MTH-4110 Operations on Algebraic Fractions: **Worksheet #1** (the only one!)

Reduce the following algebraic expressions to lowest terms, making sure to observe the order of operations. Show all the steps in the solutions.

1. $\left(\frac{2c+1}{c+3} + \frac{c^2-c}{c^2+2c-3}\right) \div \frac{3c^2+c}{c^2+c-6}$ flip

$$\frac{2c+1}{(c+3)} + \frac{c(c-1)}{(c+3)(c-1)}$$

$$\frac{2c+1+c}{(c+3)}$$

$$\frac{(3c+1)}{(c+3)} \cdot \frac{(c+3)(c-2)}{c(3c+1)}$$

$$\boxed{\frac{c-2}{c}}$$

2. $\left(\frac{k}{k-1} - \frac{k}{k+1}\right) \div \frac{k}{k^2-1}$

$$\frac{K(K+1)}{(K+1)(K-1)} - \frac{K(K-1)}{(K+1)(K-1)}$$

$$\frac{K^2+K}{C.D.} - \frac{(K^2-K)}{C.D.}$$

$$\frac{K^2+K-K^2+K}{C.D}$$

$$\frac{2K}{(K+1)(K-1)} \cdot \frac{(K-1)(K+1)}{K}$$

$$\boxed{2}$$

3. $\dfrac{h^2+2h+1}{h^2+3h+2} + \dfrac{(h+3)^2}{h^2-9} \times \dfrac{h+1}{h^2+5h+6}$

$$\frac{(h+3)(h+3)}{(h-3)(h+3)} \cdot \frac{(h+1)}{(h+2)(h+3)}$$

$$\frac{(h+1)(h+1)}{(h+2)(h+1)} + \frac{(h+1)}{(h-3)(h+2)}$$

$$\frac{h+1(h-3)}{(h+2)(h-3)} + \frac{h+1}{(h-3)(h+2)}$$

$$\frac{h^2-2h-3}{C.D} + \frac{h+1}{C.D}$$

$$\frac{h^2-h-2}{C.D}$$

$$\boxed{\frac{(h-2)(h+1)}{(h-3)(h+2)}}$$

4. $\dfrac{z^2+z-2}{z^2-9} \times \dfrac{z+3}{z+2} + \dfrac{1}{z}$

$$\frac{(z+2)(z-1)}{(z-3)(z+3)} \cdot \frac{(z+3)}{(z+2)}$$

$$\frac{z-1}{(z-3)} + \frac{1}{z}$$

$$\frac{z(z-1)}{z(z-3)} + \frac{z-3}{z(z-3)}$$

$$\frac{z^2-z+z-3}{\text{C.D.}}$$

$$\boxed{\frac{z^2-3}{z(z-3)}}$$

5. $\dfrac{-t^2}{t^2(t^2+t-2)} \div \dfrac{t}{t+2} - \dfrac{2}{t-t^2}$

$$\frac{-t^2}{t^2(t+2)(t-1)} \cdot \frac{(t+2)}{t}$$

$$\frac{-1}{t-1} \cdot \frac{1}{t}$$

$$\frac{-1}{t(t-1)} - \frac{2}{t(1-t)}$$

$$\frac{1}{t(1-t)} - \frac{2}{t(1-t)}$$

$$\boxed{\frac{-1}{t(1-t)}}$$

(or $\dfrac{1}{t(t-1)}$)

6. $\left(\frac{x^2+2x-3}{x^2-9} + \frac{x}{x-3}\right) \div \frac{(2x-1)}{1}$

$$\frac{(x+3)(x-1)}{(x-3)(x+3)} + \frac{x}{(x-3)}$$

$$\frac{x-1}{x-3} + \frac{x}{x-3}$$

$$\frac{x-1+x}{x-3}$$

$$\frac{(2x-1)}{x-3} \cdot \frac{1}{(2x-1)}$$

$$\boxed{\frac{1}{x-3}}$$

7. $\left(\frac{2}{a} - \frac{b}{a^2}\right) \div \frac{b}{a^3}$

$\frac{2 \cdot a}{a \cdot a} - \frac{b}{a^2}$

$\frac{2a}{C.D.} - \frac{b}{C.D}$

$\frac{2a-b}{C.D} \cdot \frac{a^3}{b}$

$\frac{2a-b}{a^2} \cdot \frac{a^3}{b} = \boxed{\frac{a(2a-b)}{b}}$

8. $\left(\frac{m-n}{n} \circ \frac{m}{n}\right) + \frac{m(m+n)}{n^2}$

$$\frac{m(m-n)}{n^2} + \frac{m(m+n)}{n^2}$$

$$\frac{m^2 - mn}{n^2} + \frac{m^2 + mn}{n^2}$$

$$\frac{m^2 - mn + m^2 + mn}{n^2}$$

$$\boxed{\frac{2m^2}{n^2}}$$

9. $\frac{a-b}{x-y} \cdot \frac{y-x}{b-a} + \frac{1}{a+b}$

$\frac{-1(b-a)}{(x-y)} \cdot \frac{-1(x-y)}{(b-a)}$

$\frac{1}{1} + \frac{1}{(a+b)}$

$\frac{a+b}{a+b} + \frac{1}{a+b}$

$\boxed{\frac{a+b+1}{a+b}}$

11. $\left(\frac{a^2+5a+6}{a+3} - 1 - \frac{a+b}{a}\right) \div \frac{a^2-ab}{a}$

$$\frac{(a+2)\cancel{(a+3)}}{\cancel{(a+3)}} - \frac{1}{1} - \frac{a+b}{a}$$

$$\frac{a+2}{1} - \frac{1}{1} - \frac{(a+b)}{a}$$

$$\frac{a(a+2)}{a} - \frac{a}{a} - \frac{(a+b)}{a}$$

$$\frac{a^2+2a}{a} - \frac{a}{a} - \frac{(a+b)}{a}$$

$$\frac{a^2+2a-a-a-b}{a}$$

$$\frac{\cancel{(a^2-b)}}{a} \cdot \frac{\cancel{a}}{\cancel{a}(a^2-b)} = \boxed{\frac{1}{a}}$$

12. $\dfrac{10-10y}{y^2-6y+5} + \dfrac{4y^2+4y+1}{2y^2-y-1} \div \dfrac{2y^2-9y-5}{y^2+4y-5}$

$$\frac{(2y+1)(2y+1)}{(2y+1)(y-1)} \cdot \frac{(y+5)(y-1)}{(2y+1)(y-5)}$$

$$\frac{10(1-y)}{(y-5)(y-1)} + \frac{(y+5)}{(y-5)}$$

$$\frac{-10(y-1)}{(y-5)(y-1)} + \frac{y+5}{y-5}$$

$$\frac{-10}{y-5} + \frac{y+5}{y-5}$$

$$\frac{-10+y+5}{y-5}$$

$$\frac{y-5}{y-5} = \boxed{1}$$

13. $\dfrac{pq+2q-8p-16}{(q-8)^2} \div \dfrac{q+9}{q^2+q-72} - \dfrac{3}{p}$

$pq+2q-8p-16$

$(pq+2q)+(-8p-16)$

$q(p+2)-8(p+2)$

$(q-8)(p+2)$

$$\frac{(q-8)(p+2)}{(q-8)(q-8)} \cdot \frac{(q+9)(q-8)}{(q+9)}$$

$$\frac{p+2}{1} - \frac{3}{p}$$

$$\frac{p(p+2)}{p} - \frac{3}{p}$$

$$\frac{p^2+2p-3}{p}$$

$$\boxed{\frac{(p+3)(p-1)}{p}}$$

14: ④ $\frac{2x^2-7x+3}{x^2+x-12} \div$ ① $\frac{2x^2-5x+3}{-2x^2-2x+4} \cdot$ ③ $\frac{4x^2-4}{2x^2-x-3}$

$$\frac{(2x-3)(x-1)}{2(-x+1)(x+2)} \cdot \frac{4(x-1)(x+1)}{(x+1)(2x-3)}$$

$$\frac{(x-1)}{-2(x-1)(x+2)} \cdot \frac{4(x-1)}{1}$$

$$\frac{(2x-1)(x-3)}{(x+4)(x-3)} + \frac{-2(x-1)}{(x+2)}$$

$$\frac{(x+2)\ 2x-1}{(x+2)(x+4)} + \frac{-2(x-1)(x+4)}{(x+2)(x+4)}$$

$$\frac{2x^2+3x-2}{\text{C.D.}} + \frac{-2(x^2+3x-4)}{\text{C.D.}}$$

$$\frac{2x^2+3x-2-2x^2-6x+8}{\text{C.D.}}$$

$$\frac{-3x+6}{(x+2)(x+4)}$$

$$\frac{-3(x-2)}{(x+2)(x+4)}$$

① $2x^2-5x+3$ $p=6$ $s=-5$ $-2,-3$

$2x^2-2x-3x+3$

$(2x^2-2x)+(-3x+3)$

$2x(x-1)-3(x-1)$

$(2x-3)(x-1)$

② $-2x^2-2x+4$

$p=-8$ $s=-2$

$-4, +2$

$-2x^2-4x+2x+4$

$(-2x^2-4x)+(2x+4)$

$-2x(x+2)+2(x+2)$

$(-2x+2)(x+2)$

$2(-x+1)(x+2)$

③ $2x^2-x-3$ $p=-6$ $s=-1$ $-3, +2$

$2x^2-3x+2x-3$

$(2x^2-3x)+(2x-3)$

$x(2x-3)+1(2x-3)$

$(x+1)(2x-3)$

④ $2x^2-7x+3$ $p=6$ $s=-7$

$2x^2-6x-1x+3$

$(2x^2-6x)+(-1x+3)$

$2x(x-3)-1(x-3)$

$(2x-1)(x-3)$

15. $\dfrac{a^3-ab^2}{(a-b)^2(a+2b)} \div \left(\dfrac{a+b}{a^2+ab-2b^2} - \dfrac{a+b}{a^2-2ab+b^2}\right)$

$\dfrac{a+b}{(a+2b)(a-b)} - \dfrac{a+b}{(a-b)(a-b)}$

CD $(a+2b)(a-b)^2$

$\dfrac{(a+b)(a-b)}{C.D.} - \dfrac{(a+b)(a+2b)}{C.D.}$

$\dfrac{a^2-b^2}{C.D.} - \dfrac{(a^2+3ab+2b^2)}{C.D.}$

$\dfrac{a^2-b^2-a^2-3ab-2b^2}{C.D.}$

$\dfrac{-3b^2-3ab}{C.D.}$

$\dfrac{-3b(b+a)}{(a+2b)(a-b)^2}$

$\dfrac{a(a-b)(a+b)}{(a-b)^2(a+2b)}$

$\dfrac{a(a+b)}{(a-b)(a+2b)}$

$\dfrac{a(a+b)}{(a-b)(a+2b)} \cdot \dfrac{(a+2b)(a-b)^2}{-3b(b+a)}$

$\dfrac{a(a-b)}{-3b}$

OR $\dfrac{-a(a-b)}{3b}$

$2st - 8s + 3t - 12$

$(2st - 8s) + (3t - 12)$

$2s(t-4) + 3(t-4)$

$(2s+3)(t-4)$

16. $\dfrac{2s+3}{s^2-s-2} \times \dfrac{t^2-16}{2st-8s+3t-12} - \dfrac{t+4}{s-2}$

$$\frac{2s+3}{(s-2)(s+1)} \cdot \frac{(t-4)(t+4)}{(2s+3)(t-4)}$$

$$\frac{t+4}{(s-2)(s+1)} - \frac{t+4\,(s+1)}{(s-2)(s+1)}$$

$$\frac{t+4}{\text{C.D.}} - \frac{(st+4s+t+4)}{\text{C.D.}}$$

$$\frac{t+4 - st - 4s - t - 4}{\text{C.D.}}$$

$$\frac{-st-4s}{\text{C.D.}}$$

$$\frac{-s(t+4)}{(s-2)(s+1)}$$

17. $\dfrac{x^2}{x^2-y^2} - \dfrac{y^2}{x+y} \times \dfrac{2x^2+xy-y^2}{2xy^2-y^3}$

① $2x^2+xy-y^2$

$2x^2+2xy-1xy-y^2$ $\quad p=-2$ $\quad s=+1$ $\quad +2,-1$

$(2x^2+2xy)+(-1xy-y^2)$

$2x(x+y)-y(x+y)$

$(2x-y)(x+y)$

$\dfrac{y^2}{(x+y)} \cdot \dfrac{(2x-y)(x+y)}{y^2(2x-y)}$

1

$\dfrac{x^2}{(x-y)(x+y)} - \dfrac{1}{1}$

$\dfrac{x^2}{(x-y)(x+y)} - \dfrac{(x-y)(x+y)}{(x-y)(x+y)}$

$\dfrac{x^2}{\text{C.D.}} - \dfrac{(x^2-y^2)}{\text{C.D.}}$

$\dfrac{x^2-x^2+y^2}{\text{C.D.}} = \dfrac{y^2}{(x-y)(x+y)}$

18. $\left(\frac{x}{x+2} + 3 - \frac{1}{x}\right) \times \frac{x+2}{8x^2+10x-4}$

$$\frac{x}{(x+2)} + \frac{3}{1} - \frac{1}{x}$$

C.D = $x(x+2)$

$$\frac{x \cdot x}{C.D.} + \frac{3(x)(x+2)}{C.D} - \frac{(x+2)}{x(x+2)}$$

$$\frac{x^2}{C.D.} + \frac{3x^2+6x}{C.D.} - \frac{(x+2)}{C.D.}$$

$$\frac{x^2+3x^2+6x-x-2}{C.D.}$$

$$\frac{(4x^2+5x-2)}{x(x+2)} \cdot \frac{(x+2)}{2(4x^2+5x-2)}$$

$$\boxed{\frac{1}{2x}}$$

19. $\left(\frac{x^2-x-6}{x^2-4}+\frac{x}{x-2}\right)\div(2x-3)$

$$\frac{(x-3)(x+2)}{(x-2)(x+2)}+\frac{x}{(x-2)}$$

$$\frac{x-3}{(x-2)}+\frac{x}{(x-2)}$$

$$\frac{x-3+x}{x-2}$$

$$\frac{(2x-3)}{x-2}\cdot\frac{1}{(2x-3)}=\boxed{\frac{1}{x-2}}$$

20. $\left(a - \frac{8}{a+2} + \frac{a^2-1}{a^2+a-2}\right) \cdot \frac{3a+6}{7-3a-a^2}$

$$\frac{a}{1} - \frac{8}{(a+2)} + \frac{(a-1)(a+1)}{(a+2)(a-1)}$$

$$\frac{a(a+2)}{(a+2)} - \frac{8}{(a+2)} + \frac{a+1}{(a+2)}$$

$$\frac{a^2+2a}{C.D.} - \frac{8}{C.D} + \frac{a+1}{C.D.}$$

$$\frac{a^2+2a-8+a+1}{C.D.}$$

$$\frac{a^2+3a-7}{(a+2)} \cdot \frac{3(a+2)}{-1(-7+3a+a^2)}$$

$$= \boxed{-3}$$

21. ① $\left(\frac{y}{2y^2-5y-3}-\frac{4}{y^2+2y-15}\right)$ ② $\cdot\frac{2y^2+9y+4}{y^2+5y+4}$

① $2y^2-5y-3$ $\quad s=-5$ $\quad -6,+1$

$2y^2-6y+1y-3$

$(2y^2-6y)+(y-3)$

$2y(y-3)+1(y-3)$

$(2y+1)(y-3)$

$\frac{y}{(2y+1)(y-3)}-\frac{4}{(y+5)(y-3)}$

$C.D=(2y+1)(y-3)(y+5)$

$\frac{y(y+5)}{C.D.}-\frac{4(2y+1)}{C.D.}$

$\frac{y^2+5y}{C.D.}-\frac{(8y+4)}{C.D.}$

$\frac{y^2+5y-8y-4}{C.D.}$

$\frac{y^2-3y-4}{C.D.}$

② $2y^2+9y+4$ $\quad p=8$ $\quad s=9$ $\quad 1,8$

$2y^2+1y+8y+4$

$(2y^2+1y)+(8y+4)$

$y(2y+1)+4(2y+1)$

$(y+4)(2y+1)$

$\frac{(y-4)(y+1)}{(2y+1)(y-3)(y+5)}\cdot\frac{(y+4)(2y+1)}{(y+4)(y+1)}$

$=\boxed{\frac{y-4}{(y-3)(y+5)}}$

22. $\left(\frac{2x+1}{x+3} + \frac{x}{x+3}\right) \div \frac{3x^2+x}{x^2+x-6}$

$\frac{2x+1+x}{x+3}$

$\frac{(3x+1)}{(x+3)} \cdot \frac{(x+3)(x-2)}{x(3x+1)}$

$\boxed{\frac{x-2}{x}}$

23. $\dfrac{m^2 - p^2}{p^2} \div \left(\dfrac{m}{p^2} - \dfrac{2}{p} + \dfrac{1}{m}\right)$

C.D $= mp^2$

$$\frac{m \cdot m}{\text{C.D.}} - \frac{2mp}{\text{C.D}} + \frac{p^2}{\text{C.D.}}$$

$$\frac{m^2 - 2mp + p^2}{\text{C.D.}}$$

flip

$$\frac{(m-p)(m+p)}{p^2} \cdot \frac{mp^2}{(m-p)(m-p)}$$

$$= \frac{m(m+p)}{m-p}$$

24. ② $\dfrac{2b^2+9b+4}{b^2+5b+4} + \dfrac{2b(b+2)(b-1)}{2-b-b^2} \times \dfrac{b}{b^2-1}$ ①

① $2-b-b^2$

$-b^2-b+2$ $\quad p=-2$, $s=-1$, $-2,+1$

$(-b^2-2b)+(1b+2)$

$-b(b+2)+1(b+2)$

$(-b+1)(b+2)$

$\dfrac{2b(b+2)(b-1)}{(-b+1)(b+2)} \cdot \dfrac{b}{(b-1)(b+1)}$

② $2b^2+9b+4$ $\quad p=8$, $s=9$, $1,8$

$(2b^2+1b)+(8b+4)$

$b(2b+1)+4(2b+1)$

$(b+4)(2b+1)$

$\dfrac{(b+4)(2b+1)}{(b+4)(b+1)} + \dfrac{2b^2}{(-b+1)(b+1)}$

C.D. $(b+1)(-b+1)$

$\dfrac{(2b+1)(-b+1)}{\text{C.D.}} + \dfrac{2b^2}{\text{C.D.}}$

$\dfrac{-2b^2+b+1}{\text{C.D.}} + \dfrac{2b^2}{\text{C.D}} = \dfrac{b+1}{(b+1)(-b+1)}$

$= \boxed{\dfrac{1}{-b+1}}$

25. $\dfrac{m^2+n^2}{m^2-n^2} - \dfrac{n^2}{m+2n} \circ \dfrac{15m^2+27mn-6n^2}{15mn^2-3n^3}$

① $15m^2 + 27mn - 6n^2$

$3(5m^2 + 9mn - 2n^2)$

$p = -10$

$s = +9$

$5m^2 + 10mn - 1mn - 2n^2 + 10, -1$

$(5m^2 + 10mn) + (-1mn - 2n^2)$

$5m(m+2n) - n(m+2n)$

$(5m-n)(m+2n)$

② $15mn^2 - 3n^3$

$3n^2(5m-n)$

$\dfrac{\cancel{n^2}}{\cancel{(m+2n)}} \cdot \dfrac{\cancel{3}(\cancel{5m-n})(\cancel{m+2n})}{\cancel{3n^2}(\cancel{5m-n})}$

1

$\dfrac{m^2+n^2}{(m-n)(m+n)} - \dfrac{1}{1}$

$\dfrac{m^2+n^2}{(m-n)(m+n)} - \dfrac{(m-n)(m+n)}{(m-n)(m+n)}$

$\dfrac{m^2+n^2}{C.D.} - \dfrac{(m^2-n^2)}{C.D.}$

$\dfrac{\cancel{m^2}+n^2-\cancel{m^2}+n^2}{C.D.} = \boxed{\dfrac{2n^2}{(m-n)(m+n)}}$

26. $\dfrac{x^2-3x+2}{x^2-16} - \dfrac{x^2+5x+4}{x^2-2x-3} \cdot \dfrac{2x^2-5x-3}{2x^2-32}$

$$\frac{(x+4)(x+1)}{(x-3)(x+1)} \cdot \frac{(2x+1)(x-3)}{2(x-4)(x+4)}$$

$$\frac{(x-2)(x-1)}{(x-4)(x+4)} - \frac{(2x+1)}{2(x-4)}$$

C.D. $2(x-4)(x+4)$

$$\frac{2(x-2)(x-1)}{C.D.} - \frac{(2x+1)(x+4)}{C.D.}$$

$$\frac{2x^2-6x+4}{C.D.} - \frac{(2x^2+9x+4)}{C.D.}$$

$$\frac{2x^2-6x+4-2x^2-9x-4}{C.D.}$$

$$\boxed{\frac{-15x}{2(x-4)(x+4)}}$$

27. $\left(\frac{g}{g-1} - \frac{g}{g+1}\right) \div \frac{g}{g^2-1}$

C.D. $(g-1)(g+1)$

$$\frac{g(g+1)}{C.D.} - \frac{g(g-1)}{C.D.}$$

$$\frac{g^2+g}{C.D.} - \frac{(g^2-g)}{C.D.}$$

$$g^2 + g - g^2 + g$$

$$\frac{2g}{(g-1)(g+1)} \cdot \frac{(g-1)(g+1)}{g}$$

$$= \boxed{2}$$

28. ② ③ $\dfrac{-2x^2+5x-2}{2x^2-3x+1} + \dfrac{x^2+x-12}{x^2+3x-4} \div ① \dfrac{x^2-5x+6}{2x^2-7x+6}$

$$\frac{(x+4)(x-3)}{(x+4)(x-1)} \cdot \frac{(x-2)(2x-3)}{(x-2)(x-3)}$$

$$\frac{(-x+2)(2x-1)}{(2x-1)(x-1)} + \frac{2x-3}{(x-1)}$$

$$\frac{-x+2}{(x-1)} + \frac{2x-3}{(x-1)}$$

$$\frac{-x+2+2x-3}{x-1}$$

$$\frac{x-1}{x-1} = \boxed{1}$$

① $2x^2-7x+6$ $\quad p=12$, $s=-7$, $-3,-4$

$2x^2-3x-4x+6$

$(2x^2-3x)+(-4x+6)$

$x(2x-3)-2(2x-3)$

$(x-2)(2x-3)$

② $-2x^2+5x-2$ $\quad p=4$, $s=5$, $1,4$

$(-2x^2+1x)+(4x-2)$

$-x(2x-1)+2(2x-1)$

$(-x+2)(2x-1)$

③ $2x^2-3x+1$ $\quad p=2$, $s=-3$, $-2,-1$

$2x^2-2x-1x+1$

$(2x^2-2x)+(-1x+1)$

$2x(x-1)-1(x-1)$

$(2x-1)(x-1)$

29. $\left(\dfrac{2g(2g-3k)}{2g^2+gk-6k^2} - 2 + \dfrac{3k}{g+2k}\right) \times \dfrac{2g+4k}{gk}$

① $2g^2 + gk - 6k^2$

$p = -12$

$s = +1$

$+4, -3$

$2g^2 + 4gk - 3gk - 6k^2$

$(2g^2 + 4gk) + (-3gk - 6k^2)$

$2g(g+2k) - 3k(g+2k)$

$(2g-3k)(g+2k)$

$\dfrac{2g(2g-3k)}{(2g-3k)(g+2k)} - \dfrac{2}{1} + \dfrac{3k}{(g+2k)}$

$\dfrac{2g}{(g+2k)} - \dfrac{2(g+2k)}{(g+2k)} + \dfrac{3k}{(g+2k)}$

$\dfrac{2g}{C.D.} - \dfrac{(2g+4k)}{C.D.} + \dfrac{3k}{C.D.}$

$\dfrac{2g - 2g - 4k + 3k}{C.D.}$

$\dfrac{-k}{g+2k} \cdot \dfrac{2(g+2k)}{gk} = \boxed{\dfrac{-2}{g}}$

① $-f^2+4f+16$ $p=-16$ $s=4$

✓ 30. $\left(\frac{f^2+4f}{f^2}-1-\frac{f^2}{f^2+4f}\right)\times\frac{f+4}{f^2-4f-16}$ doesn't factor

$$\frac{f(f+4)}{f^2}-\frac{1}{1}-\frac{f^2}{f(f+4)}$$

$$\frac{(f+4)}{f}-\frac{1}{1}-\frac{f}{(f+4)}$$

C.D = $f(f+4)$

$$\frac{(f+4)(f+4)}{C.D.}-\frac{f(f+4)}{f(f+4)}-\frac{f\cdot f}{C.D.}$$

$$\frac{f^2+8f+16}{C.D.}-\frac{(f^2+4f)}{C.D.}-\frac{f^2}{C.D}$$

$$\frac{f^2+8f+16-f^2-4f-f^2}{C.D.}$$

① $$\frac{-f^2+4f+16}{C.D.}$$

$$\frac{-1(f^2-4f-16)}{f(f+4)}\cdot\frac{f+4}{(f^2-4f-16)}$$

$$=\boxed{\frac{-1}{f}}$$

31. $\dfrac{10-10y}{5-6y+y^2} \div \dfrac{4y+1+4y^2}{2y^2-y-1} \div \dfrac{2y^2-9y-5}{y^2+4y-5}$

① $4y^2+4y+1$ $p=4$ $s=4$ $2,2$

$(4y^2+2y)+(2y+1)$

$2y(2y+1)+1(2y+1)$

$(2y+1)(2y+1)$

② $2y^2-y-1$ $p=-2$ $s=-1$ $-2,+1$

$2y^2-2y+1y-1$

$(2y^2-2y)+(1y-1)$

$2y(y-1)+1(y-1)$

$(2y+1)(y-1)$

③ $2y^2-9y-5$ $p=-10$ $s=-9$ $-10,+1$

$2y^2-10y+1y-5$

$(2y^2-10y)+1(y-5)$

$2y(y-5)+1(y-5)$

$(2y+1)(y-5)$

$\dfrac{(2y+1)(2y+1)}{(2y+1)(y-1)} \cdot \dfrac{(y+5)(y-1)}{(2y+1)(y-5)}$

$\dfrac{10(1-y)}{(y-5)(y-1)} + \dfrac{(y+5)}{(y-5)}$

$\dfrac{-10(y-1)}{(y-5)(y-1)} + \dfrac{y+5}{y-5}$

$\dfrac{-10}{y-5} + \dfrac{y+5}{y-5}$

$\dfrac{-10+y+5}{y-5}$

$\dfrac{y-5}{y-5} = \boxed{1}$

✓ 32. $\dfrac{4a^2-b^2}{b-a} \div \left(\dfrac{2a}{b+a} + \dfrac{3b}{b-a}\right)$

$\dfrac{2a}{(b+a)} + \dfrac{3b}{(b-a)}$ C.D. $= (b+a)(b-a)$

$\dfrac{2a(b-a)}{C.D.} + \dfrac{3b(b+a)}{C.D}$

$\dfrac{2ab-2a^2}{C.D.} + \dfrac{3b^2+3ab}{C.D.}$

$\dfrac{2ab-2a^2+3b^2+3ab}{C.D.}$

$\dfrac{-2a^2+5ab+3b^2}{C.D.}$

$p = -6$ $\quad +6, -1$
$s = 5$

$-2a^2+6ab-1ab+3b^2$

$(-2a^2+6ab)+(-1ab+3b^2)$

$2a(-a+3b)+b(-a+3b)$

$(2a+b)(-a+3b)$

$\dfrac{(2a+b)(-a+3b)}{(b+a)(b-a)}$ ⤵ flip ⤵

$\dfrac{(2a-b)(2a+b)}{(b-a)} \cdot \dfrac{(b+a)(b-a)}{(2a+b)(-a+3b)}$

$\boxed{\dfrac{(2a-b)(b+a)}{-a+3b}}$

33. $\dfrac{a^3-ab^2}{(a-b)^2(a+2b)} \div \left(\dfrac{a+b}{a^2+ab-2b^2} - \dfrac{a+b}{a^2-2ab+b^2}\right)$

$\dfrac{a+b}{(a+2b)(a-b)} - \dfrac{a+b}{(a-b)(a-b)}$ C.D. $=(a+2b)(a-b)^2$

$\dfrac{(a+b)(a-b)}{\text{C.D.}} - \dfrac{(a+b)(a+2b)}{\text{C.D.}}$

$\dfrac{a^2-b^2}{\text{C.D.}} - \dfrac{(a^2+3ab+2b^2)}{\text{C.D.}}$

$\dfrac{a^2-b^2-a^2-3ab-2b^2}{\text{C.D.}}$

$\dfrac{-3b^2-3ab}{(a+2b)(a-b)^2}$

$\dfrac{-3b(b+a)}{(a+2b)(a-b)^2}$ flip

$\dfrac{a(a-b)(a+b)}{(a-b)^2(a+2b)} \cdot \dfrac{(a+2b)(a-b)^2}{-3b(b+a)}$

$\boxed{\dfrac{a(a-b)}{-3b}}$

34. $\dfrac{(2x+2y)(x^2y)}{(2xy+2y^2)(x^2-xy)} - \dfrac{9y^2-x^2}{x^3-3x^2y} \div \dfrac{3y^2-2xy-x^2}{x^2y}$

① $3y^2-2xy-x^2$ $p=-3$ $s=-2$ $-3,+1$

$3y^2-3xy+1xy-x^2$

$(3y^2-3xy)+(1xy-x^2)$

$3y(y-x)+x(y-x)$

$(3y+x)(y-x)$

$\dfrac{(3y-x)(3y+x)}{x^2(x-3y)} \cdot \dfrac{x^2y}{(3y+x)(y-x)}$

$\dfrac{-1(-3y+x)}{(x-3y)} \cdot \dfrac{y}{(y-x)}$

$\dfrac{2(x+y)x^2y}{2y(x+y)x(x-y)} - \dfrac{-1y}{(y-x)}$

$\dfrac{x}{x-y} - \dfrac{y}{x-y}$

$\dfrac{x-y}{x-y} = \boxed{1}$

35. $(s+t) + \dfrac{15s^2t - 5st^2}{4m^2 - n^2} \div \dfrac{st}{2m-n} \cdot \dfrac{2m+n}{9s^2 - t^2}$

$$\frac{5st(3s-t)}{(2m-n)(2m+n)} \cdot \frac{(2m-n)}{st} \cdot \frac{(2m+n)}{(3s-t)(3s+t)}$$

$$\frac{s+t}{1} + \frac{5}{(3s+t)}$$

$$\frac{(s+t)(3s+t)}{(3s+t)} + \frac{5}{(3s+t)}$$

$$\frac{3s^2 + st + 3st + t^2}{\text{C.D.}} + \frac{5}{\text{C.D.}}$$

$$\boxed{\frac{3s^2 + 4st + t^2 + 5}{3s+t}}$$

36. $\frac{a^3-ab^2}{a^3-4a^2b+3ab^2} \div \frac{-2a^2+ab+3b^2}{a-3b} - \frac{a}{3ab-2b^2}$

① $a^3-4a^2b+3ab^2$

$a(a^2-4ab+3b^2)$

$a(a-3b)(a-b)$

② $-2a^2+ab+3b^2$ $\quad p=-6$, $s=+1$, $+3,-2$

$-2a^2+3ab-2ab+3b^2$

$(-2a^2+3ab)+(-2ab+3b^2)$

$a(-2a+3b)+b(-2a+3b)$

$(a+b)(-2a+3b)$

$\frac{a(a-b)(a+b)}{a(a-3b)(a-b)} \cdot \frac{(a-3b)}{(a+b)(-2a+3b)}$

$\frac{1}{(-2a+3b)} - \frac{a}{b(3a-2b)}$

$CD = b(-2a+3b)(3a-2b)$

$\frac{b(3a-2b)}{C.D.} - \frac{a(-2a+3b)}{C.D.}$

$\frac{3ab-2b^2}{C.D.} - \frac{(-2a^2+3ab)}{C.D.}$

$\frac{3ab-2b^2+2a^2-3ab}{C.D.}$

$\frac{2a^2-2b^2}{C.D} = \boxed{\frac{2(a-b)(a+b)}{b(-2a+3b)(3a-2b)}}$

37. $\left(\dfrac{2x-5}{\textcircled{1}\ 3x^2-11x-4}+\dfrac{4}{\textcircled{2}\ 2x^2-5x-12}\right)\div\dfrac{4x^2+8x-11}{\textcircled{3}\ -2x^2+5x+12}$

$$\frac{2x-5}{(3x+1)(x-4)}+\frac{4}{(x-4)(2x+3)}$$

C.D. $(3x+1)(x-4)(2x+3)$

$$\frac{(2x-5)(2x+3)}{\text{C.D.}}+\frac{4(3x+1)}{\text{C.D.}}$$

$$\frac{4x^2-4x-15}{\text{C.D.}}+\frac{12x+4}{\text{C.D.}}$$

$$\frac{4x^2-4x-15+12x+4}{\text{C.D.}}$$

$$\frac{(4x^2+8x-11)}{(3x+1)(x-4)(2x+3)}\cdot\frac{(2x+3)(-x+4)}{(4x^2+8x-11)}$$

$$\frac{1}{(3x+1)(x-4)}\cdot\frac{-1(x-4)}{1}$$

$$\boxed{\frac{-1}{3x+1}}$$

① $3x^2-11x-4$ $\quad p=-12$, $s=-11$, $-12,+1$

$3x^2-12x+1x-4$

$(3x^2-12x)+1(x-4)$

$3x(x-4)+1(x-4)$

$(3x+1)(x-4)$

② $2x^2-5x-12$ $\quad p=-24$, $s=-5$, $+3,-8$

$2x^2+3x-8x-12$

$(2x^2+3x)+(-8x-12)$

$x(2x+3)-4(2x+3)$

$(x-4)(2x+3)$

③ $-2x^2+5x+12$ $\quad p=-24$, $s=5$, $+8,-3$

$-2x^2+8x-3x+12$

$(-2x^2+8x)+(-3x+12)$

$2x(-x+4)+3(-x+4)$

$(2x+3)(-x+4)$

38. $\dfrac{a^2}{a^2-b^2} - \dfrac{b^2}{a+b} \cdot \dfrac{4a^2+2ab-2b^2}{4ab^2-2b^3}$

① $4a^2+2ab-2b^2$

$2(2a^2+ab-b^2)$

$2(2a-b)(a+b)$

$\dfrac{b^2}{(a+b)} \cdot \dfrac{2(2a-b)(a+b)}{2b^2(2a-b)}$

$\dfrac{a^2}{(a-b)(a+b)} - 1$

$\dfrac{a^2}{(a-b)(a+b)} - \dfrac{(a-b)(a+b)}{(a-b)(a+b)}$

$\dfrac{a^2}{C.D.} - \dfrac{(a^2-b^2)}{C.D}$

$\dfrac{a^2-a^2+b^2}{C.D} = \dfrac{b^2}{(a-b)(a+b)}$

39. $\dfrac{2x^2+xy}{2x^2+xy-y^2} \div \left(\dfrac{2x+y}{4x^2-4xy+y^2} - \dfrac{2x+y}{2x^2+xy-y^2}\right)$

① $4x^2-4xy+y^2$ $p=4$

$4x^2-2xy-2xy+y^2$ $s=-4$ $-2,-2$

$(4x^2-2xy)+(-2xy+y^2)$

$2x(2x-y)-y(2x-y)$

$(2x-y)(2x-y)$

$\dfrac{2x+y}{(2x-y)(2x-y)} - \dfrac{2x+y}{(2x-y)(x+y)}$

C.D. $(2x-y)^2(x+y)$

$\dfrac{x(2x+y)}{(2x-y)(x+y)}$

$\dfrac{(2x+y)(x+y)}{\text{C.D.}} - \dfrac{(2x+y)(2x-y)}{\text{C.D.}}$

$\dfrac{2x^2+3xy+y^2}{\text{C.D.}} - \dfrac{(4x^2-y^2)}{\text{C.D.}}$

$\dfrac{x(2x+y)}{(2x-y)(x+y)} \cdot \dfrac{(2x-y)^2(x+y)}{(2x+y)(-x+2y)}$

$\dfrac{x(2x-y)}{-x+2y}$

$\dfrac{2x^2+3xy+y^2-4x^2+y^2}{\text{C.D.}}$

$\dfrac{-2x^2+3xy+2y^2}{\text{C.D.}}$

$-2x^2+3xy+2y^2$

$p=-4$

$s=+3$

$+4,-1$

$-2x^2+4xy-1xy+2y^2$

$(-2x^2+4xy)+(-1xy+2y^2)$

$2x(-x+2y)+y(-x+2y)$

$(2x+y)(-x+2y)$

$\dfrac{(2x+y)(-x+2y)}{(2x-y)^2(x+y)}$

flip

40. $\left(\frac{2}{1-x^2} - \frac{1}{1+x} + 3\right) \cdot (x-1)^2$

$3(1-x)(1+x)$

$3(1-x^2)$

$3-3x^2$

$$\frac{2}{(1-x)(1+x)} - \frac{1}{(1+x)} + \frac{3}{1}$$

$$\frac{2}{C.D.} - \frac{(1-x)}{C.D.} + \frac{3(1-x)(1+x)}{C.D.}$$

$$\frac{2-1+x+3-3x^2}{C.D.}$$

$$\frac{-3x^2+x+4}{C.D.}$$

$-3x^2+x+4$ $\quad p=-12$, $s=+1$, $4,-3$

$-3x^2+4x-3x+4$

$(-3x^2+4x)+(-3x+4)$

$x(-3x+4)+1(-3x+4)$

$(x+1)(-3x+4)$

$$\frac{(x+1)(-3x+4)}{(1-x)(1+x)}$$

$$\frac{-3x+4}{(1-x)} \cdot \frac{(x-1)(x-1)}{1}$$

$$\frac{-3x+4}{1-x} \cdot \frac{-1(1-x)(x-1)}{1} = \begin{matrix} -1(x-1)(-3x+4) \\ \text{OR} \\ (1-x)(-3x+4) \end{matrix}$$