

MTH-4106 Factoring and Algebraic Fractions: **Worksheet #8**Factor the following polynomials **completely**:

e.g. 1 $3x^2 + 6x - 9$

$3(x^2 + 2x - 3)$

$$3(x+3)(x-1)$$

e.g. 2 $3x^3 - 9x^2 - 12x$

$3x(x^2 - 3x - 4)$

$$3x(x-4)(x+1)$$

e.g. 3 $2p^2 - 4pq - 48q^2$

$2(p^2 - 2pq - 24q^2)$

$$2(p-6q)(p+4q)$$

e.g. 4 $8y^4 - 2y^3z - 6y^2z^2$

$2y^2(4y^2 - yz - 3z^2)$

prod = -12

sum = -1 -4, +3

$$(4y^2 - 4yz) + (3yz - 3z^2)$$
$$4y(y-z) + 3z(y-z)$$
$$(4y+3z)(y-z)$$

$$2y^2(4y+3z)(y-z)$$

e.g. 5 $27ax^2 - 48ay^2$

$3a(9x^2 - 16y^2)$

$$3a(3x+4y)(3x-4y)$$

e.g. 6 $16a^4 - 81b^4$

$$(4a^2 - 9b^2)(4a^2 + 9b^2)$$

$$(2a-3b)(2a+3b)$$

e.g. 7 $2abc^2 + 6abc - 8abc - 24ab$

$$2ab(c^2 + 3c - 4c - 12)$$

$$(c^2 + 3c) + (-4c - 12)$$

$$c(c+3) - 4(c+3)$$

$$2ab(c-4)(c+3)$$

Now you do: $(c-4)(c+3)$

1. $3ax - 3ax^3$

$$3ax(1 - x^2)$$

$$3ax(1-x)(1+x)$$

2. $6x^3y - 72x^2 - 18x^2y + 24x^3$

$$6x^2(xy - 12 - 3y + 4x)$$

$$(xy - 3y) + (4x - 12)$$

$$y(x-3) + 4(x-3)$$

$$6x^2(y+4)(x-3)$$

3. $-4x^2y^4 + 2x^3y^3 + 12x^4y^2$

$$2x^2y^2(-2y^2 + xy + 6x^2)$$

$$\text{prod} = -12$$

$$\text{sum} = +1$$

$$+4, -3$$

$$(-2y^2 + 4xy)(3xy + 6x^2)$$

$$2y(-y + 2x) + 3x(-y + 2x)$$

$$(2y+3x)(-y+2x)$$

$$2x^2y^2(2y+3x)(-y+2x)$$

4. $24x^2 + 12x + 8xy + 4y$

$$4(6x^2 + 3x + 2xy + y)$$

$$(6x^2 + 3x) + (2xy + y)$$

$$3x(2x+1) + y(2x+1)$$

$$(3x+y)(2x+1)$$

$$4(3x+y)(2x+1)$$

5. $3d^2e^6 - 48d^2f^8$

$$3d^2(e^6 - 16f^8)$$

$$3d^2(e^3 - 4f^4)(e^3 + 4f^4)$$

6. $12a^4 + 32a^3 - 12a^2$
 $4a^2(3a^2 + 8a - 3)$

$p = -9$
 $s = 8$
 $9, -1$

7. $45x^4y^3 - 39x^3y^4 + 6x^2y^5$
 $3x^2y^3(15x^2 - 13xy + 2y^2)$

$p = 30$
 $s = -13$
 $-3, -10$

8. $16x^4y^2 - 68x^3y^3 + 16x^2y^4$
 $4x^2y^2(4x^2 - 17xy + 4y^2)$

prod = 16
sum = -17
 $-16, -1$

9. $9a^2 - 36y^4$
 $9(a^2 - 4y^4)$
 $9(a - 2y^2)(a + 2y^2)$

10. $4 - 4b^2$
 $4(1 - b^2)$
 $4(1 - b)(1 + b)$

11. $2y^3 + 6y + 2y^2 + 6$
 $2(y^3 + 3y + y^2 + 3)$
 $(y^3 + y^2) + (3y + 3)$
 $y^2(y + 1) + 3(y + 1)$
 $(y^2 + 3)(y + 1)$

3
 $(3a^2 + 9a)(-1a - 3)$
 $3a(a + 3) - 1(a + 3)$
 $(3a - 1)(a + 3)$

$4a^2(3a - 1)(a + 3)$

$(15x^2 - 3xy)(-10xy + 2y^2)$
 $3x(5x - y) - 2y(5x - y)$

$(3x - 2y)(5x - y)$

$3x^2y^3(3x - 2y)(5x - y)$

$(4x^2 - 16xy)(1xy + 4y^2)$
 $4x(x - 4y) - y(x - 4y)$

$(4x - y)(x - 4y)$

$4x^2y^2(4x - y)(x - 4y)$

$2(y^2 + 3)(y + 1)$

12.

$$6b^2 - 21b - 45$$

$$3(2b^2 - 7b - 15)$$

$\text{prod} = -30$
 $\text{sum} = -7$
 $3, -10$

$$(2b^2 + 3b)(10b - 15)$$

$$b(2b+3) - 5(2b+3)$$

$$(b-5)(2b+3)$$

$$\boxed{3(b-5)(2b+3)}$$

13.

$$3x^2 + 6xy + 3y^2$$

$$3(x^2 + 2xy + y^2)$$

$$\boxed{3(x+y)(x+y)}$$

$$\text{or } \boxed{3(x+y)^2}$$

14.

$$5x^2 - 5y^2$$

$$5(x^2 - y^2)$$

$$\boxed{5(x+y)(x-y)}$$

15.

$$5x^2 + 35xy + 30y^2$$

$$5(x^2 + 7xy + 6y^2)$$

$$\boxed{5(x+6y)(x+y)}$$

16.

$$50 - 2a^2$$

$$2(25 - a^2)$$

$$\boxed{2(5-a)(5+a)}$$

17.

$$5x^2 + 30x - 35$$

$$5(x^2 + 6x - 7)$$

$$\boxed{5(x+7)(x-1)}$$

18.

$$3x^5y^3 + x^4y^4 - 2x^3y^5$$

$$x^3y^3(3x^2 + xy - 2y^2)$$

prod = -6
sum = +1
+3, -2

$$(3x^2 + 3xy)(-2xy - 2y^2)$$

$$3x(x+y) - 2y(x+y)$$

$$(3x-2y)(x+y)$$

$$x^3y^3(3x-2y)(x+y)$$

19.

$$b^2y^2 - 4b^2$$

$$b^2(y^2 - 4)$$

$$b^2(y-2)(y+2)$$

20.

$$27a^2b^2 - 12a^4$$

$$3a^2(9b^2 - 4a^2)$$

$$3a^2(3b+2a)(3b-2a)$$

21.

$$-3x^3y + 10x^2y^2 - 3xy^3$$

$$xy(-3x^2 + 10xy - 3y^2)$$

p = 9
s = 10
9, 1

$$(-3x^2 + 9xy)(xy - 3y^2)$$

$$-3x(x-3y) + y(x-3y)$$

$$(-3x+y)(x-3y)$$

$$xy(-3x+y)(x-3y)$$

22.

$$4m^3 - 9m^2 + 2m$$

$$m(4m^2 - 9m + 2)$$

p = 8
s = -9
-8, -1

$$(4m^2 - 8m)(-1m + 2)$$

$$4m(m-2) - 1(m-2)$$

$$(4m-1)(m-2)$$

$$m(4m-1)(m-2)$$

23.

$$-3x^3 - 8x^2 + 3x$$

$$x(-3x^2 - 8x + 3)$$

p = -9
s = -8
-9, +1

$$(-3x^2 - 9x)(x+3)$$

$$-3x(x+3) + 1(x+3)$$

$$(-3x+1)(x+3)$$

$$x(-3x+1)(x+3)$$

24. $12x^2 + 4xy + 6x + 2y$

$$2(6x^2 + 2xy + 3x + y)$$

$$(3x + 6x^2) + (y + 2xy)$$

$$3x(1 + 2x) + y(1 + 2x)$$

$$(3x + y)(1 + 2x)$$

$$2(3x + y)(1 + 2x)$$

25. $2p^5q^2 - p^4q^3 - p^3q^4$

$$p^3q^2(2p^2 - pq - q^2)$$

$$p = -2$$

$$s = -1$$

$$-2, +1$$

$$(2p^2 - 2pq) + (pq - q^2)$$

$$2p(p - q) + q(p - q)$$

$$(2p + q)(p - q)$$

$$p^3q^2(2p + q)(p - q)$$

26. $4a^4b + a^3b^2 - 3a^2b^3$

$$a^2b(4a^2 + ab - 3b^2)$$

$$p = -12$$

$$s = +1$$

$$+4, -3$$

$$(4a^2 + 4ab) - (3ab - 3b^2)$$

$$4a(a + b) - 3b(a + b)$$

$$(4a - 3b)(a + b)$$

$$a^2b(4a - 3b)(a + b)$$

27. $3x^8z^3 - 75y^6z^3$

$$3z^3(x^8 - 25y^6)$$

$$3z^3(x^4 - 5y^3)(x^4 + 5y^3)$$

28. $18m^8n^2 - 2p^2n^2$

$$2n^2(9m^8 - p^2)$$

$$2n^2(3m^4 - p)(3m^4 + p)$$

29. $-18a^2x^3 + 15a^2x^2y - 3a^2xy^2$

$$3a^2x(-6x^2 + 5xy - y^2)$$

$$\text{prod} = +6$$

$$\text{sum} = +5$$

$$+2, +3$$

$$(-6x^2 + 2xy) + (3xy - y^2)$$

$$-2x(3x - y) + y(3x - y)$$

$$(-2x + y)(3x - y)$$

$$3a^2x(-2x + y)(3x - y)$$

30. $3a^2 - 9ab + 6b^2$

$$3(a^2 - 3ab + 2b^2)$$

$$\boxed{3(a-2b)(a-b)}$$

31. $32 - 2c^2$

$$2(16 - c^2)$$

$$\boxed{2(4-c)(4+c)}$$

32. $-2g^4h + 20g^3h^2 - 48g^2h^3$

$$2g^2h(-g^2 + 10gh - 24h^2)$$

$$\text{prod} = +24$$

$$\text{sum} = +10$$

$$4, 6$$

$$(-g^2 + 4gh)(6gh - 24h^2)$$

$$-g(g-4h) + 6h(g-4h)$$

$$(-g+6h)(g-4h)$$

$$\boxed{2g^2h(-g+6h)(g-4h)}$$

33. $7x^4 - 28$

$$7(x^4 - 4)$$

$$\boxed{7(x^2-2)(x^2+2)}$$

34. $12x^3 + 144x + 84x^2$

$$12x(x^2 + 12 + 7x)$$

$$12x(x^2 + 7x + 12) = \boxed{12x(x+3)(x+4)}$$

35. $(10rs - 5rt) + (20s^2 - 10st)$

$$5r(2s-t) + 10s(2s-t)$$

$$(5r + 10s)(2s-t)$$

$$\boxed{5(r+2s)(2s-t)}$$

36. $c^3 - 36a^4b^2c$

$$c(c^2 - 36a^4b^2)$$

$$\boxed{c(c - 6a^2b)(c + 6a^2b)}$$

37. $2a^4 - 20a^3b + 50a^2b^2$

$$2a^2(a^2 - 10ab + 25b^2)$$

$$\boxed{2a^2(a - 5b)(a - 5b)}$$

38. $-a^5 + 13a^4 - 42a^3$

$$a^3(-a^2 + 13a - 42)$$

$$\begin{aligned} \text{prod} &= 42 \\ \text{sum} &= 13 \\ &6, 7 \end{aligned}$$

$$\begin{aligned} &(-a^2 + 6a) + (7a - 42) \\ &-a(a - 6) + 7(a - 6) \end{aligned}$$

$$(-a + 7)(a - 6)$$

$$\boxed{a^3(-a + 7)(a - 6)}$$

39. $3 - 3t^2$

$$3(1 - t^2)$$

$$\boxed{3(1 - t)(1 + t)}$$

40. $2uvx - 4uvy - 4ux + 8uy$

$$2u(vx - 2vy - 2x + 4y)$$

$$(vx - 2vy) + (-2x + 4y)$$

$$v(x - 2y) - 2(x - 2y)$$

$$(v - 2)(x - 2y)$$

$$\boxed{2u(v - 2)(x - 2y)}$$