

MTH-4106 Factoring and Algebraic Fractions: Worksheet #7

Factor the following polynomials using the appropriate method:

1. $14a^4y^6c^2 - 7y^2cd^3 + 21a^2yc^6e - 7yc$

$$7yc(2a^4y^5c - yd^3 + 3a^2c^5e - 1)$$

2. $\underline{abc} - \underline{c^2} - \underline{abd} + cd + \underline{abe} - ce$

$$(abc - abd + abc) + (-c^2 + cd - ce)$$

$$ab(c-d+e) - c(c-d+e)$$

$$\boxed{(ab-c)(c-d+e)}$$

3. $x^2 - 5x - 14$

$$\boxed{(x-7)(x+2)}$$

4. $3a^2 - 8ab + 4b^2$

$$p = 12$$

$$s = -8$$

$$-2, -6$$

$$(3a^2 - 2ab)(6ab + 4b^2)$$

$$a(3a - 2b) - 2b(3a - 2b)$$

$$\boxed{(a - 2b)(3a - 2b)}$$

5. $-16a^4 + 9b^2$

$$9b^2 - 16a^4$$

$$\boxed{(3b - 4a^2)(3b + 4a^2)}$$

6. $-4x^6 + 3x^5 + 6x^3 - x^2 - 2x$

$$\boxed{x(-4x^5 + 3x^4 + 6x^2 - x - 2)}$$

$$\boxed{\text{OR } -x(4x^5 - 3x^4 - 6x^2 + x + 2)}$$

$$7. \underline{6ax^2 - 2a + 15x^2y - 5y}$$

$$(6ax^2 + 15x^2y) + (-2a - 5y)$$

$$\underline{3x^2(2a + 5y) - 1(2a + 5y)}$$

$$\boxed{(3x^2 - 1)(2a + 5y)}$$

$$8. \underline{2 + b - b^2}$$

$$-b^2 + b + 2$$

$$(-b^2 + 2b) + 1(b + 2)$$

$$p = -2$$

$$b(-b + 2) + 1(-b + 2)$$

$$s = +1$$

$$2, -1$$

$$\boxed{(b+1)(-b+2)}$$

$$9. \underline{3y^2 + 7y - 6}$$

$$p = -18$$

$$(3y^2 + 9y) + (-2y - 6)$$

$$s = +7$$

$$3y(y + 3) - 2(y + 3)$$

$$9, -2$$

$$\boxed{(3y - 2)(y + 3)}$$

$$10. \underline{x^{16} - 16y^4}$$

$$\boxed{(x^8 - 4y^2)(x^8 + 4y^2)}$$

$$11. \underline{8b^2x^4d^3 - 32b^3d^4x + 16byx}$$

$$\boxed{8bx(b^3d^3 - 4b^2d^4 + 2y)}$$

$$12. \underline{-2bxy + 6axy + 4ax^2 - 3by^2}$$

$$(-2bxy - 3by^2) + (4ax^2 + 6ax + 4)$$

$$-by(2x + 3y) + 2ax(2x + 3y)$$

$$\boxed{(-by + 2ax)(2x + 3y)}$$

13. $x^2 - 8x + 15$

$$\boxed{(x-3)(x-5)}$$

14. $-3x^2 + 7xy - 2y^2$

$$P = 6$$

$$S = 7$$

$$6, 1$$

$$(-3x^2 + 6xy) + (1xy - 2y^2)$$

$$-3x(x-2y) + y(x-2y)$$

$$\boxed{(-3x+y)(x-2y)}$$

15. $t^2 - 1$

$$\boxed{(t+1)(t-1)}$$

16. $-6p^3q^5 + 8p^2r^6 - 12p^5r^7 + 14r^3s^2 - 2s^3t^4$

$$2 \left(-3p^3q^5 + 4p^2r^6 - 6p^5r^7 + 7r^3s^2 - s^3t^4 \right)$$

17. $\underline{ax} - \underline{bx} + \underline{by} + \underline{cy} - \underline{cx} - \underline{ay}$

$$(ax - bx - cx) + (-ay + by + cy)$$

$$\times (a - b - c) - y(a - b - c)$$

$$\boxed{(x-y)(a-b-c)}$$

18. $2 - 3b + b^2$

$$b^2 - 3b + 2$$

$$\boxed{(b-2)(b-1)}$$

19. $21g^2 - 8gf - 5f^2$

$P = -105$

$s = -8$

$-15, +7$

$$\begin{aligned} & (21g^2 - 15gf) + (7gf - 5f^2) \\ & 3g(7g - 5f) + f(7g - 5f) \\ & \boxed{(3g+f)(7g-5f)} \end{aligned}$$

20. $4 - m^2$

$$\boxed{(2-m)(2+m)}$$

21. $xy - x^2$

$$\boxed{x(y-x)}$$

22. $a^2x + abx + 2ac + 3aby + 3b^2y + 2bc$

$$\begin{aligned} & (a^2x + abx) + (2ac + 2bc) + (3aby + 3b^2y) \\ & ax(a+b) + 2c(a+b) + 3by(a+b) \\ & \boxed{(ax+2c+3by)(a+b)} \end{aligned}$$

23. $x^2 + 9x + 20$

$$\boxed{(x+4)(x+5)}$$

24. $3t^2 + 7t - 6$

$P = -18$

$s = +7$

$-2, 9$

$$\begin{aligned} & (3t^2 - 2t) + (9t - 6) \\ & t(3t-2) + 3(3t-2) \\ & \boxed{(t+3)(3t-2)} \end{aligned}$$

25. $16ab^4 - 25b^2$

$$\boxed{b^2(16ab^2 - 25)}$$

26. $6x - 4x^2$

$$\boxed{2x(3 - 2x)}$$

27. $2a^2b^3y + 3ab^3x^2 - y + ab^3y - 3x^2 + 6a^2b^3x^2$

$$(2a^2b^3y + 6a^2b^3x^2) + (ab^3y + 3ab^3x^2) + (-y - 3x^2)$$

$$2a^2b^3(y + 3x^2) + ab^3(y + 3x^2) - 1(y + 3x^2)$$

$$\boxed{(2a^2b^3 + ab^3 - 1)(y + 3x^2)}$$

28. $x^2 - x - 42$

$$\boxed{(x - 7)(x + 6)}$$



29. $35 - 38x + 8x^2$

$$8x^2 - 38x + 35$$

$$P = 280$$

$$S = -38$$

$$-28, -10$$

30. $121 - 1.96y^6$

$$(8x^2 - 28x)^+(-10x + 35)$$

$$4x(2x - 7) - 5(2x - 7)$$

$$\boxed{(4x - 5)(2x - 7)}$$

$$\boxed{(11 + 1.4y^3)(11 - 1.4y^3)}$$

31. $4a^2 - a$

$$\boxed{a(4a-1)}$$

32. $12x^3 + 24x^2 - 5y^2 + 20xy^2 - 6x + 10x^2y^2$

$$\begin{aligned} & (-5y^2 - 6x) + (10x^2y^2 + 12x^3) + (20xy^2 + 24x^2) \\ & - 1(5y^2 + 6x) + 2x^2(5y^2 + 6x) + 4x(5y^2 + 6x) \\ & \boxed{(4x + 2x^2 - 1)(5y^2 + 6x)} \end{aligned}$$

33. $x^2 - xy - 72y^2$

$$\boxed{(x-9y)(x+8y)}$$

34. $-4x^2 + 23xy - 15y^2$

$p = 60$

$s = 23$

$20, 3$

$$\begin{aligned} & (-4x^2 + 20xy) + (3xy - 15y^2) \\ & - 4x(x - 5y) + 3y(x - 5y) \end{aligned}$$

$$\boxed{(-4x + 3y)(x - 5y)}$$

35. $\frac{25x^2}{16} - 81y^2$

$$\boxed{\left(\frac{5x}{4} - 9y\right)\left(\frac{5x}{4} + 9y\right)}$$

36. $2x^3 - 3x^2$

$$\boxed{x^2(2x - 3)}$$

7

✓

37. $12m^2 - 11m + 2$ $(12m^2 - 3m) + (-8m + 2)$

$P = 24$ $3m(4m-1) - 2(4m-1)$

$S = -11$ $\boxed{(3m-2)(4m-1)}$

-3, -8

38. $8xy^5 - 4m^2ny^3 - 3m^3n^2 + 6mnxy^2 - 3mn - 4y^3$

$$\frac{(6mnxy^2 + 8xy^5) + (-3mn - 4y^3) + (-3m^3n^2 - 4m^2ny^3)}{2xy^2(3mn + 4y^3) - 1(3mn + 4y^3) - m^2n(3mn + 4y^3)}$$

$$\boxed{(2xy^2 - 1 - m^2n)(3mn + 4y^3)}$$

39. $k^2 - 11kl - 180l^2$

$P = -180$ $\boxed{(k + 9l)(k - 20l)}$

$S = -11$

9, -20

40. $1 - c^2$

$$\boxed{(1-c)(1+c)}$$

✓

41. $-6x^2 - 11x - 4$ $(-6x^2 - 3x) + (-8x - 4)$

$P = 24$ $-3x(2x+1) - 4(2x+1)$

$S = -11$ $\boxed{(-3x-4)(2x+1)}$

-3, -8

42. $-2b^3 - 6b^2$

$\boxed{-2b^2(b+3)}$ OR $\boxed{2b^2(-b-3)}$

43. $c^2 - 27cd + 50d^2$

$$(c-2d)(c-25d)$$

44. $-a^2 + 6ab - 8b^2$

$P = 8$
 $S = 6$
 $2,4$

$$\begin{aligned} & (-a^2 + 2ab) + (4ab - 8b^2) \\ & -a(a-2b) + 4b(a-2b) \\ & \boxed{(a+4b)(a-2b)} \end{aligned}$$

45. $-x^4 + 25$

$$25 - x^4 = \boxed{(5-x^2)(5+x^2)}$$

46. $7t^2 - 17tu + 6u^2$

$P = 42$
 $S = -17$
 $-14, -3$

$$\begin{aligned} & (7t^2 - 14tu) + (-3tu + 6u^2) \\ & 7t(t-2u) - 3u(t-2u) \\ & \boxed{(7t-3u)(t-2u)} \end{aligned}$$

47. $9wz^2 - 81z^4$

$$\boxed{9z^2(w-9z^2)}$$

48. $(7a^2c^2 - 21c) + (9a^4c + 27a^2)$

$7c(a^2c - 3) - 9a^2(a^2c - 3)$

$$\boxed{(7c-9a^2)(a^2c-3)}$$

49. $6p^2 - 13pr + 2r^2$

$$P = 12$$

$$S = -13$$

$$-12, -1$$

9

$$(6p^2 - 12pr) \cancel{(-1pr + 2r^2)}$$

$$6p(p-2r) - r(p-2r)$$

$$\boxed{(6p-r)(p-2r)}$$

50. $\frac{9n^2}{36} - 49p^2$

$$\left(\frac{3n}{6} + 7p\right)\left(\frac{3n}{6} - 7p\right) = \boxed{\left(\frac{n}{2} + 7p\right)\left(\frac{n}{2} - 7p\right)}$$

51. $6y^2 + 7yz - 3z^2$

$$P = -18$$

$$S = 7$$

$$9, -2$$

$$(6y^2 + 9yz) \cancel{(-2yz - 3z^2)}$$

$$3y(2y+3z) - z(2y+3z)$$

$$\boxed{(3y-z)(2y+3z)}$$

52. $a^2 + a - 42$

$$\boxed{(a+7)(a-6)}$$

53. $225 - 1.69w^2$

$$\boxed{(15 - 1.3w)(15 + 1.3w)}$$

$$(-1)(1.3w - 15)(1.3 + 15)$$

54. $35a^6b^5c + 42a^5b^4cd^2 - 28a^7b^3 - 21a^3b^4 + 14a^4b^3 - 7a^3b^2$

$$\boxed{7a^3b^2(5a^3b^3c + 6a^2b^2cd^2 - 4a^4b - 3b^2 + 2ab - 1)}$$

55. $3by^2 - 3b - c + 6bz^3 + cy^2 + 2cz^3$

$$(2cz^3 + 6bz^3) + (cy^2 + 3by^2) + (-c - 3b)$$

$$2z^3(c + 3b) + y^2(c + 3b) - 1(c + 3b)$$

$$\boxed{(2z^3 + y^2 - 1)(c + 3b)}$$

56. $9 - x^2$

$$\boxed{(3-x)(3+x)} = (-1)(x-3)(x+3)$$



57. $9r^2 - 6r + 1$

$$P=9$$

$$S=-6$$

$$-3, -3$$

$$(9r^2 - 3r) \cancel{+} - 3r + 1 \\ 3r(3r-1) - 1(3r-1)$$

$$\boxed{(3r-1)(3r-1)}$$

58. $x^2 - 7x + 10$

$$\boxed{(x-2)(x-5)}$$

59. $2a^2 + a$

$$\boxed{a(2a+1)}$$



60. $2q^2 - 5qr + 2r^2$

$$(2q^2 - 4qr) \cancel{+} - 1qr + 2r^2$$

$$P=4$$

$$S=-5$$

$$-4, -1$$

$$2q(q-2r) - r(q-2r)$$

$$\boxed{(2q-r)(q-2r)}$$