Quadratic Functions: Word Problems

Type 1: Solve for x when $y = 0$ (Solve by factoring).

e.g. 1 Jose is practicing his baseball hitting. He hits the ball whose trajectory is defined by the equation: $y = \frac{-7}{48}(x-12)^2 + 21$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work.

$y = \frac{-7}{48}(x-12)(x-12) + 21$

$0 = \frac{-7}{48}(x^2 - 24x + 144) + 21$

$0 = \frac{-7}{48}x^2 + 3\frac{1}{2}x - \cancel{21} + \cancel{21}$

$\left(0 = -\frac{7}{48}x^2 + 3\frac{1}{2}x\right)48$

$0 = -7x^2 + 168x$

$0 = -7x(x-24)$ $x = 0$ or $x = 24$

$\boxed{\text{The ball travels 24m.}}$

Now you try:

1. While golfing, Alain hits the ball whose trajectory is defined by the equation: $y = \frac{-5}{81}(x-18)^2 + 20$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work.

$y = \frac{-5}{81}(x^2 - 36x + 324) + 20$

$0 = \frac{-5}{81}x^2 + \frac{20}{9}x - \cancel{20} + \cancel{20}$

$\left(0 = -\frac{5}{81}x^2 + \frac{20}{9}x\right)81$

$0 = -5x^2 + 180x$

$0 = -5x(x-36)$

$x = 36$

Answer: $\boxed{\text{The ball travels 36m.}}$

2. Todd kicks a soccer ball into the air. The ball's trajectory is defined by the equation: $y = \frac{-2}{49}(x-14)^2 + 8$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work.

$$y = \frac{-2}{49}(x^2 - 28x + 196) + 8$$

$$0 = \frac{-2}{49}x^2 + \frac{8}{7}x - \cancel{8} + \cancel{8}$$

$$\left(0 = \frac{-2}{49}x^2 + \frac{8}{7}x\right)49$$

$$0 = -2x^2 + 56x$$

$$0 = -2x(x-28)$$

$$\boxed{x = 28 \} \text{ The ball travels } 28\text{m}}$$

3. Vanessa throws a javelin, whose trajectory is defined by the equation: $y = \frac{-3}{13}(x-13)^2 + 39$, where y represents the height of the javelin and x, the distance it travels. The variables x and y are expressed in metres. How far does the javelin travel? Clearly show all your work.

$$y = \frac{-3}{13}(x^2 - 26x + 169) + 39$$

$$0 = \frac{-3}{13}x^2 + 6x - \cancel{39} + \cancel{39}$$

$$\left(0 = \frac{-3}{13}x^2 + 6x\right)13$$

$$0 = -3x^2 + 78x$$

$$0 = -3x(x - 26)$$

$$x = 26$$

$$\boxed{\text{The javelin travels } 26\text{m}}$$

4. While golfing at Mont Tremblant, Chris hits a ball whose trajectory is defined by the equation: $y = \frac{-5}{32}(x-16)^2 + 40$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work.

$$y = -\frac{5}{32}(x^2 - 32x + 256) + 40$$

$$0 = -\frac{5}{32}x^2 + 5x - \cancel{40} + \cancel{40}$$

$$\left(0 = -\frac{5}{32}x^2 + 5x\right)32$$

$$0 = -5x^2 + 160x$$

$$0 = -5x(x - 32)$$

$x = 32$ } The ball travels 32m

5. Jennifer hits a softball whose trajectory is defined by the equation: $y = \frac{-4}{17}(x-17)^2 + 68$, where y represents the height of the ball and x, the distance it travels. The variables x and y are expressed in metres. How far does the ball travel? Clearly show all your work.

$$y = -\frac{4}{17}(x^2 - 34x + 289) + 68$$

$$0 = \frac{-4}{17}x^2 + 8x - \cancel{68} + \cancel{68}$$

$$\left(0 = \frac{-4}{17}x^2 + 8x\right)17$$

$$0 = -4x^2 + 136x$$

$$0 = -4x(x - 34)$$

$x = 34$ } The ball travels 34m

Type 2: Calculating the abscissa of the vertex: $x = \dfrac{-b}{2a}$

e.g. 1 After x seconds, the height in metres of a ball thrown up into the air is $y = 30x - 5x^2$. Calculate the time required for the ball to reach its maximum height. $y = -5x^2 + 30x$

$$x = \frac{-b}{2a} = \frac{-30}{2(-5)} = \frac{-30}{-10}$$

$$= 3$$

3 seconds

Now you try:

1. The height in metres reached by an object x seconds after being thrown into the air is represented by $y = -5x^2 + 20x + 30$. Calculate the time required for this object to reach its maximum height.

$$x = \frac{-b}{2a} = \frac{-20}{2(-5)} = \frac{-20}{-10}$$

$= 2$ seconds

2. Alka, the CFO of RGA Insurance, tracked the price of her company's shares over a three-week period. She discovered that the share price fluctuated according to the equation $y = \frac{x^2}{6} - 3x + 11.5$, where x represents a specific day during the observation period and y, the share price in \$. On which day did the share price reach its lowest point? Clearly show all your work.

$$x = \frac{-b}{2a} = \frac{3}{2\left(\frac{1}{6}\right)} = \frac{3}{\frac{1}{3}} = 9$$

Day 9

3. An object is thrown from the top of a building 15m high. The height in metres reached by the object with respect to the time in seconds is represented by the following equation:

$$y = -\frac{18}{5}x^2 + \frac{144}{5}x + 15$$

Determine how long it will take the object to reach its maximum height.

$$x = \frac{-b}{2a} = \frac{-\frac{144}{5}}{2\left(-\frac{18}{5}\right)} = \frac{-\frac{144}{5}}{-7\frac{1}{5}}$$

$= 4$ seconds

4. During an economical slump, the equation $y = \frac{x^2}{4} - 4x + 14.2$ represents the profit earned by a company over x number of days. On which day did the profit reach its lowest point? Clearly show all your work.

$$x = \frac{-b}{2a}$$

$$= \frac{4}{2(\frac{1}{4})} = \frac{4}{\frac{1}{2}}$$

$$= 8$$

Day 8

5. An object is thrown from the top of a building 14m high. The height in metres reached by the object with respect to the time in seconds is represented by the following equation:

$$y = -\frac{25}{7}x^2 + \frac{250}{7}x + 14$$

Determine how long it will take the object to reach its maximum height.

$$x = \frac{-b}{2a} = \frac{-\frac{250}{7}}{2(-\frac{25}{7})} = \frac{-\frac{250}{7}}{-7\frac{1}{7}}$$

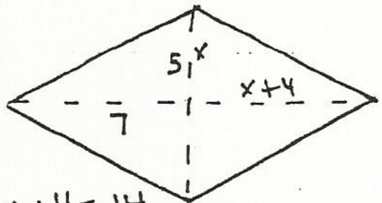
$$= 5 \text{ seconds}$$

Type 3: Determining the perimeter of a rhombus

e.g. 1 A rhombus has an area of $70cm^2$. Given that the long diagonal measures 4cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.

Let x = short diagonal

$x+4$ = long diagonal



$x = 10, \ x+4 = 14$

$$\frac{D \cdot d}{2} = \text{Area}$$

$$\frac{(x+4)x}{2} = 70$$

$$\frac{x^2+4x}{2} = 70$$

$$\tfrac{1}{2}x^2 + 2x - 70 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-4(\frac{1}{2})(-70)}}{1}$$

$$= -2 \pm 12 = 10 \text{ OR } -14$$

$$c^2 = a^2 + b^2$$
$$= 5^2 + 7^2 = 74$$
$$c = 8.6$$
$$4c = 34.4cm$$

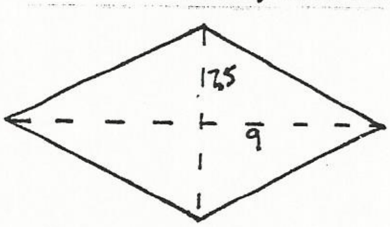
Now you try:

1. A rhombus has an area of $135cm^2$. Given that the long diagonal measures 3 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.

P.8

Let x = short diagonal

$x+3$ = long diagonal



$$\frac{D \cdot d}{2} = \text{area}$$

$$\frac{(x+3)x}{2} = 135$$

$$\tfrac{1}{2}x^2 + \tfrac{3}{2}x - 135 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= -\tfrac{3}{2} \pm \sqrt{(\tfrac{3}{2})^2 - 4(\tfrac{1}{2})(-135)}$$

$$= -\tfrac{3}{2} \pm \sqrt{2.25 + 270}$$

$$= -\tfrac{3}{2} \pm 16.5$$

$$x = 15$$

$x + 3$

$15 + 3$

$= 18$

$15 \div 2 = 7.5$

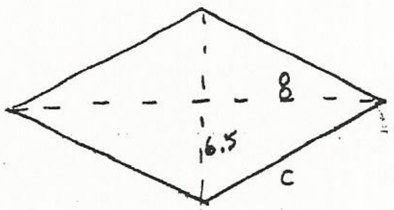
$18 \div 2 = 9$

$X = 15$

$x + 3 = 18$

$$c^2 = (7.5)^2 + (9)^2$$
$$= 56.25 + 81$$
$$c = 11.7cm \qquad 4c = 46.9cm$$

2. A rhombus has an area of 104cm^2. Given that the long diagonal measures 3 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.



let x = short diagonal

$x + 3$ = long diagonal

$$\frac{D \cdot d}{2} = \text{area}$$

$$\frac{(x+3)x}{2} = 104$$

$$\frac{1}{2}x^2 + \frac{3}{2}x - 104 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 4\left(\frac{1}{2}\right)(-104)}}{1}$$

$$= -\frac{3}{2} \pm \sqrt{2.25 + 208}$$

$$= -\frac{3}{2} \pm 14.5$$

$$= 13$$

$x = 13$

$x + 3 = 16$

$$^2 = (6.5)^2 + (8)^2$$

$$= 42.25 + 64$$

$$= 106.25$$

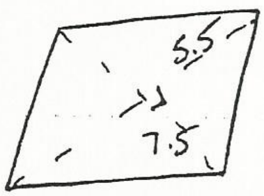
$c = 10.3$

$4c = 41.2$ cm

3. A rhombus has an area of 82.5 cm^2. Given that the long diagonal measures 4 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.

Let x = short diagonal

$x + 4$ = long diagonal



$$\frac{D \cdot d}{2} = \text{area}$$

$x = 11$

$x + 4 = 15$

$$\frac{(x+4)x}{2} = 82.5$$

$$\frac{1}{2}x^2 + 2x - 82.5 = 0$$

$$c^2 = 5.5^2 + 7.5^2$$

$$= 30.25 + 56.25$$

$$= 86.5$$

$$c = 9.3$$

$$4c = 37.2 \text{ cm}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -2 \pm \sqrt{4 - 4(\tfrac{1}{2})(-82.5)}$$

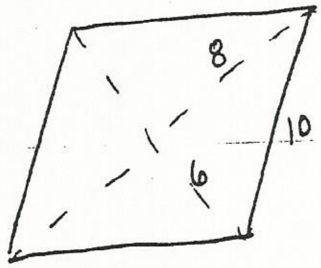
$$= \frac{-2 \pm 13}{1}$$

$$= 11$$

4. A rhombus has an area of $96cm^2$. Given that the long diagonal measures 4 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.

Let $x =$ short diagonal

$x+4 =$ long diagonal



$\frac{D \cdot d}{2} = $ area

$x = 12$

$x+4 = 16$

$\frac{(x+4)x}{2} = 96cm^2$

$c^2 = a^2 + b^2$

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

$c = 10$

$4c = 40 \text{ cm}$

$\frac{1}{2}x^2 + 2x - 96 = 0$

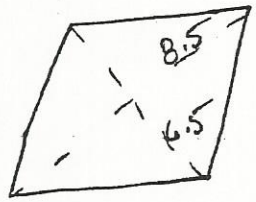
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4 - 4(\frac{1}{2})(-96)}}{1}$

$= -2 \pm 14$

$= 12$

5. A rhombus has an area of 110.5 cm^2. Given that the long diagonal measures 4 cm more than the short diagonal, determine the perimeter of the rhombus. Clearly show all your work.



let x = short diagonal

$x+4$ = long diagonal

$$\frac{D \cdot d}{2} = 110.5$$

$$\frac{(x+4)x}{2} = 110.5$$

$$\frac{1}{2}x^2 + 2x - 110.5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4\left(\frac{1}{2}\right)(-110.5)}}{1}$$

$$= -2 \pm 15$$

$$= 13$$

$x = 13$

$x + 4 = 17$

$$c^2 = (8.5)^2 + (6.5)^2$$

$$= (72.25) + (42.25)$$

$$= 114.5$$

$c = 10.7$

$4c = 42.8\text{cm}$

Type 4: time = $\frac{\text{distance}}{\text{velocity}}$

e.g. #1 A school bus must travel 550 km to take some students to a sports tournament. If the coach, driving his car, takes the same route, but travels 40 km/h faster than the school bus, the travel time could be reduced by 2.3 hours. What is the speed of the school bus? Round off your answer to the nearest unit.
Clearly show all your work.

Let x = speed of school bus

$x+40$ = speed of coach's car ⤳ faster ∴ less time

School bus time − coach's time = 2.3

$$\frac{\text{bus dist}}{\text{velocity}} - \frac{\text{coach dist}}{\text{velocity}} = 2.3$$

$$\frac{550}{x} - \frac{550}{x+40} = 2.3$$

c.d = $x(x+40)$

$$\frac{550(x+40)}{x(x+40)} - \frac{550x}{x(x+40)} = 2.3$$

$$\frac{550x+22000}{\text{c.d.}} - \frac{550x}{\text{c.d.}} = 2.3$$

$$\frac{22000}{x(x+40)} = \frac{2.3}{1}$$

$$\frac{22000}{x^2+40x} = \frac{2.3}{1}$$

$$2.3x^2 + 92x = 22000$$

$$2.3x^2+92x-22000 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-92 \pm \sqrt{92^2-4(2.3)(-22000)}}{2(2.3)}$$

$$= \frac{-92 \pm \sqrt{8464+202400}}{4.6}$$

$$= \frac{-92 \pm 459.2}{4.6}$$

$= 79.8$ km/h = speed of bus.

Now you try:

1. A bus must travel a distance of 510 km. If a train were to take the same route, but travel 45 km/h faster than the bus, the travel time could be reduced by 1.7 hours. What is the speed of the bus? Round off your answer to the nearest unit.

Let x = speed of bus

$x+45$ = speed of train → faster ∴ less time

$$\frac{\text{dist}_{\text{bus}}}{\text{velocity}} - \frac{\text{dist}_{\text{train}}}{\text{velocity}} = 1.7$$

$$\frac{510}{x} - \frac{510}{x+45} = 1.7 \qquad \text{c.d} = x(x+45)$$

$$\frac{510(x+45)}{x(x+45)} - \frac{510x}{x(x+45)} = 1.7$$

$$\frac{510x + 22950}{\text{c.d.}} - \frac{510x}{\text{c.d.}} = 1.7$$

$$\frac{22950}{x(x+45)} = 1.7$$

$$\frac{22950}{x^2+45x} = \frac{1.7}{1}$$

$$1.7x^2 + 76.5x = 22950$$

$$1.7x^2 + 76.5x - 22950 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-76.5 \pm \sqrt{(76.5)^2 - 4(1.7)(-22950)}}{2(1.7)}$$

$$= \frac{-76.5 \pm \sqrt{5852.25 + 156060}}{3.4}$$

$$= \frac{-76.5 \pm 402.38}{3.4}$$

$$= 95.8 \text{ Km/h} = \text{speed of bus}$$

2. Andrew must drive his Excel a distance of 620 km. If Cole, in his Mustang, takes the same route, but travels 27 km/h faster than Andrew, the travel time could be reduced by 1.4 hours. What is Andrew's speed in the Excel? Round off your answer to the nearest unit.

Let x = Excel speed

$x + 27$ = Mustang Speed (faster, smaller time)

$$\frac{620}{x} - \frac{620}{x+27} = 1.4$$

$$\frac{620(x+27)}{x(x+27)} - \frac{620x}{x(x+27)} = 1.4$$

$$\frac{620x + 16740}{c.d.} - \frac{620x}{c.d.} = 1.4$$

$$\frac{16740}{x(x+27)} = \frac{1.4}{1}$$

$$\frac{16740}{x^2 + 27x} = \frac{1.4}{1}$$

$$1.4x^2 + 37.8x = 16740$$

$$1.4x^2 + 37.8x - 16740 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-37.8 \pm \sqrt{(37.8)^2 - 4(1.4)(-16740)}}{2(1.4)}$$

$$= \frac{-37.8 \pm \sqrt{1428.84 + 93744}}{2.8}$$

$$= \frac{-37.8 \pm 308.50}{2.8}$$

$$= \boxed{96.7 \text{ km/h} = \text{speed of Excel}}$$

3. A freight train must travel a distance of ~~480~~ 620 km. If a truck were to take the same Route, but travel 50 km/h faster than the freight train, the travel time could be reduced by 7.2 hours. What is the speed of the freight train? Round off your answer to the nearest unit.

Let x = speed of train

$x + 50$ = speed of truck

$$\frac{620}{x} - \frac{620}{x+50} = 7.2$$

$$\frac{620(x+50)}{c.d.} - \frac{620x}{c.d.} = 7.2$$

$$\frac{620x + 31000}{c.d.} - \frac{620x}{c.d.} = 7.2$$

$$\frac{31\,000}{x(x+50)} = \frac{7.2}{1}$$

$$\frac{31\,000}{x^2 + 50x} = \frac{7.2}{1}$$

$$7.2x^2 + 360x = 31\,000$$

$$7.2x^2 + 360x - 31\,000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-360 \pm \sqrt{(360)^2 - 4(7.2)(-31\,000)}}{2(7.2)}$$

$$= \frac{-360 \pm \sqrt{129600 + 892800}}{14.4}$$

$$= \frac{-360 \pm 1011.138}{14.4}$$

= 45.2 km/h = speed of train

4. Trevor bikes an 80-km scenic route along the ocean. Sean drives his car through the same route, and saves 1.7 hours, compared to Trevor's biking time. If Sean's speed is 50 km/h faster than Trevor's, then what is Trevor's speed?

Let x = Trevor's speed

$x + 50$ = Sean's speed

$$\frac{80}{x} - \frac{80}{x+50} = 1.7$$

$$\frac{80(x+50)}{c.d.} - \frac{80x}{c.d.} = 1.7$$

$$\frac{80x + 4000}{c.d.} - \frac{80x}{c.d.} = 1.7$$

$$\frac{4000}{x(x+50)} = 1.7$$

$$\frac{4000}{x^2+50x} = \frac{1.7}{1}$$

$$1.7x^2 + 85x = 4000$$

$$1.7x^2 + 85x - 4000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-85 \pm \sqrt{(85)^2 - 4(1.7)(-4000)}}{2(1.7)}$$

$$= \frac{-85 \pm \sqrt{7225 + 27200}}{3.4}$$

$$= \frac{-85 \pm 185.54}{3.4}$$

$$= \boxed{29.6 \frac{km}{h} = \text{Trevor's speed}}$$

5. A car and a bus both travel through the same 714-km road trip. The car travels 20 km/h faster than the bus, and so the car does the trip in 1.4 hours less than the bus. What is the speed of the bus?

Let x = speed of bus

$x+20$ = speed of car

$$\frac{714}{x} - \frac{714}{x+20} = 1.4$$

$$\frac{714(x+20)}{c.d.} - \frac{714x}{c.d.} = 1.4$$

$$\frac{714x + 14280}{c.d.} - \frac{714x}{c.d.} = 1.4$$

$$\frac{14280}{x(x+20)} = \frac{1.4}{1}$$

$$\frac{14280}{x^2+20x} = \frac{1.4}{1}$$

$$1.4x^2 + 28x = 14280$$

$$1.4x^2 + 28x - 14280 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(1.4)(-14280)}}{2(1.4)}$$

$$= \frac{-28 \pm \sqrt{784 + 79968}}{2.8}$$

$$= \frac{-28 \pm 284.17}{2.8}$$

$= 91.5$ km/h = speed of bus

Type 5: time = $\frac{\text{volume}}{\text{rate of flow}}$

e.g.1 In a factory, 75-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 44 litres per minute, it would take 45 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x + 44$ = hypothetical new rate of flow

$$\frac{75}{x} - \frac{75}{x+44} = 0.75$$

*must be in minutes

$$\frac{75(x+44)}{\text{c.d.}} - \frac{75x}{\text{c.d.}} = 0.75$$

$$\frac{75x + 3300}{\text{c.d.}} - \frac{75x}{\text{c.d.}} = 0.75$$

$$\frac{3300}{\text{c.d.}} = \frac{0.75}{1}$$

$$\frac{3300}{x(x+44)} = \frac{0.75}{1}$$

$$\frac{3300}{x^2 + 44x} = \frac{0.75}{1}$$

$$0.75x^2 + 33x = 3300$$

$$0.75x^2 + 33x - 3300 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-33 \pm \sqrt{(33)^2 - 4(0.75)(-3300)}}{2(0.75)}$$

$$= \frac{-33 \pm \sqrt{1089 + 9900}}{1.5}$$

$$= \frac{-33 \pm 104.83}{1.5}$$

$$= 47.9$$

$$\sim \boxed{48 \text{ L/min} = \text{flow from tap}}$$

Now you try:

1. In a factory, 120-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 17 litres per minute, it would take 30 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x + 17$ = hypothetical new rate of flow

$$\frac{120}{x} - \frac{120}{x+17} = 0.5$$

$$\frac{120(x+17)}{c.d.} - \frac{120x}{c.d.} = 0.5$$

$$\frac{120x + 2040}{c.d.} - \frac{120x}{c.d.} = 0.5$$

$$\frac{2040}{x(x+17)} = 0.5$$

$$\frac{2040}{x^2 + 17x} = \frac{0.5}{1}$$

$$0.5x^2 + 8.5x = 2040$$

$$0.5x^2 + 8.5x - 2040 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8.5 \pm \sqrt{(8.5)^2 - 4(0.5)(-2040)}}{2(0.5)}$$

$$= \frac{-8.5 \pm \sqrt{72.25 + 4080}}{1}$$

$$= -8.5 \pm 64.44$$

$$= 55.9$$

$$\approx \boxed{56 \text{ L/min} = \text{flow from tap}}$$

2. In a factory, 100-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 8 litres per minute, it would take 15 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x+8$ = hypothetical new rate of flow

$$\frac{100}{x} - \frac{100}{x+8} = 0.25$$

$$\frac{100(x+8)}{c.d.} - \frac{100x}{c.d.} = 0.25$$

$$\frac{100x+800}{c.d.} - \frac{100x}{c.d.} = 0.25$$

$$\frac{800}{x(x+8)} = 0.25$$

$$\frac{800}{x^2+8x} = \frac{0.25}{1}$$

$$0.25x^2+2x = 800$$

$$0.25x^2+2x-800 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2-4(0.25)(-800)}}{2(0.25)}$$

$$= \frac{-2 \pm \sqrt{4+800}}{0.5}$$

$$= \frac{-2 \pm 28.35}{0.5}$$

$$= 52.7$$

$$\approx \boxed{53\ L/min = \text{flow from tap}}$$

3. In a factory, 80-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 94 litres per minute, it would take 90 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x + 94$ = hypothetical new rate of flow

$$\frac{80}{x} - \frac{80}{x+94} = 1.5$$

$$\frac{80(x+94)}{c.d.} - \frac{80x}{c.d.} = 1.5$$

$$\frac{80x + 7520}{c.d.} - \frac{80x}{c.d.} = 1.5$$

$$\frac{7520}{x(x+94)} = 1.5$$

$$\frac{7520}{x^2+94x} = \frac{1.5}{1}$$

$$1.5x^2 + 141x = 7520$$

$$1.5x^2 + 141x - 7520 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-141 \pm \sqrt{(141)^2 - 4(1.5)(-7520)}}{2(1.5)}$$

$$= \frac{-141 \pm \sqrt{19881 + 45120}}{3}$$

$$= \frac{-141 \pm 254.95}{3}$$

$$= 37.98$$

$$\approx \boxed{38 \text{ L/min} = \text{flow from tap}}$$

4. In a factory, 140-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 27 litres per minute, it would take 75 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

Let x = rate of flow (L/min)

$x+27$ = hypothetical new rate of flow

$$\frac{140}{x} - \frac{140}{x+27} = 1.25$$

$$\frac{140(x+27)}{c.d.} - \frac{140x}{c.d.} = 1.25$$

$$\frac{140x+3780}{c.d.} - \frac{140x}{c.d.} = 1.25$$

$$\frac{3780}{x(x+27)} = 1.25$$

$$\frac{3780}{x^2+27x} = \frac{1.25}{1}$$

$$1.25x^2 + 33.75x = 3780$$

$$1.25x^2 + 33.75x - 3780 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-33.75 \pm \sqrt{(33.75)^2 - 4(1.25)(-3780)}}{2(1.25)}$$

$$= \frac{-33.75 \pm \sqrt{1139.0625 + 18900}}{2.5}$$

$$= \frac{-33.75 \pm 141.56}{2.5}$$

$$= 43.12$$

$\approx \boxed{43 \text{ L/min} = \text{flow from tap}}$

5. In a factory, 110-litre containers are placed on a conveyor belt and filled with liquid as they pass under a tap one by one. If the flow from the tap was increased by 45 litres per minute, it would take 45 fewer seconds to fill each container. What is the flow from the tap to the nearest litre? Clearly show all your work.

let x = rate of flow (L/min)

$x+45$ = hypothetical new rate of flow

$$\frac{110}{x} - \frac{110}{x+45} = 0.75$$

$$\frac{110(x+45)}{c.d.} - \frac{110x}{c.d.} = 0.75$$

$$\frac{110x + 4950}{c.d.} - \frac{110x}{c.d.} = 0.75$$

$$\frac{4950}{x(x+45)} = 0.75$$

$$\frac{4950}{x^2+45x} = \frac{0.75}{1}$$

$$0.75x^2 + 33.75x = 4950$$

$$0.75x^2 + 33.75x - 4950 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-33.75 \pm \sqrt{(33.75)^2 - 4(.75)(-4950)}}{2(0.75)}$$

$$= \frac{-33.75 \pm \sqrt{1139.06 + 14850}}{1.5}$$

$$= \frac{-33.75 \pm 126.45}{1.5}$$

$$= 61.80$$

$$\approx \boxed{62\ L/min = \text{flow from tap}}$$

Type 6: Writing Second-Degree Equations for Word Problems (no solving!)

e.g.1 The area of a rectangle is 520 cm^2. Given that the length of the rectangle is 5 cm longer than the width, determine the second-degree equation of the form $ax^2 + bx + c = 0$.

Let x = width

$x + 5$ = length

$x(x+5) = 520$

$x^2 + 5x = 520$

$\boxed{x^2 + 5x - 520 = 0}$

You try:

1. The area of a rectangle is 315 cm^2. Given that the length of the rectangle is 11 cm longer than the width, determine the second-degree equation of the form $ax^2 + bx + c = 0$.

x = width

$x + 11$ = length

$x(x+11) = 315$

$x^2 + 11x = 315$

$\boxed{x^2 + 11x - 315 = 0}$

2. The area of a rectangle is 283 cm^2. Given that the length of the rectangle is 7 cm longer than the width, determine the second-degree equation of the form $ax^2 + bx + c = 0$.

x = width

$x + 7$ = length

$x(x+7) = 283$

$x^2 + 7x = 283$

$\boxed{x^2 + 7x - 283 = 0}$

3. Mary measured the front of her refrigerator. She found that it measures 13 860 cm^2. Given that the refrigerator is 81 cm longer than it is wide, determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

x = width

$x + 81$ = length

$x(x+81) = 13\ 860$

$\boxed{x^2 + 81x - 13860 = 0}$

4. Sam measured the area of the top of his laptop. He found that it measures 775 cm^2. Given that the laptop is 6 cm longer than it is wide, determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

x = width

$x + 6$ = length

$x(x+6) = 775$

$\boxed{x^2 + 6x - 775 = 0}$

5. Jacob measured the area of the front of his television. He found that it measures 4636 cm^2. Given that the television is 15 cm longer than it is wide, determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

x = width

$x + 15$ = length

$x(x+15) = 4636$

$\boxed{x^2 + 15x - 4636 = 0}$

e.g.2 The square of a number increased by twice that number is equal to 195. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 2x = 195$$

$$\boxed{x^2 + 2x - 195 = 0}$$

You try:

1. The square of a number increased by three times that number is equal to 108. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 3x = 108$$

$$\boxed{x^2 + 3x - 108 = 0}$$

2. The square of a number increased by twice that number is equal to 255. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 2x = 255$$

$$\boxed{x^2 + 2x - 255 = 0}$$

3. The square of a number increased by four times that number is equal to 165. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 4x = 165$$

$$\boxed{x^2 + 4x - 165 = 0}$$

4. The square of a number increased by five times that number is equal to 104. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 5x = 104$$

$$\boxed{x^2 + 5x - 104 = 0}$$

5. The square of a number increased by twice that number is equal to 323. Determine the second-degree equation of the form $ax^2 + bx + c = 0$ that describes this situation.

$$x^2 + 2x = 323$$

$$\boxed{x^2 + 2x - 323 = 0}$$