

MTH-4106 Factoring and Algebraic Fractions: Worksheet #10

e.g. 1 $\frac{8}{x^2-36} \times \frac{x+6}{4x+4}$

$$\frac{8}{(x-6)(x+6)} \cdot \frac{(x+6)}{4(x+1)}$$

$$\frac{2\cancel{8}}{(x-6)\cancel{(x+6)}} \cdot \frac{\cancel{(x+6)}}{4(x+1)}$$

$$\boxed{\frac{2}{(x-6)(x+1)}}$$

e.g. 2 $\frac{y^2+6y+5}{7y^2-63} \times \frac{7y+21}{(5+y)^2} \times \frac{3-y}{1+y}$

$$\frac{\cancel{(y+5)}\cancel{(y+1)}}{7\cancel{(y+3)}\cancel{(y-3)}} \cdot \frac{7\cancel{(y+3)}}{\cancel{(5+y)}(5+y)} \cdot \frac{\cancel{(3-y)}}{(1+y)}$$

$$\boxed{\frac{-1}{5+y}}$$

e.g. 3 $\frac{2x+y}{y^2} \times \frac{3y^2-3xy}{xy+2x^2} \times \frac{5xy-10x^2}{y^2-3xy+2x^2}$

$$\frac{\cancel{2x+y}}{y^2} \cdot \frac{3y(\cancel{y-x})}{x(y+2x)} \cdot \frac{5x(\cancel{y-2x})}{(y-2x)(\cancel{y-x})}$$

$$\boxed{\frac{15}{y}}$$

e.g. 4 $\frac{-x-y}{4x} \times \frac{-x^2+y^2}{-a} \times \frac{-2xa}{(x+y)^2}$

$$\frac{\cancel{-x-y}}{4x} \cdot \frac{(y-x)(y+x)}{-a} \cdot \frac{-2xa}{(x+y)(x+y)}$$

$$\boxed{\frac{-(y-x)}{2} \text{ or } \frac{-y+x}{2} \text{ or } \frac{x-y}{2}}$$

$$\text{e.g. 5} \quad \frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{12}{15} = \frac{4}{5}$$

$$\text{e.g. 6} \quad \frac{5a^2}{b^2-36} \div \frac{25ab-25a}{b^2-7b+6} = \frac{5a^2}{b^2-36} \cdot \frac{b^2-7b+6}{25ab-25a}$$

$$\frac{5a^2}{(b-6)(b+6)} \cdot \frac{(b-6)(b-1)}{25a(b-1)}$$

$$\boxed{\frac{a}{5(b+6)}}$$

$$\text{e.g. 7} \quad \frac{2-x}{b} \div \frac{x^2+2x}{4a^2-a^2x^2}$$

$$\frac{2-x}{b} \cdot \frac{4a^2-a^2x^2}{x^2+2x}$$

$$\frac{2-x}{b} \cdot \frac{a^2(2-x)(2+x)}{x(x+2)}$$

$$\boxed{\frac{a^2(2+x)^2}{bx}}$$

e.g. 8 $\frac{x^2-8x-9}{x^2-17x+72} \times \frac{x^2-25}{x^2-1} \div \frac{x^2+4x-5}{x^2-9x+8}$

$$\frac{x^2-8x-9}{x^2-17x+72} \cdot \frac{x^2-25}{x^2-1} \cdot \frac{x^2-9x+8}{x^2+4x-5}$$

$$\frac{(x-9)(x+1)}{(x-9)(x-8)} \cdot \frac{(x+5)(x-5)}{(x+1)(x-1)} \cdot \frac{(x-8)(x-1)}{(x+5)(x-1)}$$

$$\boxed{\frac{x-5}{x-1}}$$

e.g. 9 $\frac{b^2-b-20}{b^2-25} \div \frac{b^2+2b-8}{b^2-b-2} \div \frac{b+1}{b^2+5b}$

$$\frac{b^2-b-20}{b^2-25} \cdot \frac{b^2-b-2}{b^2+2b-8} \cdot \frac{b^2+5b}{b+1}$$

$$\frac{(b-5)(b+4)}{(b-5)(b+5)} \cdot \frac{(b-2)(b+1)}{(b+4)(b-2)} \cdot \frac{b(b+5)}{(b+1)} = \boxed{b}$$

$$1. \frac{a-2b}{2} \times \frac{12}{6b-3a} = \frac{a-2b}{2} \cdot \frac{12}{3(2b-a)}$$

① $6q^2 - 11q - 10$ $p = -60$
 $(6q^2 - 15q) + (4q - 10)$ $s = -11$
 $-15, +4$
 $3q(2q-5) + 2(2q-5)$
 $(3q+2)(2q-5)$

$$= \frac{-1(-a+2b)}{2} \cdot \frac{12}{3(2b-a)}$$

② $2q^2 + q - 15$ $p = -30$
 $(2q^2 + 6q) - 5q - 15$ $s = +1$
 $+6, -5$
 $2q(q+3) - 5(q+3)$
 $(2q-5)(q+3)$

$$= \frac{-1(-a+2b)}{2} \cdot \frac{12}{3(2b-a)}$$

$$= \boxed{-2}$$

$$2. \frac{x^2-x-12}{x-2} \times \frac{x^2+4x-12}{x-4} = \frac{(x-4)(x+3)}{(x-2)} \cdot \frac{(x+6)(x-2)}{(x-4)}$$

$$= \frac{(x-4)(x+3)}{(x-2)} \cdot \frac{(x+6)(x-2)}{(x-4)}$$

$$= \boxed{(x+3)(x+6)}$$

① $3q^2 + 8q - 3$ $p = -9$
 $(3q^2 + 9q) - 1q - 3$ $s = 8$
 $9, -1$
 $3q(q+3) - 1(q+3)$
 $(3q-1)(q+3)$

④ $-3q^2 + 4q - 1$ $p = +3$
 $(-3q^2 + 3q) + 1q - 1$ $s = +4$
 $+3, +1$
 $-3q(q-1) + 1(q-1)$
 $(-3q+1)(q-1)$

① $3q^2 + 8q - 3$
 ② $2q^2 + q - 15$
 ③ $6q^2 - 11q - 10$
 ④ $-3q^2 + 4q - 1$

$$= \frac{(3q-1)(q+3)}{(2q-5)(q+3)} \cdot \frac{(3q+2)(2q-5)}{(-3q+1)(q-1)}$$

$$= \frac{-1(-3q+1)}{(3q-1)(q+3)} \cdot \frac{(3q+2)(2q-5)}{(-3q+1)(q-1)}$$

$$= \boxed{\frac{-1(3q+2)}{(q-1)}}$$

Product-sum method (not shown) ⁶

4. $\frac{m^2 - 2mn + n^2}{4mn + 4n} \times \frac{(m+1)^2}{2m^2 - 2mn} \times \frac{8mn}{m^2 - 1}$

$$\frac{(m-n)(m-n)}{4n(m+1)} \cdot \frac{(m+1)(m+1)}{2m(m-n)} \cdot \frac{8mn}{(m-1)(m+1)}$$

$$\frac{\cancel{(m-n)}\cancel{(m-n)}}{\cancel{4n}\cancel{(m+1)}} \cdot \frac{\cancel{(m+1)}\cancel{(m+1)}}{\cancel{2m}\cancel{(m-n)}} \cdot \frac{\cancel{8mn}}{\cancel{(m-1)}\cancel{(m+1)}}$$

$$\boxed{\frac{m-n}{m-1}}$$

5. $\frac{s^2 + 4s - 5}{r+2} \times \frac{s^2 + 6s + 8}{r+5} \times \frac{s^2 + s - 2}{(r-1)^2}$

$$\frac{(s+5)(s-1)}{(r+2)} \cdot \frac{(s+2)(s+4)}{(r+5)} \cdot \frac{(s+2)(s-1)}{(r-1)(r-1)}$$

$$\frac{(s+5)(s-1)}{(r+2)} \cdot \frac{(s+2)(s+4)}{(r+5)} \cdot \frac{(s+2)(s-1)}{(r-1)(r-1)}$$

$$= \boxed{\frac{(s+5)(s-1)^2(s+2)^2(s+4)}{(r+2)(r+5)(r-1)^2}}$$

* nothing cancels!

6. $\frac{3a-2b}{5a+2b} \times \frac{8b+20a}{6b-9a}$

$$= \frac{(3a-2b)}{(5a+2b)} \cdot \frac{4(2b+5a)}{3(2b-3a)}$$

$$= \frac{-1(\cancel{-3a+2b})}{(5a+2b)} \cdot \frac{4(\cancel{2b+5a})}{3(\cancel{2b-3a})} = \boxed{\frac{-4}{3}}$$

$$7. \frac{z^2 - 49}{z^2 - 5z - 14} \div \frac{z+7}{2z^2 - 13z - 7} = \frac{z^2 - 49}{z^2 - 5z - 14} \cdot \frac{2z^2 - 13z - 7}{z+7}$$

$$\textcircled{1} 2z^2 - 13z - 7$$

$P = -14$
 $S = -13$
 $-14, +1$

$$(2z^2 - 14z) + (1z - 7)$$

$$2z(z-7) + 1(z-7)$$

$$(2z+1)(z-7)$$

$$= \frac{(z-7)(z+7)}{(z-7)(z+2)} \cdot \frac{(2z+1)(z-7)}{(z+7)}$$

$$= \frac{\cancel{(z-7)}\cancel{(z+7)}}{\cancel{(z-7)}(z+2)} \cdot \frac{(2z+1)\cancel{(z-7)}}{\cancel{(z+7)}}$$

$$= \boxed{\frac{(2z+1)(z-7)}{z+2}}$$

$$8. \textcircled{1} \frac{6x^2 - ax - 2a^2}{ax - a^2} \times \frac{x-a}{9x^2 - 4a^2} \times \frac{3ax + 2a^2}{2x+a}$$

$$= \frac{(2x+a)(3x-2a)}{a(x-a)} \cdot \frac{(x-a)}{(3x-2a)(3x+2a)} \cdot \frac{a(3x+2a)}{(2x+a)}$$

$$= \frac{\cancel{(2x+a)}\cancel{(3x-2a)}}{a\cancel{(x-a)}} \cdot \frac{\cancel{(x-a)}}{\cancel{(3x-2a)}(3x+2a)} \cdot \frac{a\cancel{(3x+2a)}}{\cancel{(2x+a)}}$$

$$\textcircled{1} 6x^2 - ax - 2a^2$$

$P = -12$
 $S = -1$
 $-4, +3$

$$(6x^2 - 4ax) + (3ax - 2a^2)$$

$$2x(3x-2a) + a(3x-2a)$$

$$(2x+a)(3x-2a)$$

$$= \boxed{1}$$

$$9. \frac{r^2 + 5r + 6}{r^2 - 1} \div \frac{r^2 - 9}{r^2 - 2r - 3} = \frac{r^2 + 5r + 6}{r^2 - 1} \cdot \frac{r^2 - 2r - 3}{r^2 - 9}$$

$$= \frac{(r+2)(r+3)}{(r-1)(r+1)} \cdot \frac{(r-3)(r+1)}{(r-3)(r+3)}$$

$$= \frac{(r+2)\cancel{(r+3)}}{(r-1)\cancel{(r+1)}} \cdot \frac{\cancel{(r-3)}\cancel{(r+1)}}{\cancel{(r-3)}(r+3)}$$

$$= \boxed{\frac{r+2}{r-1}}$$

$$10. \frac{c^2 - 18c + 80}{c^2 - 5c - 50} \times \frac{c^2 - 6c - 7}{c^2 - 15c + 56} \times \frac{c+5}{c-1}$$

$$\frac{(c-8)(c-10)}{(c-10)(c+5)} \cdot \frac{(c-7)(c+1)}{(c-7)(c-8)} \cdot \frac{(c+5)}{(c-1)}$$

$$\frac{\cancel{(c-8)}\cancel{(c-10)}}{\cancel{(c-10)}\cancel{(c+5)}} \cdot \frac{\cancel{(c-7)}(c+1)}{\cancel{(c-7)}\cancel{(c-8)}} \cdot \frac{\cancel{(c+5)}}{(c-1)} = \frac{c+1}{c-1}$$

$$11. \frac{4x+5y}{4x-5y} \div \frac{16x^2-25y^2}{4x-5y} = \frac{4x+5y}{4x-5y} \cdot \frac{4x-5y}{16x^2-25y^2}$$

$$= \frac{(4x+5y)}{(4x-5y)} \cdot \frac{(4x-5y)}{(4x-5y)(4x+5y)}$$

$$= \frac{\cancel{(4x+5y)}}{(4x-5y)} \cdot \frac{\cancel{(4x-5y)}}{\cancel{(4x-5y)}\cancel{(4x+5y)}}$$

$$= \boxed{\frac{1}{4x-5y}}$$

$$12. \frac{-m-n}{4m} \times \frac{-2ms}{(m+n)^2} \times \frac{-m^2+n^2}{-s}$$

$$= \frac{-m-n}{4m} \cdot \frac{-2ms}{(m+n)(m+n)} \cdot \frac{n^2-m^2}{(-1)(s)}$$

$$= \frac{-1(m+n)}{(4)(m)} \cdot \frac{(-2)(m)(s)}{(m+n)(m+n)} \cdot \frac{(n-m)(n+m)}{(-1)(s)}$$

$$= \frac{\cancel{-1}\cancel{(m+n)}}{(4)\cancel{(m)}} \cdot \frac{\cancel{(-2)}\cancel{(m)}(s)}{\cancel{(m+n)}\cancel{(m+n)}} \cdot \frac{(n-m)\cancel{(n+m)}}{\cancel{(-1)}(s)} = \frac{-2(n-m)}{4} = \boxed{\frac{-(n-m)}{2}}$$

$$b^2(a^2-4) = b^2(a-2)(a+2)$$

13. $\frac{4-a^2}{a^2+4} \div \frac{(b^2a^2-4b^2)}{1} = \frac{4-a^2}{a^2+4} \cdot \frac{1}{b^2a^2-4b^2}$

$$= \frac{(2-a)(2+a)}{(a^2+4)} \cdot \frac{1}{b^2(a-2)(a+2)}$$

$$= \frac{\cancel{(2-a)}\cancel{(2+a)}}{(a^2+4)} \cdot \frac{1}{b^2(-1)(-a+2)(a+2)}$$

$$= \boxed{\frac{1}{-b^2(a^2+4)}}$$

14. $\frac{b^2-4ab}{b-a} \div \frac{16a^2b^2-b^4}{4a^2-3ab-b^2}$

$$= \frac{b^2-4ab}{b-a} \cdot \frac{4a^2-3ab-b^2}{16a^2b^2-b^4}$$

$$= \frac{b(b-4a)}{(b-a)} \cdot \frac{(4a+b)(a-b)}{b^2(4a-b)(4a+b)}$$

$$= \frac{b(-1)(-b+4a)}{\cancel{(b-a)}} \cdot \frac{(4a+b)(-1)(-a+b)}{b^2(4a-b)(4a+b)}$$

① $4a^2-3ab-b^2$ $p=-4$
 $(4a^2-4ab)+(ab-b^2)$ $s=-3$
 $4a(a-b)+b(a-b)$ $-4, +1$
 $(4a+b)(a-b)$

② $16a^2b^2-b^4$
 $b^2(16a^2-b^2)$
 $b^2(4a-b)(4a+b)$

$$\rightarrow = \frac{b}{b^2} = \boxed{\frac{1}{b}}$$

15. $\frac{2c+d}{d^2} \times \frac{3d^2-3cd}{cd+2c^2} \times \frac{5cd-10c^2}{d^2-3cd+2c^2}$

$$\frac{2c+d}{d^2} \cdot \frac{3d(d-c)}{c(d+2c)} \cdot \frac{5c(d-2c)}{(d-2c)(d-c)}$$

$$= \frac{\cancel{(2c+d)}}{d^2} \cdot \frac{3d\cancel{(d-c)}}{c(d+2c)} \cdot \frac{5c\cancel{(d-2c)}}{\cancel{(d-2c)}\cancel{(d-c)}} = \frac{15cd}{cd^2} = \boxed{\frac{15}{d}}$$

$$\begin{aligned}
 16. \quad \frac{25-x^2}{x} \times \frac{5x-10}{-25x-5x^2} &= \frac{(5-x)(5+x)}{x} \cdot \frac{5(x-2)}{-5x(5+x)} \\
 &= \frac{(5-x)\cancel{(5+x)}}{x} \cdot \frac{\cancel{5}(x-2)}{-\cancel{5}x(5+x)} \\
 &= \boxed{\frac{(5-x)(x-2)}{-x^2}}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{3p+p^2}{12+3p} \div \frac{9p-p^3}{12-p-p^2} &\rightarrow \begin{array}{l} \textcircled{1} -p^2-p+12 \quad p=-12 \\ (-p^2-4p)(3p+12) \quad s=-1 \\ -4, +3 \\ -p(p+4)+3(p+4) \\ (-p+3)(p+4) \\ \textcircled{2} 9p-p^3 \\ p(9-p^2) \\ p(3-p)(3+p) \end{array} \\
 \frac{3p+p^2}{12+3p} \times \frac{\textcircled{1} -p^2-p+12}{\textcircled{2} 9p-p^3} &= \frac{p(3+p)}{3(4+p)} \cdot \frac{(-p+3)(p+4)}{p(3-p)(3+p)} \\
 &= \boxed{\frac{1}{3}}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{9-a^2}{a-3} \div \frac{a^2-a-12}{a-4} &= \frac{9-a^2}{a-3} \cdot \frac{a-4}{a^2-a-12} \\
 &= \frac{(3-a)(3+a)}{(a-3)} \cdot \frac{(a-4)}{(a-4)(a+3)} \\
 &= \frac{(3-a)\cancel{(3+a)}}{(a-3)} \cdot \frac{\cancel{(a-4)}}{\cancel{(a-4)}(a+3)} \\
 &= \frac{-1(-3+a)}{\cancel{(a-3)}} = \boxed{-1}
 \end{aligned}$$

$$19. \frac{64p^2q^2 - z^2}{x^2 - 4} \times \frac{(x-2)^2}{8pq+z} \div \frac{x^2-4}{(x+2)^2}$$

$$\frac{64p^2q^2 - z^2}{x^2 - 4} \cdot \frac{(x-2)(x-2)}{8pq+z} \cdot \frac{(x+2)(x+2)}{x^2-4}$$

$$\frac{(8pq-z)(8pq+z)}{(x-2)(x+2)} \cdot \frac{(x-2)(x-2)}{(8pq+z)} \cdot \frac{(x+2)(x+2)}{(x+2)(x-2)}$$

$$= \boxed{8pq - z}$$

$$20. \frac{r^2 - s^2}{3-r} \div \frac{4s-4r}{r^2+3rs} \div \frac{r^2+4rs+3s^2}{rs-3s}$$

$$\frac{r^2 - s^2}{3-r} \cdot \frac{r^2+3rs}{4s-4r} \cdot \frac{rs-3s}{r^2+4rs+3s^2}$$

$$\frac{(r-s)(r+s)}{(3-r)} \cdot \frac{r(r+3s)}{4(s-r)} \cdot \frac{s(r-3)}{(r+3s)(r+s)}$$

$$\frac{(-1)(-r+s)}{(3-r)} \cdot \frac{r}{4(s-r)} \cdot \frac{s(-1)(-r+3)}{1} = \frac{(-1)(-1)(r)(s)}{4} = \boxed{\frac{rs}{4}}$$

$$21. \textcircled{1} \frac{9x^2 - 6x + 1}{x-5} \times \frac{x^2 - 7x + 10}{3x^2 - 7x + 2}$$

$$\frac{(3x-1)(3x-1)}{(x-5)} \cdot \frac{(x-2)(x-5)}{(3x-1)(x-2)}$$

$$\frac{(3x-1)(\cancel{3x-1})}{(\cancel{x-5})} \cdot \frac{(\cancel{x-2})(\cancel{x-5})}{(3x-1)(\cancel{x-2})}$$

$$\boxed{3x-1}$$

$$\textcircled{1} 9x^2 - 6x + 1$$

$$(9x^2 - 3x) + (-3x + 1)$$

$$3x(3x-1) - 1(3x-1)$$

$$(3x-1)(3x-1)$$

$$p=9$$

$$s=-6$$

$$-3, -3$$

$$\textcircled{2} 3x^2 - 7x + 2$$

$$(3x^2 - 6x) + (-1x + 2)$$

$$3x(x-2) - 1(x-2)$$

$$(3x-1)(x-2)$$

$$p=6$$

$$s=-7$$

$$-6, -1$$

$$22. \quad \textcircled{1} \frac{6b^2 - ab - 2a^2}{ab - a^2} \times \frac{b-a}{9b^2 - 4a^2} \times \frac{3ab + 2a^2}{2b+a}$$

$$= \frac{(2b+a)(3b-2a)}{a(b-a)} \cdot \frac{(b-a)}{(3b-2a)(3b+2a)} \cdot \frac{a(3b+2a)}{(2b+a)}$$

$$= \boxed{1}$$

$$\begin{aligned} \textcircled{1} \quad & 6b^2 - 1ab - 2a^2 \quad p = -12 \\ & (6b^2 - 4ab) + (3ab - 2a^2) \quad s = -1 \\ & 2b(3b-2a) + a(3b-2a) \quad -4, +3 \\ & (2b+a)(3b-2a) \end{aligned}$$

$$23. \quad \textcircled{1} \frac{4c^2 + c - 14}{6cd - 14d} \times \frac{4c^2}{c^2 - 4} \times \frac{c-2}{4c-7} \div \textcircled{2} \frac{2c^2 + 4c}{3c^2 - c - 14}$$

$$= \frac{4c^2 + c - 14}{6cd - 14d} \cdot \frac{4c^2}{c^2 - 4} \cdot \frac{c-2}{4c-7} \cdot \frac{3c^2 - c - 14}{2c^2 + 4c}$$

$$= \frac{(4c-7)(c+2)}{2d(3c-7)} \cdot \frac{4c^2}{(c-2)(c+2)} \cdot \frac{(c-2)}{(4c-7)} \cdot \frac{(3c-7)(c+2)}{2c(c+2)}$$

$$= \frac{(4c-7)(c+2)}{2d(3c-7)} \cdot \frac{4c^2}{(c-2)(c+2)} \cdot \frac{(c-2)}{(4c-7)} \cdot \frac{(3c-7)(c+2)}{2c(c+2)}$$

$$= \frac{4c^2}{4cd} = \boxed{\frac{c}{d}}$$

$$24. \quad \frac{6x - 4x^2}{2x^2 - 5xy + 3y^2} \div \frac{2x^3 - 3x^2}{xy - x^2} = \textcircled{1} \frac{6x - 4x^2}{2x^2 - 5xy + 3y^2} \cdot \frac{xy - x^2}{2x^3 - 3x^2}$$

$$= \frac{2x(3-2x)}{(2x-3y)(x-y)} \cdot \frac{x(y-x)}{x^2(2x-3)}$$

$$= \frac{2x(-1)(-3+2x)}{(2x-3y)(x-y)} \cdot \frac{x(-1)(-y+x)}{x^2(2x-3)}$$

$$= \frac{2x^2(-1)(-1)}{(2x-3y)x^2} = \boxed{\frac{2}{2x-3y}}$$

$$\begin{aligned} \textcircled{1} \quad & 4c^2 + c - 14 \quad p = -56 \\ & (4c^2 + 8c) + (-7c - 14) \quad s = +1 \\ & 4c(c+2) - 7(c+2) \quad 8, -7 \\ & (4c-7)(c+2) \\ \textcircled{2} \quad & 3c^2 - 1c - 14 \quad p = -42 \\ & (3c^2 + 6c) + (-7c - 14) \quad s = -1 \\ & 3c(c+2) - 7(c+2) \quad 6, -7 \\ & (3c-7)(c+2) \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad & 2x^2 - 5xy + 3y^2 \quad p = 6 \\ & (2x^2 - 2xy) + (-3xy + 3y^2) \quad s = -5 \\ & 2x(x-y) - 3y(x-y) \quad -2, -3 \\ & (2x-3y)(x-y) \end{aligned}$$

$$25. \frac{-x^4 + 25}{5x - 15} \times \frac{2x^3 + 2x^2 + 6x + 6}{x^4 - 2x^2 - 15} \times \frac{x^3 - 3x^2}{x^4 + 5x^2}$$

$$\begin{aligned} &\rightarrow (2x^3 + 2x^2) + (6x + 6) \\ &2x^2(x+1) + 6(x+1) \\ &(2x^2 + 6)(x+1) \\ &2(x^2 + 3)(x+1) \end{aligned}$$

hint... the 4-step trinomial method (product-sum method) works for the second denominator

$$\begin{aligned} &\frac{25 - x^4}{5x - 15} \cdot \frac{2(x^2 + 3)(x+1)}{(x^2 + 3)(x^2 - 5)} \cdot \frac{x^2(x-3)}{x^2(x^2 + 5)} \\ &\frac{(5 - x^2)(5 + x^2)}{5(x-3)} \cdot \frac{2(x^2 + 3)(x+1)}{(x^2 + 3)(x^2 - 5)} \cdot \frac{x^2(x-3)}{x^2(x^2 + 5)} \end{aligned}$$

$$\frac{(-1)(-5 + x^2)(2)(x+1)}{(5)(x^2 - 5)} = \boxed{\frac{-2(x+1)}{5}}$$

$$\begin{aligned} &p = -15, s = -2 \\ &\quad -5, +3 \\ &(x^4 - 5x^2)(3x^2 - 15) \\ &x^2(x^2 - 5) + 3(x^2 - 5) \\ &(x^2 + 3)(x^2 - 5) \end{aligned}$$

$$26. \frac{1 - x^2}{x^2 + xy} \times \frac{x^2 - y^2}{x - 1} \div \frac{x^2 - xy + x - y}{x}$$

$$\begin{aligned} &\rightarrow (x^2 - xy) + (x - y) \\ &x(x - y) + 1(x - y) \\ &(x+1)(x-y) \end{aligned}$$

$$\frac{1 - x^2}{x^2 + xy} \cdot \frac{x^2 - y^2}{x - 1} \cdot \frac{x}{x^2 - xy + x - y}$$

$$\frac{(1-x)(1+x)}{x(x+y)} \cdot \frac{(x-y)(x+y)}{(x-1)} \cdot \frac{x}{(x+1)(x-y)}$$

$$\frac{(-1)(-1+x)(1+x)}{x(x+y)} \cdot \frac{(x-y)(x+y)}{(x-1)} \cdot \frac{x}{(x+1)(x-y)} = \boxed{-1}$$

$$27. \frac{3x^2 - 13x + 4}{-3x^2 + 16x - 5} \div \frac{x^2 - 16}{-x^2 + 9x - 20}$$

$$\textcircled{1} \begin{array}{l} 3x^2 - 13x + 4 \quad p=12 \\ (3x^2 - 12x)(1x+4) \quad s=-13 \\ \quad \quad \quad \quad \quad -12, -1 \end{array}$$

$$3x(x-4) - 1(x-4)$$

$$(3x-1)(x-4)$$

$$\textcircled{2} \begin{array}{l} -3x^2 + 16x - 5 \quad p=15 \\ -3x^2 + 15x + 1x - 5 \quad s=16 \\ \quad \quad \quad \quad \quad 15, 1 \end{array}$$

$$-3x(x-5) + 1(x-5)$$

$$(-3x+1)(x-5)$$

$$\textcircled{3} \begin{array}{l} -x^2 + 9x - 20 \quad p=20 \\ (-x^2 + 4x)(5x-20) \quad s=9 \\ \quad \quad \quad \quad \quad 4, 5 \end{array}$$

$$-x(x-4) + 5(x-4)$$

$$(-x+5)(x-4) \quad b^2 - b - 2$$

$$28. \frac{-2-b+b^2}{2b^2+6b-3bc-9c} \times \frac{3b-2c}{b^2-2b} \times \frac{-2b^3-6b^2}{b^2+b}$$

$$\hookrightarrow (2b^2+6b) + (-3bc-9c)$$

$$2b(b+3) - 3c(b+3)$$

$$(2b-3c)(b+3)$$

$$= \frac{(b-2)(b+1)}{(2b-3c)(b+3)} \cdot \frac{(3b-2c)}{b(b-2)} \cdot \frac{-2b^2(b+3)}{b(b+1)}$$

$$= \frac{\cancel{(b-2)}\cancel{(b+1)}}{(2b-3c)\cancel{(b+3)}} \cdot \frac{(3b-2c)}{b\cancel{(b-2)}} \cdot \frac{-2b^2\cancel{(b+3)}}{b\cancel{(b+1)}}$$

$$= \frac{-2b^2(3b-2c)}{b^2(2b-3c)} = \boxed{\frac{-2(3b-2c)}{2b-3c}}$$

$$\textcircled{1} \frac{3x^2 - 13x + 4}{-3x^2 + 16x - 5} \cdot \textcircled{3} \frac{-x^2 + 9x - 20}{x^2 - 16}$$

$$= \frac{(3x-1)(x-4)}{(-3x+1)(x-5)} \cdot \frac{(-x+5)(x-4)}{(x-4)(x+4)}$$

$$= \frac{(-1)\cancel{(-3x+1)}(x-4)}{\cancel{(-3x+1)}(x-5)} \cdot \frac{(-1)\cancel{(x-5)}(x-4)}{\cancel{(x-4)}(x+4)}$$

$$= \boxed{\frac{x-4}{x+4}}$$

$$\begin{aligned} & \textcircled{1} (2h-8g) + (hg-4g^2) \\ & 2(h-4g) + g(h-4g) \\ & (2+g)(h-4g) \end{aligned}$$

$$29. \frac{h^2-16g^2}{4-4g+g^2} \times \frac{4-g^2}{2h-8g+hg-4g^2} \div \frac{3(h+4g)}{4(2-g)}$$

$$\frac{h^2-16g^2}{g^2-4g+4} \cdot \frac{4-g^2}{2h-8g+hg-4g^2} \cdot \frac{4(2-g)}{3(h+4g)}$$

$$\frac{(h-4g)(h+4g)}{(g-2)(g-2)} \cdot \frac{(2-g)(2+g)}{(2+g)(h-4g)} \cdot \frac{4(2-g)}{3(h+4g)}$$

$$\frac{\cancel{(h-4g)}\cancel{(h+4g)}}{\cancel{(g-2)}\cancel{(g-2)}} \cdot \frac{-1(-2+g)\cancel{(2+g)}}{\cancel{(2+g)}\cancel{(h-4g)}} \cdot \frac{4(-1)\cancel{(-2+g)}}{3\cancel{(h+4g)}}$$

$$\boxed{\frac{4}{3}}$$

$$30. \frac{16a^2-9b^2}{2a-8a^2} \div \frac{3b-4a}{4a^2-a} = \frac{16a^2-9b^2}{2a-8a^2} \cdot \frac{4a^2-a}{3b-4a}$$

$$= \frac{(4a-3b)(4a+3b)}{2a(1-4a)} \cdot \frac{a(4a-1)}{3b-4a}$$

$$= \frac{(-1)\cancel{(-4a+3b)}(4a+3b)}{2\cancel{a}(1-4a)} \cdot \frac{\cancel{a}(-1)\cancel{(-4a+1)}}{(3b-4a)}$$

$$= \boxed{\frac{4a+3b}{2}}$$

① $-2a^2 + 7ab - 6b^2$
 $(-2a^2 + 3ab) + (4ab - 6b^2)$ $P=12$
 $-a(2a-3b) + 2b(2a-3b)$ $S=7$
 $(-a+2b)(2a-3b)$ $3,4$

31. ① $\frac{2a^2 - 4ab}{-2a^2 + 7ab - 6b^2} \div \frac{2b}{2a-3b}$

$= \frac{2a(a-2b)}{(-a+2b)(2a-3b)} \cdot \frac{2a-3b}{2b}$

$= \frac{2a(-1)(-a+2b)}{(-a+2b)(2a-3b)} \cdot \frac{2a-3b}{2b}$

$= \boxed{\frac{-a}{b}}$

$p^3 - 64p = p(p^2 - 64) = p(p-8)(p+8)$
 $-p^2 - p + 12$ $P=-12$
 $(-p^2 - 4p)(3p+12)$ $S=-1$
 $-p(p+4) + 3(p+4)$ $-4, 3$
 $(-p+3)(p+4)$

32. $\frac{3p+p^2}{12+3p} \div \frac{9p-p^3}{12-p-p^2} \times \frac{p^3-64p}{2p^2+16p} \div \frac{p^2-9p+8}{p^2+4p-5}$

$9p-p^3$
 $p(9-p^2)$
 $p(3-p)(3+p)$

① $\frac{3p+p^2}{12+3p} \cdot \frac{-p^2-p+12}{9p-p^3} \cdot \frac{p^3-64p}{2p^2+16p} \cdot \frac{p^2+4p-5}{p^2-9p+8}$

$\frac{p(3+p)}{3(4+p)} \cdot \frac{(-p+3)(p+4)}{p(3-p)(3+p)} \cdot \frac{p(p-8)(p+8)}{2p(p+8)} \cdot \frac{(p+5)(p-1)}{(p-8)(p-1)}$

$\frac{p(3+p)}{3(4+p)} \cdot \frac{(-p+3)(p+4)}{p(3-p)(3+p)} \cdot \frac{p(p-8)(p+8)}{2p(p+8)} \cdot \frac{(p+5)(p-1)}{(p-8)(p-1)}$

33. $\frac{(v-w)^2}{bw-bv} \times \frac{b^2}{3w-3v} \div \frac{16b^2-9}{5v+5w} \div \frac{15w+5v}{8b+6}$

$\frac{(v-w)^2}{bw-bv} \cdot \frac{b^2}{3w-3v} \cdot \frac{5v+5w}{16b^2-9} \cdot \frac{8b+6}{15w+5v}$

$\frac{(v-w)(v-w)}{b(w-v)} \cdot \frac{b^2}{3(w-v)} \cdot \frac{5(v+w)}{(4b-3)(4b+3)} \cdot \frac{2(4b+3)}{5(3w+v)}$

$\frac{(-1)(-v+w)(-1)(-v+w)}{b(w-v)} \cdot \frac{b^2}{3(w-v)} \cdot \frac{5(v+w)}{(4b-3)(4b+3)} \cdot \frac{2(4b+3)}{5(3w+v)}$

$= \frac{2b^2(v+w)}{3b(4b-3)(3w+v)} = \boxed{\frac{2b(v+w)}{3(4b-3)(3w+v)}}$

$= \frac{p(p+5)}{6p} = \boxed{\frac{p+5}{6}}$

$$34. \quad \frac{4x^2 - 16x + 15}{2x^2 + 3x + 1} \div \frac{2x^2 - 17x + 21}{x^2 - 6x - 7} \div \frac{1}{5x^2 - 2x^3} \times \frac{4x^2 - 1}{4x^2 - 20x + 25}$$

$$\frac{4x^2 - 16x + 15}{2x^2 + 3x + 1} \cdot \frac{x^2 - 6x - 7}{2x^2 - 17x + 21} \cdot \frac{5x^2 - 2x^3}{1} \cdot \frac{4x^2 - 1}{4x^2 - 20x + 25}$$

$$\frac{(2x-3)(2x-5)}{(2x+1)(x+1)} \cdot \frac{(x-7)(x+1)}{(x-7)(2x-3)} \cdot \frac{x^2(5-2x)}{1} \cdot \frac{(2x-1)(2x+1)}{(2x-5)(2x-5)}$$

$$\frac{\cancel{(2x-3)}\cancel{(2x-5)}}{\cancel{(2x+1)}\cancel{(x+1)}} \cdot \frac{\cancel{(x-7)}\cancel{(x+1)}}{\cancel{(x-7)}\cancel{(2x-3)}} \cdot \frac{-x^2\cancel{(2x-5)}}{1} \cdot \frac{(2x-1)\cancel{(2x+1)}}{\cancel{(2x-5)}\cancel{(2x-5)}}$$

$$= -x^2(2x-1)$$

$$35. \quad \frac{36a^4 - 49b^2}{6a^2 + 9} \times \frac{5b^3}{3 - 2a^2} \div \frac{6a^2b^2 + 7b^3}{3} \times \frac{20a^4 - 45}{25b}$$

$$\frac{36a^4 - 49b^2}{6a^2 + 9} \cdot \frac{5b^3}{3 - 2a^2} \cdot \frac{3}{6a^2b^2 + 7b^3} \cdot \frac{20a^4 - 45}{25b}$$

$$\frac{(6a^2 - 7b)(6a^2 + 7b)}{3(2a^2 + 3)} \cdot \frac{5b^3}{(3 - 2a^2)} \cdot \frac{3}{b^2(6a^2 + 7b)} \cdot \frac{5(4a^4 - 9)}{25b}$$

$$\frac{(6a^2 - 7b)\cancel{(6a^2 + 7b)}}{3\cancel{(2a^2 + 3)}} \cdot \frac{5b^3}{(3 - 2a^2)} \cdot \frac{3}{b^2\cancel{(6a^2 + 7b)}} \cdot \frac{5(2a^2 - 3)\cancel{(2a^2 + 3)}}{25b}$$

$$\frac{(6a^2 - 7b)}{1} \cdot \frac{5b^3}{\cancel{(3 - 2a^2)}} \cdot \frac{1}{b^2} \cdot \frac{-5\cancel{(-2a^2 + 3)}}{25b}$$

$$\frac{-25b^3(6a^2 - 7b)}{25b^3} = -(6a^2 - 7b)$$

OR $-6a^2 + 7b$ OR $7b - 6a^2$

36. $\frac{16y^2 - 12y + 2}{4y^2 - 4} \div \frac{10y^2 + 15y - 10}{2y + 10} \times \frac{y^2 - y - 2}{4y^2 + 19y - 5}$

$$\frac{\textcircled{1} 16y^2 - 12y + 2}{\textcircled{2} 4y^2 - 4} \cdot \frac{2y + 10}{\textcircled{3} 10y^2 + 15y - 10} \cdot \frac{y^2 - y - 2}{\textcircled{4} 4y^2 + 19y - 5}$$

$$\frac{2(2y-1)(4y-1)}{4(y-1)(y+1)} \cdot \frac{2(y+5)}{5(2y-1)(y+2)} \cdot \frac{(y-2)(y+1)}{(4y-1)(y+5)}$$

$$\frac{\cancel{2(2y-1)}(\cancel{4y-1})}{4(y-1)(y+1)} \cdot \frac{2(y+5)}{5(\cancel{2y-1})(y+2)} \cdot \frac{(y-2)(y+1)}{(\cancel{4y-1})(y+5)}$$

$$\frac{4(y-2)}{20(y-1)(y+2)} = \frac{(y-2)}{5(y-1)(y+2)}$$

$$\textcircled{4} 4y^2 + 19y - 5 \quad p = -20$$

$$(4y^2 + 20y)(-1y - 5) \quad s = 19$$

$$4y(y+5) - 1(y+5) \quad 20, -1$$

$$(4y-1)(y+5)$$

37. $\frac{x^2 - 12x + 36}{x^2 + x} \times \frac{2xy}{x^2 - 7x + 6} \div \frac{xy - 6y}{x^2 - 1}$

$$\frac{x^2 - 12x + 36}{x^2 + x} \cdot \frac{2xy}{x^2 - 7x + 6} \cdot \frac{x^2 - 1}{xy - 6y}$$

$$\frac{(x-6)(x-6)}{x(x+1)} \cdot \frac{2xy}{(x-6)(x-1)} \cdot \frac{(x-1)(x+1)}{y(x-6)}$$

$$\frac{\cancel{(x-6)}(\cancel{x-6})}{x(x+1)} \cdot \frac{2\cancel{x}y}{(\cancel{x-6})(x-1)} \cdot \frac{(x-1)(x+1)}{y(x-6)}$$

$$= 2$$

$$\textcircled{1} 16y^2 - 12y + 2$$

$$2(8y^2 - 6y + 1)$$

prod = 8
 sum = -6
 -2, -4
 $(8y^2 - 2y)(-4y + 1)$
 $2y(4y-1) - 1(4y-1)$
 $2(2y-1)(4y-1)$

$$\textcircled{2} 4y^2 - 4$$

$$4(y^2 - 1)$$

$$4(y-1)(y+1)$$

$$\textcircled{3} 10y^2 + 15y - 10$$

$$5(2y^2 + 3y - 2)$$

prod = -10
 sum = 3
 +4, -1
 $(2y^2 + 4y)(-1y - 2)$
 $2y(y+2) - 1(y+2)$
 $5(2y-1)(y+2)$

$$38. \frac{(x+y)^2}{(x-y)^3} \times \frac{(x^2-y^2)^2}{x^4-y^4} \div \frac{(x+y)^3}{x^2+y^2}$$

$$\frac{(x+y)^2}{(x-y)^3} \cdot \frac{(x^2-y^2)^2}{x^4-y^4} \cdot \frac{x^2+y^2}{(x+y)^3}$$

$$\frac{(x+y)(x+y)}{(x-y)(x-y)(x-y)} \cdot \frac{(x^2-y^2)(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} \cdot \frac{(x^2+y^2)}{(x+y)(x+y)(x+y)}$$

$$\frac{\cancel{(x+y)}\cancel{(x+y)}}{(x-y)(x-y)(x-y)} \cdot \frac{\cancel{(x-y)}\cancel{(x+y)}\cancel{(x-y)}\cancel{(x+y)}}{\cancel{(x-y)}\cancel{(x+y)}(x^2+y^2)} \cdot \frac{\cancel{(x^2+y^2)}}{\cancel{(x+y)}\cancel{(x+y)}\cancel{(x+y)}} = \boxed{\frac{1}{(x-y)^2}}$$

$$39. \frac{x^2-25}{-x^2-7x-10} \div \frac{5y-xy}{xy+2y}$$

$$\textcircled{1} \frac{x^2-25}{-x^2-7x-10} \cdot \frac{xy+2y}{5y-xy}$$

$$\frac{(x-5)(x+5)}{(x+5)(-x-2)} \cdot \frac{y(x+2)}{y(5-x)}$$

$$\frac{\cancel{(-1)}\cancel{(-x+5)}\cancel{(x+5)}}{(x+5)\cancel{(-1)}\cancel{(x+2)}} \cdot \frac{\cancel{y}\cancel{(x+2)}}{\cancel{y}\cancel{(5-x)}} = \boxed{1}$$

$$40. \textcircled{1} \frac{3x^2+11x-4}{12x^2-4x} \times \textcircled{2} \frac{4x^2-28x+40}{x^2-x-20}$$

$$\frac{(3x-1)(x+4)}{4x(3x-1)} \cdot \frac{4(x-2)(x-5)}{(x-5)(x+4)}$$

$$\frac{\cancel{(3x-1)}\cancel{(x+4)}}{\cancel{4x}\cancel{(3x-1)}} \cdot \frac{\cancel{4}\cancel{(x-2)}\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x+4)}}$$

$$\boxed{\frac{x-2}{x}}$$

$$\textcircled{1} -x^2-7x-10 \quad \text{prod} = +10$$

$$(-x^2-2x)(-5x-10) \quad \text{sum} = -7$$

$$x(-x-2)+5(-x-2) \quad -2, -5$$

$$(x+5)(-x-2)$$

$$\textcircled{1} 3x^2+11x-4 \quad p = -12$$

$$(3x^2+12x)(-1x-4) \quad s = +11$$

$$3x(x+4)-1(x+4) \quad +12, -1$$

$$(3x-1)(x+4)$$

$$\textcircled{2} 4x^2-28x+40$$

$$4(x^2-7x+10)$$

$$4(x-2)(x-5)$$

$$41. \frac{2y+x}{6y+1} \div \frac{x^2-4y^2}{2+9y-18y^2} \div \frac{4-9y^2}{6y^2-3xy+4y-2x}$$

$$\frac{(2y+x)}{(6y+1)} \cdot \frac{2+9y-18y^2}{x^2-4y^2} \cdot \frac{6y^2-3xy+4y-2x}{4-9y^2}$$

$$\frac{(2y+x)}{(6y+1)} \cdot \frac{(-3y+2)(6y+1)}{(x-2y)(x+2y)} \cdot \frac{(3y+2)(2y-x)}{(2-3y)(2+3y)}$$

$$\frac{\cancel{(2y+x)}}{\cancel{(6y+1)}} \cdot \frac{\cancel{(-3y+2)}\cancel{(6y+1)}}{\cancel{(x-2y)}\cancel{(x+2y)}} \cdot \frac{\cancel{(3y+2)}\cancel{(-1)}\cancel{(-2y+x)}}{\cancel{(2-3y)}\cancel{(2+3y)}} = \boxed{-1}$$

$$42. \frac{2x^3-10x^2}{-2x^3+3x^2} \div \frac{6x^2-21x-45}{9-4x^2}$$

$$\frac{2x^3-10x^2}{-2x^3+3x^2} \cdot \frac{9-4x^2}{6x^2-21x-45}$$

$$\frac{2x^2(x-5)}{x^2(-2x+3)} \cdot \frac{(3-2x)(3+2x)}{3(x-5)(2x+3)}$$

$$\frac{\cancel{2x^2}\cancel{(x-5)}}{\cancel{x^2}\cancel{(-2x+3)}} \cdot \frac{\cancel{(3-2x)}\cancel{(3+2x)}}{\cancel{3}\cancel{(x-5)}\cancel{(2x+3)}} = \boxed{\frac{2}{3}}$$

$$\begin{aligned} & \textcircled{1} -18y^2+9y+2 \\ & (-18y^2-3y)+(12y+2) \\ & -3y(6y+1)+2(6y+1) \end{aligned} \left. \begin{array}{l} p=-36 \\ s=9 \\ -3, 12 \end{array} \right\}$$

$$\begin{aligned} & (-3y+2)(6y+1) \\ & \textcircled{2} (6y^2-3xy)+(4y-2x) \\ & 3y(2y-x)+2(2y-x) \\ & (3y+2)(2y-x) \end{aligned}$$

$$\textcircled{1} 6x^2-21x-45$$

$$\begin{aligned} & 3(2x^2-7x-15) \quad \text{prod}=-30 \\ & \quad \quad \quad \text{sum}=-7 \\ & (2x^2+3x)(-10x-15) \quad 3, -10 \\ & x(2x+3) - 5(2x+3) \\ & \quad \quad \quad \downarrow \\ & \quad \quad \quad 3(x-5)(2x+3) \end{aligned}$$

$$43. \frac{2b^4+3b^3}{9-4b^2} \times \frac{2b^3-3b^2}{3b^2+2b^3} \times \frac{2b^2+9b+9}{-b^2+2b^3}$$

$$\frac{b^3(2b+3)}{(3-2b)(3+2b)} \cdot \frac{b^2(2b-3)}{b^2(3+2b)} \cdot \frac{(b+3)(2b+3)}{b^2(-1+2b)}$$

$$\frac{\cancel{b^3}\cancel{(2b+3)}}{\cancel{(3-2b)}\cancel{(3+2b)}} \cdot \frac{\cancel{b^2}\cancel{(-1)}\cancel{(-2b+3)}}{\cancel{b^2}\cancel{(3+2b)}} \cdot \frac{\cancel{(b+3)}\cancel{(2b+3)}}{\cancel{b^2}\cancel{(2b-1)}}$$

$$\frac{(-1)b^3(b+3)}{b^2(2b-1)} = \boxed{\frac{-b(b+3)}{2b-1}}$$

$$\begin{aligned} & \textcircled{1} 2b^2+9b+9 \quad \text{prod}=18 \\ & (2b^2+3b)+(6b+9) \quad \text{sum}=9 \\ & b(2b+3)+3(2b+3) \quad 3, 6 \\ & (b+3)(2b+3) \end{aligned}$$

44. $\frac{4}{5} \frac{144-4b^2}{b^2+35b+30} \times \frac{5}{3} \frac{5b^2+30b-35}{b^2+6b-7} \times \frac{b^4-b^2}{12b^3-2b^4}$

21 $\frac{b^2(b^2-1)}{b^2(b-1)(b+1)}$

$$\frac{4(b-b)(b+b)}{5(b+1)(b+6)} \cdot \frac{5(b-1)(b+7)}{(b-1)(b+7)} \cdot \frac{b^2(b-1)(b+1)}{2b^3(b-b)}$$

$$\frac{4(b-b)(b+b)}{5(b+1)(b+6)} \cdot \frac{5(b-1)(b+7)}{(b-1)(b+7)} \cdot \frac{b^2(b-1)(b+1)}{2b^3(b-b)}$$

$$\frac{4b^2(b-1)}{2b^3} = \boxed{\frac{2(b-1)}{b}}$$

① $5b^2+35b+30$
 $5(b^2+7b+6)$ prod=6 sum=7
 $(b^2+6b)+(b+6)$
 $b(b+6)+1(b+6)$
 $5(b+1)(b+6)$

② $5b^2+30b-35$ prod=-7 sum=6
 $5(b^2+6b-7)$ 7, -1
 $(b^2+7b)+(-1b-7)$
 $b(b+7)-1(b+7)$
 $5(b-1)(b+7)$

③ b^2+6b-7
 $(b-1)(b+7)$

④ $144-4b^2$
 $4(36-b^2)$
 $4(b-b)(b+b)$

45. $\frac{4x^2+12xy+9y^2}{4x^2-9y^2} \div \frac{(4x^2+6xy)}{1}$

$$\frac{4x^2+12xy+9y^2}{4x^2-9y^2} \cdot \frac{1}{4x^2+6xy}$$

$$\frac{(2x+3y)(2x+3y)}{(2x-3y)(2x+3y)} \cdot \frac{1}{2x(2x+3y)}$$

$$\frac{(2x+3y)(2x+3y)}{(2x-3y)(2x+3y)} \cdot \frac{1}{2x(2x+3y)} = \boxed{\frac{1}{2x(2x-3y)}}$$

① $4x^2+12xy+9y^2$
 $(4x^2+6xy)+(6xy+9y^2)$ prod=36 sum=12
 $2x(2x+3y)+3y(2x+3y)$ 6,6
 $(2x+3y)(2x+3y)$

46. ① $\frac{-x^2 - 14x - 40}{x-5} \div \frac{19x+5-4x^2}{x^2-25} \times \frac{1-16x^2}{4x+16}$

$$\frac{-x^2 - 14x - 40}{x-5} \cdot \frac{x^2 - 25}{-4x^2 + 19x + 5} \cdot \frac{1 - 16x^2}{4x + 16}$$

$$\frac{(x+10)(-x-4)}{(x-5)} \cdot \frac{(x-5)(x+5)}{(4x+1)(-x+5)} \cdot \frac{(1-4x)(1+4x)}{4(x+4)}$$

$$\frac{\cancel{(x+10)}(-1)\cancel{(x+4)}}{\cancel{(x-5)}} \cdot \frac{\cancel{(x-5)}(x+5)}{(4x+1)(-x+5)} \cdot \frac{(1-4x)\cancel{(1+4x)}}{4\cancel{(x+4)}}$$

$$\boxed{\frac{-1(x+10)(x+5)(1-4x)}{4(-x+5)}}$$

47. ② $\frac{y-x}{-9y^2+4x^2} \times \frac{3xy+2x^2}{x^2-xy} \times \frac{-6y^2+xy+2x^2}{-x-2y}$

$$\frac{-1(-y+x)}{(2x-3y)(2x+3y)} \cdot \frac{x(3y+2x)}{x(x-y)} \cdot \frac{(2y+x)(-3y+2x)}{-1(x+2y)}$$

$$\frac{-1\cancel{(x-y)}}{\cancel{(2x-3y)}(2x+3y)} \cdot \frac{\cancel{x}(3y+2x)}{\cancel{x}(x-y)} \cdot \frac{\cancel{(2y+x)}(2x-3y)}{-1\cancel{(x+2y)}}$$

$$= \boxed{1}$$

① $-x^2 - 14x - 40$ prod = 40
 $(-x^2 - 4x)(-10x - 40)$ sum = -14
 $x(-x-4) + 10(-x-4)$ -4, -10
 $(x+10)(-x-4)$

② $-4x^2 + 19x + 5$ prod = -20
 $(-4x^2 + 20x)(-1x + 5)$ sum = 19
 $4x(-x+5) + 1(-x+5)$ 20, -1
 $(4x+1)(-x+5)$

(Same thing as the answer in the back of your book - just looks different!)

① $-6y^2 + xy + 2x^2$ prod = -12
 sum = 1
 +4, -3
 $(-6y^2 + 4xy)(-3xy + 2x^2)$
 $2y(-3xy + 2x^2) + x(-3xy + 2x^2)$
 $(2y+x)(-3y+2x)$

② $-9y^2 + 4x^2$
 $4x^2 - 9y^2$
 $(2x-3y)(2x+3y)$

48. $\frac{2a^2+3a-2}{2a^2+5a-3} \times \frac{2a^2+7a+3}{4-a^2} \div \frac{2a^2+5a+2}{3a^2-7a+2}$

$$\frac{2a^2+3a-2}{2a^2+5a-3} \cdot \frac{2a^2+7a+3}{4-a^2} \cdot \frac{3a^2-7a+2}{2a^2+5a+2}$$

$$\frac{(2a-1)(a+2)}{(2a-1)(a+3)} \cdot \frac{(2a+1)(a+3)}{(2-a)(2+a)} \cdot \frac{(3a-1)(a-2)}{(2a+1)(a+2)}$$

$$\frac{\cancel{(2a-1)}(a+2)}{\cancel{(2a-1)}(a+3)} \cdot \frac{\cancel{(2a+1)}(a+3)}{(2-a)(2+a)} \cdot \frac{(3a-1)(a-2)}{\cancel{(2a+1)}(a+2)}$$

$$\frac{(3a-1)(-1)(-a+2)}{\cancel{(2-a)}(a+2)} = \frac{-(3a-1)}{a+2}$$

49. $\frac{6a+6b}{a^2+a-30} \div \frac{2a+2b}{a-5}$

$$\frac{6a+6b}{a^2+a-30} \cdot \frac{(a-5)}{2a+2b}$$

$$\frac{6(a+b)}{(a-5)(a+6)} \cdot \frac{\cancel{(a-5)}}{2(a+b)}$$

$$\frac{3}{a+6}$$

① a^2+a-30
 $(a^2+6a)(-5a-30)$ $P=-30$
 $S=+1$
 $a(a+6)-5(a+6)+6, -5$
 $(a-5)(a+6)$

50. $\frac{25-x^2}{-x^2-7x-10} \div \frac{5y-xy}{xy+2y}$

$$\frac{25-x^2}{-x^2-7x-10} \cdot \frac{xy+2y}{5y-xy}$$

$$\frac{(5-x)(5+x)}{(-x-5)(x+2)} \cdot \frac{y(x+2)}{y(5-x)}$$

$$\frac{\cancel{(5-x)}(5+x)}{-1(\cancel{x+5})(x+2)} \cdot \frac{\cancel{y}(x+2)}{\cancel{y}(5-x)} = -1$$

① $2a^2+3a-2$

$$(2a^2+4a)(-1a-2)$$

prod = -4
sum = 3
4, -1

$$2a(a+2) - 1(a+2)$$

$$(2a-1)(a+2)$$

② $2a^2+5a-3$ $P=-6$ $S=5$
6, -1

$$(2a^2+6a)(-1a-3)$$

$$2a(a+3) - 1(a+3)$$

$$(2a-1)(a+3)$$

③ $2a^2+7a+3$ $P=6$ $S=7$ 6, 1

$$(2a^2+6a)(1a+3)$$

$$2a(a+3) + 1(a+3)$$

$$(2a+1)(a+3)$$

④ $2a^2+5a+2$ $P=4$ $S=5$
4, 1

$$(2a^2+4a)(1a+2)$$

$$2a(a+2) + 1(a+2)$$

$$(2a+1)(a+2)$$

⑤ $3a^2-7a+2$ $P=6$ $S=-7$
-6, -1

$$(3a^2-6a)(-1a+2)$$

$$3a(a-2) - 1(a-2)$$

$$(3a-1)(a-2)$$

① $-x^2-7x-10$ $P=+10$ $S=-7$
-2, -5

$$(-x^2-2x)(-5x-10)$$

$$-x(x+2) - 5(x+2)$$

$$(-x-5)(x+2)$$