(9+9)^2+(0+1)^2}$

$= \sqrt{0^2+1^2} = \sqrt{1} = 1$

$\overline{BA}$: $\sqrt{(5+9)^2+(-2+1)^2}$

$= \sqrt{14^2+(-1)^2}$

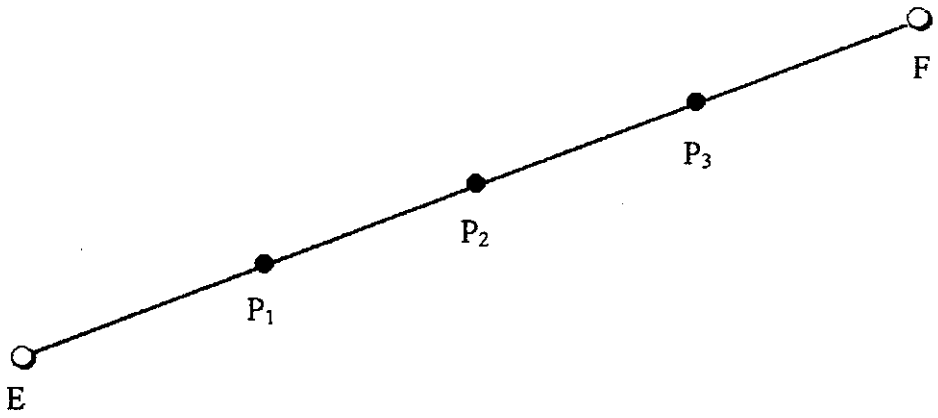
$= \sqrt{14^2+(-1)^2}$

$\overline{BC}$: $\sqrt{(-9-5)^2+(0+2)^2}$

$= \sqrt{14^2+2^2}$

Question 6

I. Points P_1, P_2, and P_3 divide segment EF into four equal parts.



Determine the point that corresponds to each statement below.

a) Divides $\overline{EP_3}$ in a ratio of $\frac{1}{2}$. P_1

b) Is located $\frac{3}{4}$ of the way along $\overline{FE}$. P_1

c) Is located at the midpoint of $\overline{FE}$. P_2

d) Divides $\overline{FP_1}$ in a ratio of $\frac{2}{1}$. P_2

e) Is located two-thirds of the way along $\overline{P_1F}$. P_3

f) Divides $\overline{FE}$ in a ratio of $\frac{3}{1}$. P_1

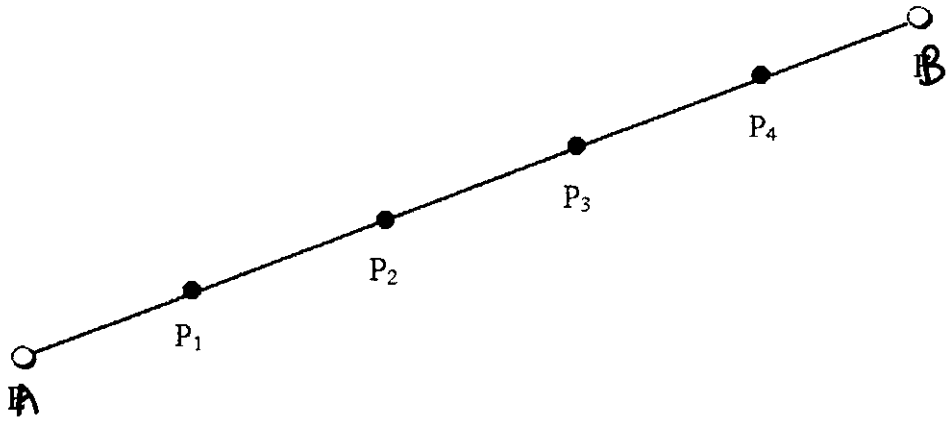
g) Is located at the midpoint of $\overline{P_3P_1}$. P_2

h) Divides $\overline{P_1F}$ in a ratio of $\frac{1}{2}$. P_2

i) Is located one-third of the way along $\overline{FP_1}$. P_3

j) Divides $\overline{EF}$ in a ratio of $\frac{1}{1}$. P_2

II. Points P_1, P_2, P_3, and P_4 divide segment AB into five equal parts.



Determine the point that corresponds to each statement below.

a) Is located one-quarter of the way along $\overline{P_4A}$. P_3

b) Divides $\overline{P_2B}$ in a ratio of $\frac{2}{1}$. P_4

c) Is located three-fifths of the way along $\overline{BA}$. P_2

d) Divides $\overline{AP_4}$ in a ratio of $\frac{1}{3}$. P_1

e) Is located at the midpoint of $\overline{AP_4}$. P_2

f) Divides $\overline{BA}$ in a ratio of $\frac{2}{3}$. P_3

g) Is located three-quarters of the way along $\overline{AP_4}$. P_3

h) Divides $\overline{P_4P_1}$ in a ratio of $\frac{2}{1}$. P_2

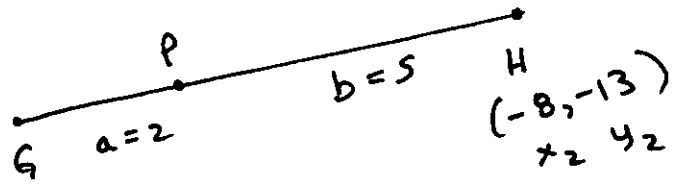
i) Is located two-thirds of the way along $\overline{P_4P_1}$. P_2

j) Divides $\overline{AB}$ in a ratio of $\frac{1}{4}$. P_1

Question 7

I. Calculate the coordinates of the point that divides segment $\overline{GH}$ in a ratio of $\frac{2}{5}$.

The coordinates of point G are (6,5) and those of point H are (-8,-13). Show all the steps in the solution.



$(6,5)$
x_1, y_1

(x, y)

$\frac{ax_2 + bx_1}{a+b}$ $\quad$ $\frac{ay_2 + by_1}{a+b}$

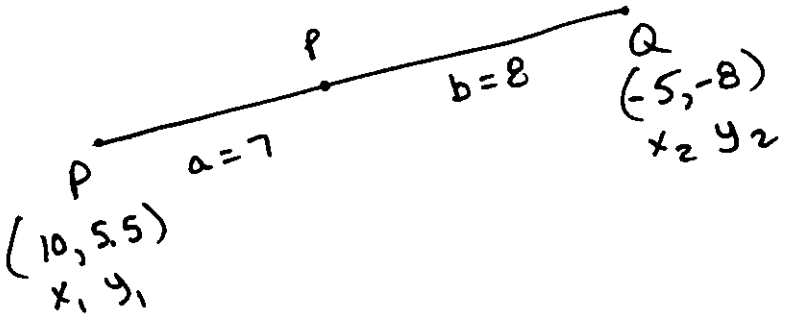
$\frac{(2)(-8) + (5)(6)}{2+5}$ $\quad$ $\frac{(2)(-13) + (5)(5)}{2+5}$

$\frac{-16 + 30}{7}$ $\quad$ $\frac{-26 + 25}{7}$

$\frac{14}{7}$ $\quad$ $\frac{-1}{7}$

answer: $\left(2, -\frac{1}{7}\right)$

II. Calculate the coordinates of the point that divides segment $\overline{PQ}$ in a ratio of $\frac{7}{8}$. The coordinates of point P are (10,5.5) and those of point Q are (-5,-8). Show all the steps in the solution.



$(\ x\ ,\ y\ )$

$\frac{ax_2 + bx_1}{a+b}$ $\qquad$ $\frac{ay_2 + by_1}{a+b}$

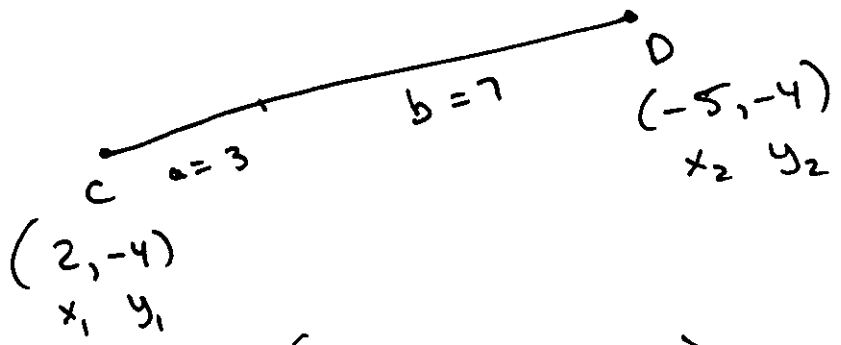
$\frac{(7)(-5) + (8)(10)}{7+8}$ $\qquad$ $\frac{(7)(-8) + (8)(5.5)}{7+8}$

$\frac{-35 + 80}{15}$ $\qquad$ $\frac{-56 + 44}{15}$

$\frac{45}{15}$ $\qquad$ $\frac{-12}{15}$

answer: $\left(3, -\frac{4}{5}\right)$

III. Calculate the coordinates of the point that divides segment $\overline{CD}$ in a ratio of $\frac{3}{7}$. The coordinates of point C are (2,-4) and those of point D are (-5,-4). Show all the steps in the solution.



(x, y)

$$\frac{ax_2 + bx_1}{a+b} \qquad \frac{ay_2 + by_1}{a+b}$$

$$\frac{(3)(-5)+(7)(2)}{3+7} \qquad \frac{(3)(-4)+(7)(-4)}{3+7}$$

$$\frac{-15+14}{10} \qquad \frac{-12+-28}{10}$$

$$\frac{-1}{10} \qquad \frac{-40}{10}$$

answer: $\left(-\frac{1}{10}, -4\right)$

Question 8

I. The Dunne family (from Montreal) made two stops on the way to their vacation destination, Niagara Falls. They made their first stop (S_1) one-quarter of the way to Niagara Falls and the second stop (S_2) after covering $\frac{3}{5}$ of the total distance.

One unit corresponds to 25 Km. Determine the distance between the two stops $(S_1 - S_2)$ by identifying the coordinates of points S_1 and S_2. Clearly show all your work.

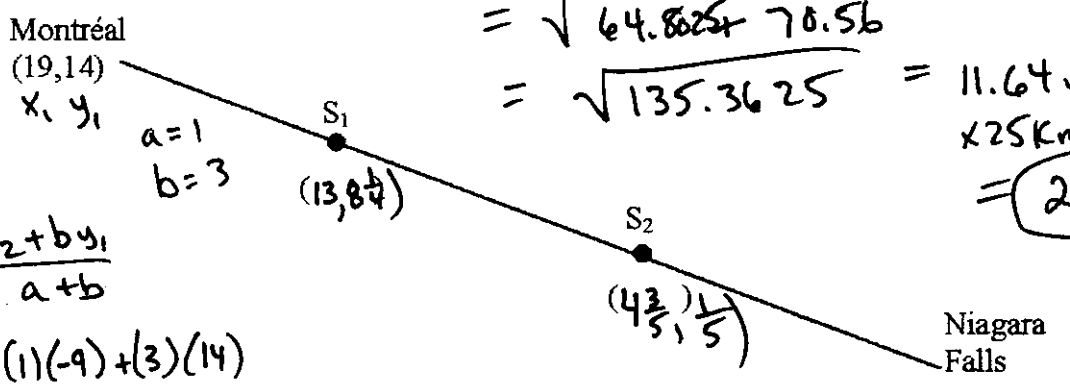
$$d = \sqrt{\left(\tfrac{1}{5} - 8\tfrac{1}{4}\right)^2 + \left(4\tfrac{3}{5} - 13\right)^2}$$

$$= \sqrt{64.8025 + 70.56}$$

$$= \sqrt{135.3625} = 11.64 \text{ u}$$

$\times 25 \text{ Km}$

$= 290.9 \text{ Km}$



S_1 $\quad a = 1$, $b = 3$

$$\frac{ax_2 + bx_1}{a+b}, \frac{ay_2 + by_1}{a+b}$$

$$\frac{(1)(-5) + (3)(19)}{4}, \frac{(1)(-9) + (3)(14)}{4}$$

$$\frac{-5 + 57}{4} \quad \frac{-9 + 42}{4}$$

$$\frac{52}{4} \quad \frac{33}{4}$$

$$\left(13, 8\tfrac{1}{4}\right)$$

S_2 $\quad a = 3$, $b = 2$

$$\frac{ax_2 + bx_1}{a+b}, \frac{ay_2 + by_1}{a+b}$$

$$\frac{(3)(-5) + (2)(19)}{5}, \frac{(3)(-9) + (2)(14)}{5}$$

$$\frac{-15 + 38}{5} \quad \frac{-27 + 28}{5}$$

$$\frac{23}{5} \quad \frac{1}{5}$$

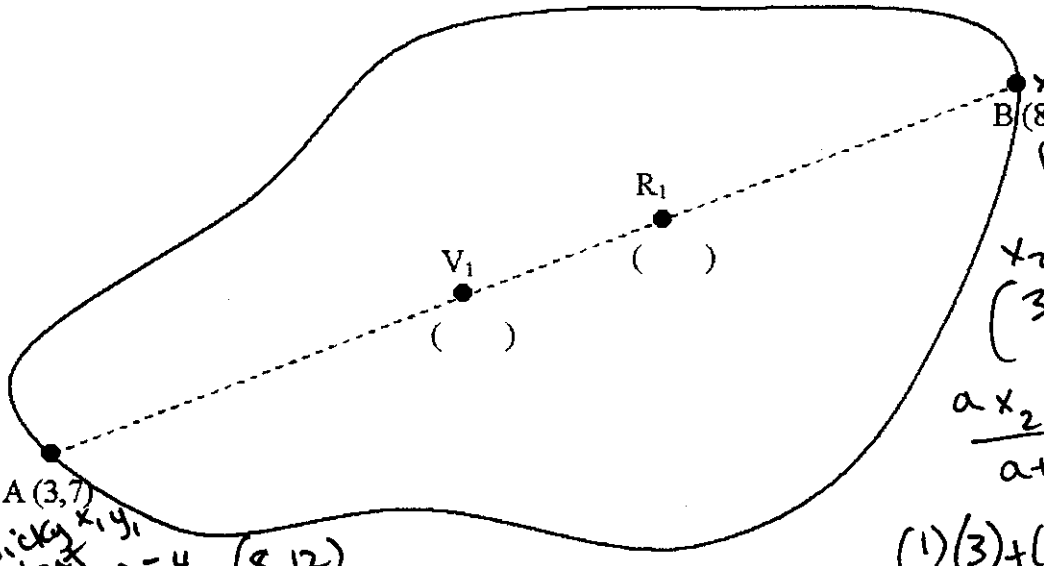
$$\left(4\tfrac{3}{5}, \tfrac{1}{5}\right)$$

II. Vicky and Rosemary are at opposite sides of a lake as shown below. They decide to swim straight toward each other to meet somewhere in the middle of the lake.

The two women leave at the same time and after 10 minutes, Vicky has covered $\frac{4}{9}$ of the total distance. She has travelled from point A to point V_1. During this same time, Rosemary has swum from point B to point R_1. Point R_1 divides the distance across the lake in a ratio of $\frac{1}{2}$.

One unit corresponds to 140 m. Determine the distance between the two swimmers after they have been swimming for 10 minutes (identify the coordinates of points V_1 and R_1).

Clearly show all your work.



Vicky x_1, y_1 start (3,7) $a=4$ $b=5$ (8,12) $x_2\ y_2$

Rosemary start x_1, y_1 (8,12) $a=1$ $b=2$ $x_2 y_2$ (3,7)

Vicky $\rightarrow V_1$

$$\frac{ax_2+bx_1}{a+b},\ \frac{ay_2+by_1}{a+b}$$

$$\frac{(4)(8)+(5)(3)}{4+5},\ \frac{(4)(12)+(5)(7)}{9}$$

$$\frac{32+15}{9},\ \frac{48+35}{9}$$

$$\frac{47}{9},\ \frac{83}{9}$$

$$\left(5\tfrac{2}{9},\ 9\tfrac{2}{9}\right)$$

Rosemary

$$\frac{(1)(3)+(2)(8)}{3},\ \frac{(1)(7)+(2)(12)}{3}$$

$$\frac{3+16}{3},\ \frac{7+24}{3}$$

$$\left(\frac{19}{3},\ \frac{31}{3}\right)$$

$$d=\sqrt{\left(\tfrac{31}{3}-9\tfrac{2}{9}\right)^2+\left(\tfrac{19}{3}-5\tfrac{2}{9}\right)^2}$$

$$=\sqrt{1.2346+1.23456}$$

$$=1.5713\text{ u}\times 140\text{ m}=220\text{ m}$$

Question 9

I. The Brown family leaves Montreal to drive to Ottawa. After 35 minutes of driving, they reach point X, which divides the total distance in a ratio of $\frac{2}{5}$.

After an additional 20 minutes of driving, they cover one-quarter of the remaining distance to reach point Y.

One unit corresponds to 17 Km. Determine the distance they have left to cover to reach Ottawa $(Y - Ottawa)$ by identifying the coordinates of points X and Y.

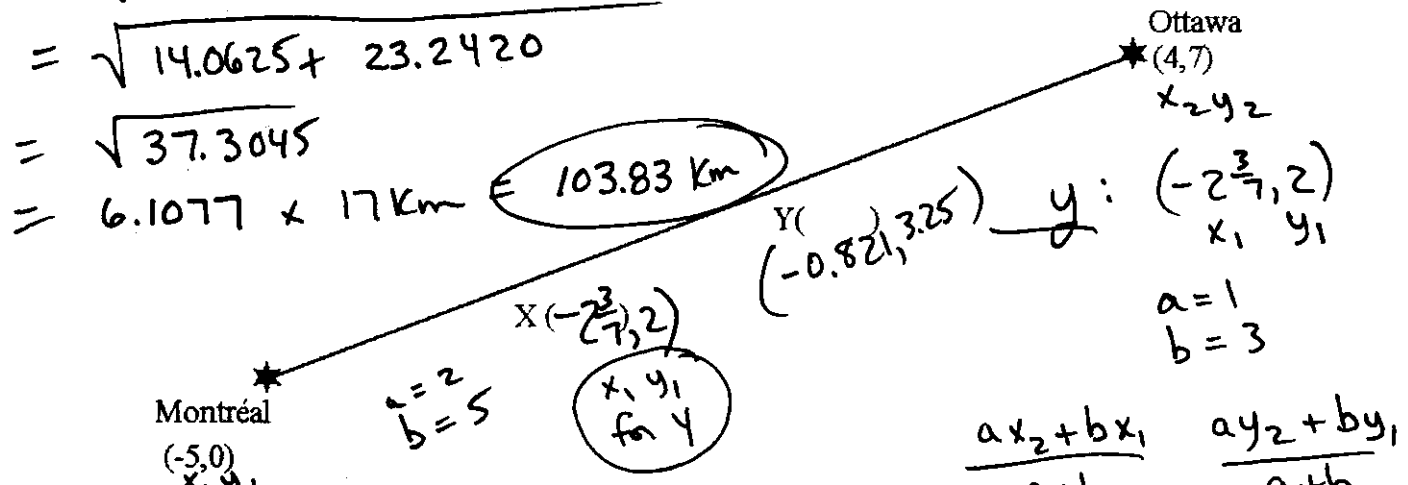
Clearly show all your work.

$d = \sqrt{(7-3.25)^2+(4+0.821)}$

$= \sqrt{14.0625 + 23.2420}$

$= \sqrt{37.3045}$

$= 6.1077 \times 17\text{Km} = 103.83\text{ Km}$



$x_2 y_2$

y: $(-2\frac{3}{7}, 2)$ x_1 y_1

$a = 1$, $b = 3$

$a = 2$, $b = 5$

x_1, y_1 for Y

x_1, y_1

X: $\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}$

$\frac{(2)(4)+(5)(-5)}{7}, \frac{(2)(7)+(5)(0)}{7}$

$\frac{8-25}{7}, \frac{14+0}{7}$

$-\frac{17}{7}, \frac{14}{7}$

$(-2\frac{3}{7}, 2)$

$\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}$

$\frac{(1)(4)+(3)(-2\frac{3}{7})}{4}$

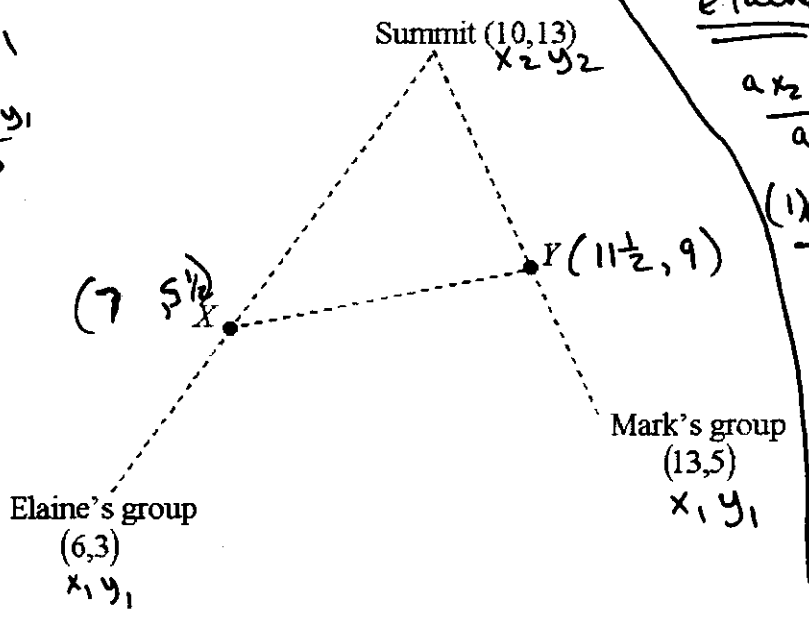
$\frac{4-7\frac{3}{7}}{4}, \frac{(1)(7)+(3)(2)}{4}$

$\left(\frac{-23}{28}, \frac{7+6}{4}\right)$

$(-0.821, 3.25)$

II. Mount McKinley, in Alaska, is the highest mountain in North America.

Two groups of hikers are hiking to the top of Mount McKinley. The diagram below shows the path each group is taking as well as the coordinates of their starting points and of the summit.



Mark's
X : $a = 1$ $b = 1$

$\frac{ax_2 + bx_1}{a+b}, \frac{ay_2 + by_1}{a+b}$

$\frac{10+13}{2}, \frac{13+5}{2}$

$\frac{23}{2}, \frac{18}{2}$

$(11\frac{1}{2}, 9)$

Elaine's X: $a = 1$ $b = 3$

$\frac{ax_2 + bx_1}{a+b}, \frac{ay_2 + by_1}{a+b}$

$\frac{(1)(10)+(3)(6)}{4}, \frac{(1)(13)+(3)(3)}{4}$

$\frac{10+18}{4}, \frac{13+9}{4}$

$\frac{28}{4}, \frac{22}{4}$

$(7, 5\frac{1}{2})$

The first week, Mark's group has covered half the distance to the summit. Elaine's group doesn't make out as well. Elaine realizes that the next week her group will have to hike three times the distance already covered in order to reach the summit before the end of the second week. $a = 1$ $b = 3$

One unit corresponds to 2 Km. Determine the distance between the two groups at the end of the first week by identifying the coordinates of points X and Y.

Clearly show all your work.

$$d = \sqrt{(9-5\tfrac{1}{2})^2 + (11\tfrac{1}{2} - 7)^2}$$

$$= \sqrt{12.25 + 20.25}$$

$$= 5.701 \times 2\text{Km} = 11.4\text{ Km}$$

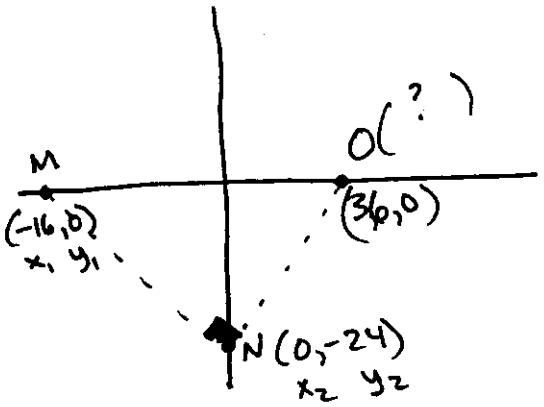
Question 10

a) In triangle *MNO*, angle N is a right angle.

The coordinates of M are $(-16,0)$ and those of N are $(0,-24)$. Determine the length of base MO given that point O is located on the x-axis.

Clearly show all your work. Show all the steps in the solution.

Hint: You've got two perpendicular lines here!



MN

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-24-0}{0+16} = \frac{-24}{16} = \frac{-3}{2}$$

NO

$m = \frac{2}{3}$

Eqn: $y = \frac{2}{3}x - 24$

Let $y = 0$:

$$0 = \frac{2}{3}x - 24$$

$$\left(\frac{-3}{2}\right) -\frac{2}{3}x = -24\left(\frac{-3}{2}\right)$$

$$x = 36$$

O: (36,0)

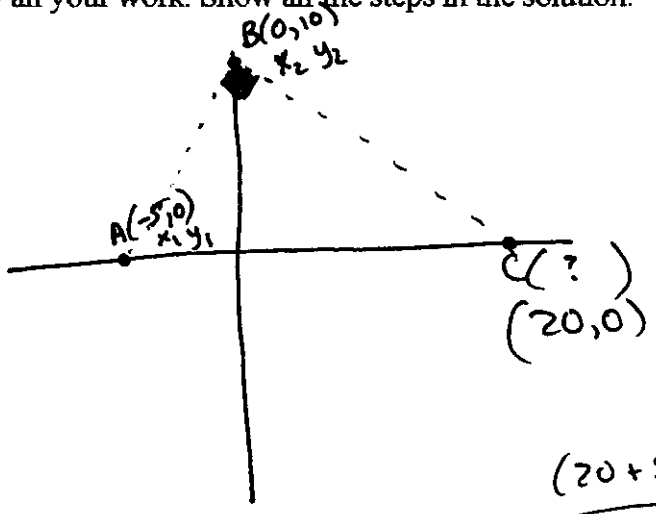
base MO = 36 + 16

= 52 units

b) In triangle *ABC*, angle B is a right angle.

The coordinates of A are $(-5,0)$ and those of B are $(0,10)$. Determine the length of base AC given that point C is located on the x-axis.

Clearly show all your work. Show all the steps in the solution.



$\overline{AB}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10 - 0}{0 + 5} = \frac{10}{5} = 2$

$\overline{BC}$: $m = -\frac{1}{2}$

Eqn : $y = -\frac{1}{2}x + 10$

Let $y = 0$

$0 = -\frac{1}{2}x + 10$

$\left(\frac{2}{1}\right)\frac{1}{2}x = 10\left(\frac{2}{1}\right)$

$x = 20$

C : (20,0)

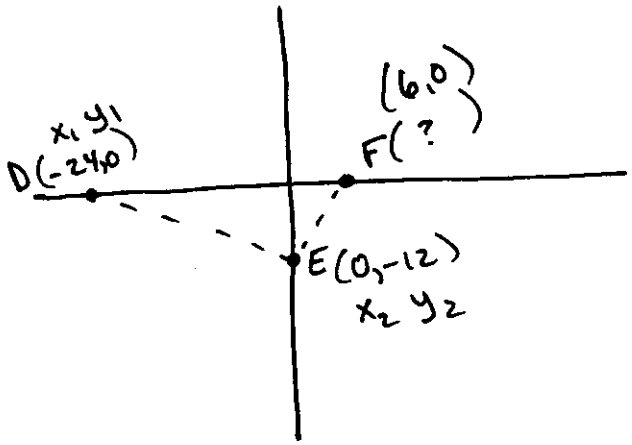
$(20+5)$ 2

$\overline{AC}$: 25 units

c) In triangle *DEF*, angle E is a right angle.

The coordinates of D are $(-24,0)$ and those of E are $(0,-12)$. Determine the length of base DF given that point F is located on the x-axis.

Clearly show all your work. Show all the steps in the solution.



DE

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 0}{0 + 24} = \frac{-12}{24} = \frac{-1}{2}$$

EF

$m = 2$

Eqn: $y = 2x - 12$

let $y = 0$

$0 = 2x - 12$

$\frac{-2x}{-2} = \frac{-12}{-2}$

$x = 6$

$F : (6,0)$

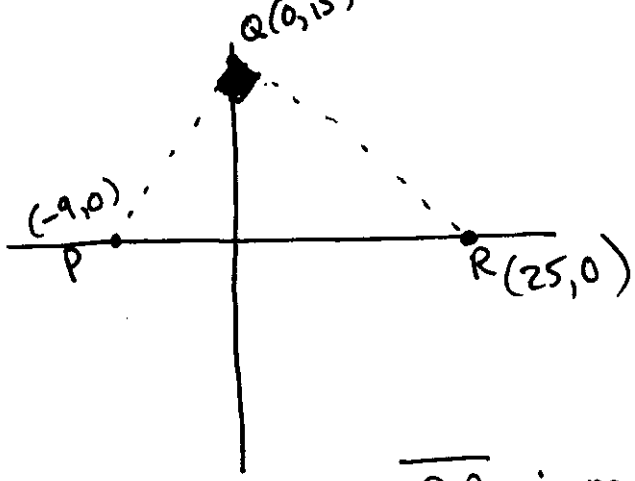
length DF = 24 + 6

= 30 units

d) In triangle *PQR*, angle Q is a right angle.

The coordinates of P are $(-9,0)$ and those of Q are $(0,15)$. Determine the length of base PR given that point R is located on the x-axis.

Clearly show all your work. Show all the steps in the solution.



$\overline{PQ}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{15-0}{0+9} = \frac{15}{9}$

$\overline{QR}$: $m = -\frac{9}{15}$

Eqn: $y = -\frac{9}{15}x + 15$

Let $y = 0$

$0 = -\frac{9}{15}x + 15$

$\left(\frac{+15}{9}\right)\frac{9}{15}x = 15\left(\frac{15}{9}\right)$

$x = 25$

length $\overline{PR} = 9 + 25$

$=$ 34 units

Question 11

a) Lines l_1 and l_2 are perpendicular to line l_3, whose equation is $5x+10y-14=0$. They intersect l_3 at its intercepts.

Determine the equation of the line that is equidistant from the two parallel lines.

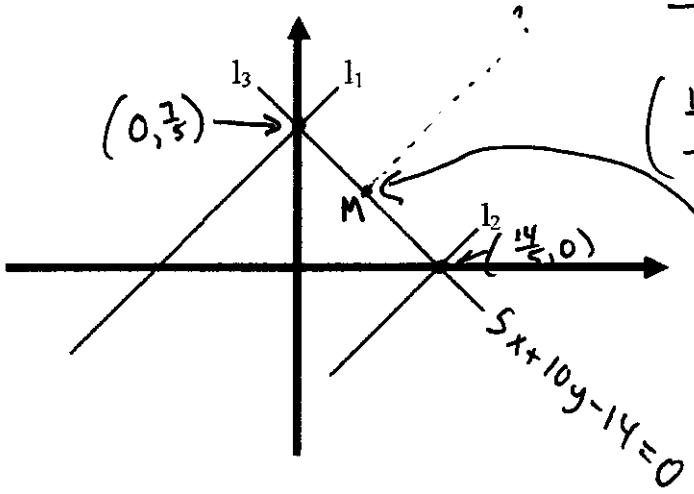
Clearly show all your work.

midpoint (M) : a = 1 b = 1

$\frac{ax_2+bx_1}{2}, \frac{ay_2+by_1}{2}$

$\left(\frac{\frac{14}{5}+0}{2}, \frac{0+\frac{7}{5}}{2}\right)$

$\left(1\frac{2}{5}, \frac{7}{10}\right)$



l_3

$5x+10y-14=0$

$\frac{10y}{10} = \frac{-5x}{10} + \frac{14}{10}$

$y = -\frac{1}{2}x + \frac{7}{5}$

$m = -\frac{1}{2}$ ($\frac{7}{5}$ ↑ b)

$l_1 \perp l_2$

$m = 2$

Unknown line : $m = 2$

contains point $\left(1\frac{2}{5}, \frac{7}{10}\right)$

$y = mx+b$

$\frac{7}{10} = (2)\left(1\frac{2}{5}\right) + b$

$\frac{7}{10} = 2\frac{4}{5} + b$

$\frac{7}{10} - 2\frac{4}{5} = b$

$-2\frac{1}{10} = b$

l_3

let $y=0$ to find x-int :

$5x + 10(0) - 14 = 0$

$5x - 14 = 0$

$\frac{5x}{5} = \frac{14}{5}$

$x = \frac{14}{5}$

Eqn: $y = 2x - \frac{21}{10}$

b) Lines l_1 and l_2 are perpendicular to line l_3, whose equation is $9x+6y-32=0$. They intersect l_3 at its intercepts.

Determine the equation of the line that is equidistant from the two parallel lines.

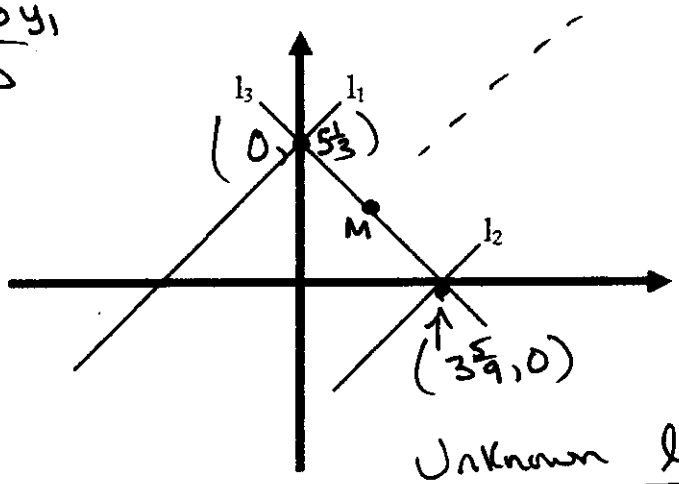
Clearly show all your work.

midpoint (M)

$\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}$

$\frac{3\frac{5}{9}}{2}, \frac{5\frac{1}{3}}{2}$

$(1\frac{7}{9}, 2\frac{2}{3})$



l_3:

$9x+6y-32=0$

$\frac{6y}{6} = \frac{-9x}{6} + \frac{32}{6}$

$y = -\frac{3x}{2} + 5\frac{1}{3}$ (b)

$m = -\frac{3}{2}$

let $y=0$

$9x+6(0)-32=0$

$9x-32=0$

$\frac{9x}{9} = \frac{32}{9}$

$x = 3\frac{5}{9}$

Unknown line

$m = \frac{2}{3}$

Contains: $(1\frac{7}{9}, 2\frac{2}{3})$

$2\frac{2}{3} = (\frac{2}{3})(1\frac{7}{9}) + b$

$2\frac{2}{3} = 1\frac{5}{27} + b$

$2\frac{2}{3} - 1\frac{5}{27} = b$

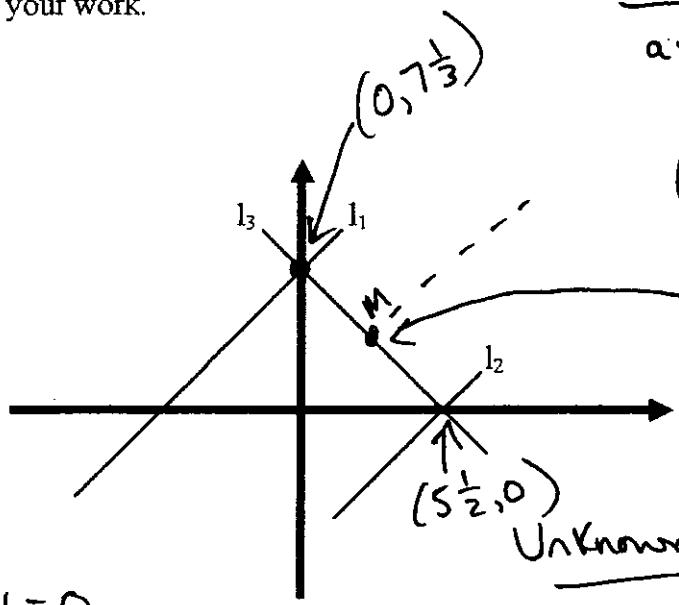
$1\frac{13}{27} = b$

$y = \frac{2}{3}x + \frac{40}{27}$

c) Lines l_1 and l_2 are perpendicular to line l_3, whose equation is $8x + 6y - 44 = 0$. They intersect l_3 at its intercepts.

Determine the equation of the line that is equidistant from the two parallel lines.

Clearly show all your work.



M: $a = 1$ $b = 1$

$\frac{ax_2 + bx_1}{2}, \frac{ay_2 + by_1}{2}$

$\left(\frac{5\frac{1}{2}}{2}, \frac{7\frac{1}{3}}{2}\right)$

$\left(2\frac{3}{4}, 3\frac{2}{3}\right)$

Unknown line

$m = \frac{3}{4}$

Contains $\left(2\frac{3}{4}, 3\frac{2}{3}\right)$

$y = mx + b$

$3\frac{2}{3} = \left(\frac{3}{4}\right)\left(2\frac{3}{4}\right) + b$

$3\frac{2}{3} = 2\frac{1}{16} + b$

$3\frac{2}{3} - 2\frac{1}{16} = b$

$1\frac{29}{48} = b$ or $\frac{77}{48}$

l_3: $8x + 6y - 44 = 0$

$\frac{6y}{6} = \frac{-8x}{6} + \frac{44}{6}$

$y = \frac{-4}{3}x + 7\frac{1}{3}$

m $\quad$ b

let $y = 0$

$0 = \frac{-4}{3}x + 7\frac{1}{3}$

$\left(\frac{3}{4}\right)\frac{4}{3}x = 7\frac{1}{3}\left(\frac{3}{4}\right)$

$x = 5\frac{1}{2}$

$y = \frac{3}{4}x + \frac{77}{48}$

d) Lines l_1 and l_2 are perpendicular to line l_3, whose equation is $4x + 12y - 42 = 0$. They intersect l_3 at its intercepts.

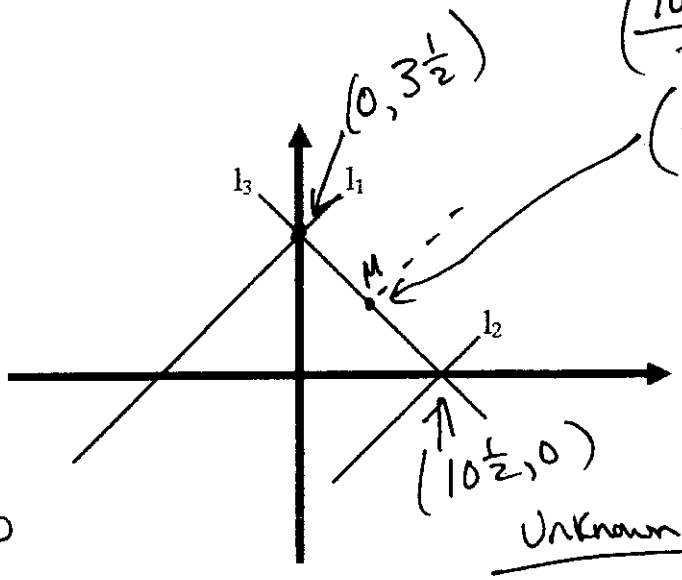
Determine the equation of the line that is equidistant from the two parallel lines.

Clearly show all your work.

Point M:

$\left(\frac{10\frac{1}{2}}{2}, \frac{3\frac{1}{2}}{2}\right)$

$\left(5\frac{1}{4}, 1\frac{3}{4}\right)$



l_3

$4x + 12y - 42 = 0$

$\frac{12y}{12} = \frac{-4x}{12} + \frac{42}{12}$

$y = -\frac{1}{3}x + 3\frac{1}{2}$

b

let $y = 0$

$0 = -\frac{1}{3}x + 3\frac{1}{2}$

$\left(+\frac{3}{1}\right)\frac{1}{3}x = 3\frac{1}{2}\left(\frac{3}{1}\right)$

$x = 10\frac{1}{2}$

Unknown line

$m = 3$

$\left(5\frac{1}{4}, 1\frac{3}{4}\right)$

$1\frac{3}{4} = (3)\left(5\frac{1}{4}\right) + b$

$1\frac{3}{4} = 15\frac{3}{4} + b$

$1\frac{3}{4} - 15\frac{3}{4} = b$

$-14 = b$

$y = 3x - 14$