

MTH-4106 Factoring and Algebraic Fractions: **Worksheet #2**

Factor the following polynomials by **grouping**:

1.  $3a^2c + 5c + 6a^2b + 10b$

$$\begin{aligned} & (3a^2c + 5c) + (6a^2b + 10b) \\ & c(3a^2 + 5) + 2b(3a^2 + 5) \\ & = \boxed{(c + 2b)(3a^2 + 5)} \end{aligned}$$

Check:

$$\begin{aligned} & \overbrace{(c + 2b)(3a^2 + 5)} \\ & = 3a^2c + 5c + 6a^2b + 10b \quad \checkmark \end{aligned}$$

2.  $mx - my + nx - ny$

$$\begin{aligned} & (mx - my) + (nx - ny) \\ & m(x - y) + n(x - y) \\ & = \boxed{(m + n)(x - y)} \end{aligned}$$

Check:

$$\begin{aligned} & \overbrace{(m + n)(x - y)} \\ & mx - my + nx - ny \quad \checkmark \end{aligned}$$

3.  $b^2 + 4b + bc + 4c$

$$\begin{aligned} & (b^2 + 4b) + (bc + 4c) \\ & b(b + 4) + c(b + 4) \\ & = \boxed{(b + c)(b + 4)} \end{aligned}$$

Check:

$$\begin{aligned} & \overbrace{(b + c)(b + 4)} \\ & b^2 + 4b + bc + 4c \quad \checkmark \end{aligned}$$

4.  $2x^4 - x^3 + 4x - 2$

$$\begin{aligned} & (2x^4 - x^3) + (4x - 2) \\ & x^3(2x - 1) + 2(2x - 1) \\ & = \boxed{(x^3 + 2)(2x - 1)} \end{aligned}$$

Check:

$$\begin{aligned} & \overbrace{(x^3 + 2)(2x - 1)} \\ & 2x^4 - x^3 + 4x - 2 \quad \checkmark \end{aligned}$$

5.  $bcy + c^2z + cd + b^2y + bd + bcz$

$$\begin{aligned} & (bcy + b^2y) + (c^2z + bcz) + (cd + bd) \\ & by(c + b) + cz(c + b) + d(c + b) \\ & = \boxed{(by + cz + d)(c + b)} \end{aligned}$$

Check:

$$\begin{aligned} & \overbrace{(by + cz + d)(c + b)} \\ & bcy + b^2y + c^2z + bcz + dc + bd \quad \checkmark \end{aligned}$$

6.  $4x^2 + 6x + 10x + 15$

$$(4x^2 + 6x) + (10x + 15)$$

$$2x(2x+3) + 5(2x+3)$$

$$= \boxed{(2x+5)(2x+3)}$$

Check

$$(2x+5)(2x+3)$$

$$4x^2 + 6x + 10x + 15 \checkmark$$

or  $4x^2 + 16x + 15$

7.  $2ac + 3c - 4ab - 6b$

$$(2ac + 3c) + (-4ab - 6b)$$

$$c(2a+3) - 2b(2a+3)$$

$$= \boxed{(c-2b)(2a+3)}$$

Check

$$(c-2b)(2a+3)$$

$$2ac + 3c - 4ab - 6b \checkmark$$

8.  $np + 12m^2 - 4mn - 3mp$

$$(np - 4mn) + (12m^2 - 3mp)$$

$$n(p-4m) - 3m(-4m+p)$$

$$(n-3m)(p-4m)$$

Check

$$(n-3m)(p-4m)$$

$$np - 4mn - 3mp + 12m^2 \checkmark$$

9.  $9ax^2 - by - 3bx + 3axy$

$$(-by + 3axy) + (9ax^2 - 3bx)$$

$$y(-b+3ax) + 3x(3ax-b)$$

$$\boxed{(y+3x)(3ax-b)}$$

Check

$$(y+3x)(3ax-b)$$

$$3axy - by + 9ax^2 - 3bx \checkmark$$

10.  $f^2x^2 + g^2x^2 - ag^2 - af^2$

$$(f^2x^2 + g^2x^2) + (-ag^2 - af^2)$$

$$x^2(f^2+g^2) - a(g^2+f^2)$$

$$\boxed{(x^2-a)(f^2+g^2)}$$

Check

$$(x^2-a)(f^2+g^2)$$

$$f^2x^2 + g^2x^2 - af^2 - ag^2 \checkmark$$

11.  $m^2x + nx + ny + hy + hx + m^2y$

$$\begin{aligned} & (m^2x + m^2y) + (nx + ny) + (hy + hx) \\ & m^2(x+y) + n(x+y) + h(y+x) \\ & = \boxed{(m^2 + n + h)(x+y)} \end{aligned}$$

Check

$$\begin{aligned} & (m^2 + n + h)(x+y) \\ & m^2x + m^2y + nx + ny + \\ & \quad hx + hy \\ & \quad \checkmark \end{aligned}$$

12.  $6a^2nx - 3bcmz + 2acnz + 4abny - 6b^2my - 9abmx$

$$\begin{aligned} & (-3bcmz + 2acnz) + (-6b^2my + 4abny) + (-6a^2nx - 9abmx) \\ & cz(-3bm + 2an) + 2by(-3bm + 2an) + 3ax(2an - 3bm) \\ & = \boxed{(cz + 2by + 3ax)(-3bm + 2an)} \end{aligned}$$

13.  $x^4 - x^3 + x - 1$

$$\begin{aligned} & (x^4 - x^3) + (x - 1) \\ & x^3(x-1) + 1(x-1) \\ & = \boxed{(x^3 + 1)(x-1)} \end{aligned}$$

Check

$$\begin{aligned} & (x^3 + 1)(x-1) \\ & x^4 - x^3 + x - 1 \quad \checkmark \end{aligned}$$

14.  $2a^2x + 2py - a^2y - 4px + 5a^2z - 10pz$

$$\begin{aligned} & (2py - 4px - 10pz) + (2a^2x - a^2y + 5a^2z) \\ & 2p(y - 2x - 5z) - a^2(-2x + y - 5z) \\ & = \boxed{(2p - a^2)(-2x + y - 5z)} \end{aligned}$$

I'm skipping  
"check" due  
to lack  
of space!

15.  $-4y^4 + 1 - 2y^3 + 2y$

$$\begin{aligned} & (-4y^4 - 2y^3) + (1 + 2y) \\ & -2y^3(2y + 1) + 1(1 + 2y) \\ & = \boxed{(-2y^3 + 1)(2y + 1)} \end{aligned}$$

Check

$$\begin{aligned} & (-2y^3 + 1)(2y + 1) \\ & -4y^4 - 2y^3 + 2y + 1 \\ & \quad \checkmark \end{aligned}$$

$$16. \quad ay^2 - a + 3by^2 - 3b + 6ay^4 + 18by^4$$

$$\begin{aligned} & (-a - 3b) + (ay^2 + 3by^2) + (6ay^4 + 18by^4) \\ & -1(a + 3b) + y^2(a + 3b) + 6y^4(a + 3b) \\ & = \boxed{(-1 + y^2 + 6y^4)(a + 3b)} \end{aligned}$$

$$17. \quad 5m^3p - 5p - w + m^3w + 8m^4w + 40m^4p$$

$$\begin{aligned} & (-5p - w) + (5m^3p + m^3w) + (8m^4w + 40m^4p) \\ & -1(5p + w) + m^3(5p + w) + 8m^4(w + 5p) \\ & = \boxed{(-1 + m^3 + 8m^4)(5p + w)} \end{aligned}$$

$$18. \quad 21aq^5 - p - 7a + 7at^4 + pt^4 + 3pq^5$$

$$\begin{aligned} & (-p - 7a) + (pt^4 + 7at^4) + (3pq^5 + 21aq^5) \\ & -1(p + 7a) + t^4(p + 7a) + 3q^5(p + 7a) \\ & = \boxed{(-1 + t^4 + 3q^5)(p + 7a)} \end{aligned}$$

$$19. \quad -4a^5d + c + a^5c - 4d - 6b^7c + 24b^7d$$

$$\begin{aligned} & (c - 4d) + (-4a^5d + a^5c) + (-6b^7c + 24b^7d) \\ & 1(c - 4d) + a^5(-4d + c) - 6b^7(c - 4d) \\ & = \boxed{(1 + a^5 - 6b^7)(c - 4d)} \end{aligned}$$

$$20. \quad 3d^2x^2 + 5a - d^2 - 2d^2y^3 + 10ay^3 - 15ax^2$$

$$\begin{aligned} & (3d^2x^2 - d^2 - 2d^2y^3) + (5a + 10ay^3 - 15ax^2) \\ & d^2(3x^2 - 1 - 2y^3) - 5a(-1 - 2y^3 + 3x^2) \\ & = \boxed{(d^2 - 5a)(3x^2 - 1 - 2y^3)} \end{aligned}$$