MTH–4111–2 **Solving a System of Equations Algebraically** (Using Substitution and the Quadratic Formula)

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Solve the following systems of equations. Round off your answers to the nearest tenth. Clearly show all the steps in your **algebraic** solutions.

All final answers to 1 decimal place!

1. ① $y = -2.5x^2$ (Y = ↓)

② $3x + \frac{2}{5}y - 1 = 0$

Step 1:

$\frac{5}{2}\left(\frac{2}{5}y\right) = \frac{5}{2}(-3x + 1)$

$y = -\frac{15}{2}x + \frac{5}{2}$

$y = -7.5x + 2.5$

Step 2: $Y = Y$

$-2.5x^2 = -7.5x + 2.5$

$-2.5x^2 + 7.5x - 2.5 = 0$

a b c

Step 3:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-7.5 \pm \sqrt{(7.5)^2 - 4(-2.5)(-2.5)}}{2(-2.5)}$

$= \frac{-7.5 \pm \sqrt{56.25 - 25}}{-5}$

$= \frac{-7.5 \pm 5.59}{-5}$

(+) → $0.382 = x_1$

(−) → $2.618 = x_2$

Step 4: Find y "partners".

$y = -2.5x^2$ ($x \approx 0.4$)

$y = -2.5(0.382)^2$

$= -0.365 \approx -0.4$

Solution 1: $(0.4, -0.4)$

$y = -2.5x^2$ ($x \approx 2.6$)

$= -2.5(2.618)^2$

$= -17.13 \approx -17.1$

Solution 2: $(2.6, -17.1)$

2. $y = \frac{-x^2}{2} = -\frac{1}{2}x^2$

Step 1: $2x + \frac{2}{3}y - 3 = 0$

$\frac{3}{2}\left(\frac{2}{3}y\right) = \frac{3}{2}(-2x + 3)$

$y = -3x + 4.5$

Step 2: $Y = Y$

$-\frac{1}{2}x^2 = -3x + 4.5$

$-\frac{1}{2}x^2 + 3x - 4.5 = 0$

a b c

Step 3: Plug in:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2 - 4(-\frac{1}{2})(-4.5)}}{2(-\frac{1}{2})}$

$= \frac{-3 \pm \sqrt{9 - 9}}{-1}$ Note that discriminant = 0 ∴ 1 solution!

$= \frac{-3 \pm 0}{-1} = 3 = x$

Step 4: Find y for $x = 3$

$y = -\frac{1}{2}x^2$

$= -\frac{1}{2}(3)^2$

$= -4.5$

Solution: $(3, -4.5)$

Only 1 solution!

Both y's already isolated:

3. $y = -2x^2 - 5x - 3$

$y = \frac{2}{3}x + 4.5$

$y = y$

$-2x^2 - 5x - 3 = \frac{2}{3}x + 4.5$

$-2x^2 - 5x - \frac{2}{3}x - 3 - 4.5 = 0$

$-2x^2 - 5\frac{2}{3}x - 7.5 = 0$

a b c

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5\frac{2}{3} \pm \sqrt{(-5\frac{2}{3})^2 - 4(-2)(-7.5)}}{2(-2)}$

$= \frac{5\frac{2}{3} \pm \sqrt{32.11 - 60}}{-4}$

$= \frac{5\frac{2}{3} \pm \sqrt{-27.89}}{-4}$

↑ The discriminant is < 0. Therefore there is no solution.

(You can't √ a negative number)

* The parabola and the line never touch.

e.g.

ANSWER: NO SOLUTION

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4. $y = -0.08x^2$

$x + \frac{4}{3}y - 2 = 0$

$\frac{3}{4}\left(\frac{4}{3}y\right) = \frac{3}{4}(-x + 2)$

$y = -\frac{3}{4}x + \frac{6}{4}$

$y = -\frac{3}{4}x + \frac{3}{2}$

$y = y$

$-0.08x^2 = -\frac{3}{4}x + \frac{3}{2}$

$-0.08x^2 + \frac{3}{4}x - \frac{3}{2} = 0$

a b c

So, for the rest of the problems I'm not going to show the work for the quadratic formula (I'm thinking that won't be an issue since you've done the Quadratic Functions module + have lots of practice with the quadratic formula.

So, I'll just give the x values "pumped out" by the quadratic formula:

$x_1 = 2.89$

$x_2 = 6.48$

} Plug into one of the equations

$y = -0.08x^2$

$y = -0.08(2.89)^2$

$= -0.67$

$y = -0.08x^2$

$= -0.08(6.48)^2$

$= -3.36$

2 solutions: (2.9, −0.7) and (6.5, −3.4)

5. $y = x^2 - 2x - 7$

$y = \frac{3}{4}x + 3.5$

$Y = Y$

$x^2 - 2x - 7 = \frac{3}{4}x + 3.5$

$x^2 - 2x - \frac{3}{4}x - 7 - 3.5 = 0$

$1x^2 - 2.75x - 10.5 = 0$

$a \quad b \quad c$

→ Plug into quadratic formula:

$x_1 = 4.9$

$x_2 = -2.1$

Plug x_1 into either equation:

$Y = \frac{3}{4}x + 3.5$

$Y = \frac{3}{4}(4.9) + 3.5 = 7.2$

Plug x_2 into either equation:

$Y = \frac{3}{4}x + 3.5$

$Y = \frac{3}{4}(-2.1) + 3.5 = 1.9$

The two solutions are:

(4.9, 7.2) and (-2.1, 1.9)

6. $y = -\frac{2}{3}x^2 - 3x - 6$

$y = \frac{1}{2}x + 1.5$

$-\frac{2}{3}x^2 - 3x - 6 = \frac{1}{2}x + 1.5$

$-\frac{2}{3}x^2 - 3x - \frac{1}{2}x - 6 - 1.5 = 0$

$-\frac{2}{3}x^2 - 3.5x - 7.5 = 0$

$a = -0.67 \quad b \quad c$

→ Plug into quadratic formula

→ discriminant (Δ) = 0

∴ No solution

7. $y = 3.25x^2$

$6x + \frac{3}{2}y - 3 = 0$

$6x + 1.5y - 3 = 0$

$\frac{1.5y}{1.5} = \frac{-6x}{1.5} + \frac{3}{1.5}$

$y = -4x + 2$

$y = y$

$3.25x^2 = -4x + 2$

$3.25x^2 + 4x - 2 = 0$

a b c

Plug into quadratic formula:

$x_1 = 0.4$

$x_2 = -1.6$

Then, plug each x (x_1 and x_2) into either equation to get their "y" partners. You will end up with: 2 solutions:

(0.4, 0.5) and (-1.6, 8.3)

Remember

For all solutions on this worksheet your "x" and "y" values must be rounded to 1 decimal place!!

8. $y = -3x^2 - x - 1$

$y = \frac{3}{2}x + 1.5$ ($\frac{3}{2} = 1.5$)

$-3x^2 - x - 1 = 1.5x + 1.5$

$-3x^2 - 1x - 1.5x - 1 - 1.5 = 0$

$-3x^2 - 2.5x - 2.5 = 0$

a b c

Plug into quadratic formula:

discriminant (Δ) $= 0$

$\therefore$ **No solution**

9. $y = \frac{5}{2}x^2$

$7x + \frac{7}{3}y - 14 = 0$

$\frac{3}{7}\left(\frac{7}{3}y\right) = \frac{3}{7}\left(-\frac{7}{1}x + \frac{14}{1}\right)$

$y = -3x + 6$

$y = y$

$\frac{5}{2}x^2 = -3x + 42$

$2.5x^2 = -3x + 6$

$2.5x^2 + 3x - 6 = 0$

a b c

Plug into quadratic formula:

$x_1 = 1.06 \approx 1.1$

$x_2 = -2.26 \approx -2.3$

After you plug the x_1 and x_2 into one of the equations you'll end up with 2 solutions:

(1.1, 3.0) and (−2.3, 13.2)

10. $y = -\frac{x^2}{5}$ } $y = -\frac{1}{5}x^2$

$9x + 3y - 6 = 0$

$\frac{3y}{3} = -\frac{9x}{3} + \frac{6}{3}$

$y = -3x + 2$

$y = y$

$-\frac{1}{5}x^2 = -3x + 2$

$-\frac{1}{5}x^2 + 3x - 2 = 0$

−0.2 → a b c

Plug into quadratic formula:

$x_1 = 0.7$

$x_2 = 14.3$

Two solutions will result:

(0.7, −0.1) and (14.3, −40.9)

11. $y = 1.5x^2$

$4x + \frac{8}{3}y - 3 = 0$

$\frac{3}{8}\left(\frac{8}{3}y\right) = \frac{3}{8}\left(-\frac{4}{1}x + \frac{3}{1}\right)$

$y = -\frac{12}{8}x + \frac{9}{8}$ (÷4)

$y = -\frac{3}{2}x + \frac{9}{8}$

$y = y$

$1.5x^2 = -\frac{3}{2}x + \frac{9}{8}$

$1.5x^2 + \frac{3}{2}x - \frac{9}{8} = 0$

$1.5x^2 + 1.5x - 1.125 = 0$

a b c

Plug into quadratic formula:

$x_1 = 0.5$

$x_2 = -1.5$

Two solutions: (0.5, 0.4) and (−1.5, 3.4)

12. $y = -0.4x^2$

$1.25x + \frac{y}{4} - \frac{3}{4} = 0$

$1.25x + \frac{1}{4}y - \frac{3}{4} = 0$

$\frac{4}{1}\left(\frac{1}{4}y\right) = \frac{4}{1}\left(-1.25x + \frac{3}{4}\right)$

$y = -5x + 3$

$y = y$

$-0.4x^2 = -5x + 3$

$-0.4x^2 + 5x - 3 = 0$

a b c

Plug into quadratic formula:

$x_1 = 0.63 \approx 0.6$

$x_2 = 11.9$

Two solutions:

(0.6, −0.1) and (11.9, −56.6)

13. $y = 0.75x^2$

$\frac{5}{2}x + \frac{y}{2} - \frac{3}{2} = 0$

$\frac{5}{2}x + \frac{1}{2}y - \frac{3}{2} = 0$

$\frac{2}{1}\left(\frac{1}{2}y\right) = \frac{2}{1}\left(-\frac{5}{2}x + \frac{3}{2}\right)$

$y = -\frac{10}{2}x + \frac{6}{2}$

$y = -5x + 3$

$Y = Y$

$0.75x^2 = -5x + 3$

$0.75x^2 + 5x - 3 = 0$

a b c

Plug into quadratic formula:

$x_1 = 0.55 \approx 0.6$

$x_2 = -7.2$

Two solutions:

(0.6, 0.3) and (−7.2, 38.9)

14. $y = 3x^2 - x - 1$

$y = \frac{x}{4} + 1.75$

$Y = Y$

$3x^2 - x - 1 = \frac{1}{4}x + 1.75$

$3x^2 - 1x - \frac{1}{4}x - 1 - 1.75 = 0$

$3x^2 - 1\frac{1}{4}x - 2.75 = 0$

$3x^2 - 1.25x - 2.75 = 0$

a b c

Plug into quadratic formula

$x_1 = 1.188 \approx 1.2$

$x_2 = -0.77 \approx -0.8$

Two solutions:

(1.2, 2.1)

and (−0.8, 1.6)

15. $y = -0.75x^2$

$\frac{2}{3}x + \frac{y}{3} - \frac{1}{3} = 0$

$\frac{2}{3}x + \frac{1}{3}y - \frac{1}{3} = 0$

$\frac{3}{1}\left(\frac{1}{3}y\right) = \frac{3}{1}\left(-\frac{2}{3}x + \frac{1}{3}\right)$

$y = -2x + 1$

$y = y$

$-0.75x^2 = -2x + 1$

$-0.75x^2 + 2x - 1 = 0$

a b c

Plug into quadratic formula:

$x_1 = 0.7$

$x_2 = 2$

Two solutions:

(0.7, −0.4) and (2, −3)

16. $y = -6.75x^2$

$3x + \frac{y}{3} - 1 = 0$

$3x + \frac{1}{3}y - 1 = 0$

$\frac{3}{1}\left(\frac{1}{3}y\right) = \frac{3}{1}(-3x + 1)$

$y = -9x + 3$

$y = y$

$-6.75x^2 = -9x + 3$

$-6.75x^2 + 9x - 3 = 0$

a b c

Plug into quadratic formula

(note that the discriminant (Δ) $= 0$,

$\therefore$ only one solution ...

$x = 0.67 \approx 0.7$

One solution: (0.7, −3.3)