Answer key

MTH-4107: Worksheet#4 **Finding Equations of Parallel and Perpendicular Lines**

1. Find the equation of the line which passes through $(8, \frac{1}{3})$ and is parallel to the line whose equation is $\frac{x}{4} = \frac{-y}{3}$.

$m_{l_1} = m_{l_2}$

$l_1$

$\frac{x}{4} = \frac{-y}{3}$

$\frac{y}{3} = \frac{-x}{4}$

$\frac{1}{3}y = -\frac{1}{4}x$

$\frac{3}{1}\left(\frac{1}{3}y\right) = \frac{3}{1}\left(-\frac{1}{4}x\right)$

$y = -\frac{3}{4}x$

$m = -\frac{3}{4}$

$l_2$ $\quad m = -\frac{3}{4}$ $\quad (8, \frac{1}{3})$

$y = mx + b$

$\frac{1}{3} = \left(-\frac{3}{4}\right)\left(\frac{8}{1}\right) + b$

$\frac{1}{3} = -6 + b$

$\frac{1}{3} + 6 = b$

$6\frac{1}{3} = b$

You could also do $\left(-\frac{3}{4}\right)\left(\frac{8}{1}\right) = \frac{-24}{4} = -6$

Equation: $y = -\frac{3}{4}x + 6\frac{1}{3}$ OR $y = -\frac{3}{4}x + \frac{19}{3}$

2. Find the equation of the line which passes through $(-8, -\frac{1}{4})$ and is perpendicular to the line whose equation is $x + \frac{y}{4} - 2 = 0$.

$l_1$

$x + \frac{y}{4} - 2 = 0$

$\frac{1}{4}y = -x + 2$

$\frac{4}{1}\left(\frac{1}{4}y\right) = \frac{4}{1}(-x + 2)$

$y = -4x + 8$

$m = -4$

$m_{l_1} \perp m_{l_2}$

$m_{l_2} = +\frac{1}{4}$

$l_2$ $\quad m = \frac{1}{4}$ $\quad (-8, -\frac{1}{4})$

$y = mx + b$

$-\frac{1}{4} = \left(\frac{1}{4}\right)\left(\frac{-8}{1}\right) + b$

$-\frac{1}{4} = -\frac{8}{4} + b$

$-\frac{1}{4} + \frac{8}{4} = b$

$\frac{7}{4} = b$

Equation: $y = \frac{1}{4}x + \frac{7}{4}$ OR $y = \frac{1}{4}x + 1\frac{3}{4}$

3. Find the equation of the line which passes through point $(-\frac{6}{5}, -\frac{4}{5})$ and is parallel to the line whose equation is $\frac{1}{3}x + 2y - 2 = 0$.

$l_1$

$\frac{1}{3}x + 2y - 2 = 0$

$2y = -\frac{1}{3}x + 2$

$\frac{2y}{2} = \frac{-\frac{1}{3}x}{2} + \frac{2}{2}$

$-\frac{1}{3} \div \frac{2}{1} = -\frac{1}{3} \times \frac{1}{2} = \frac{-1}{6}$

$y = -\frac{1}{6}x + 1$

$m = -\frac{1}{6}$

$m_{l_1} = m_{l_2}$

$l_2$ $m = -\frac{1}{6}$ $(-\frac{6}{5}, -\frac{4}{5})$

$y = mx + b$

$-\frac{4}{5} = \left(-\frac{1}{6}\right)\left(-\frac{6}{5}\right) + b$

$-\frac{4}{5} = \frac{6}{30} + b$

$-\frac{4}{5} = \frac{1}{5} + b$

$-\frac{4}{5} - \frac{1}{5} = b$

$-1 = -\frac{5}{5} = b$

Equation:

$y = -\frac{1}{6}x - 1$

4. Find the equation of the line which passes through point $(\frac{5}{6}, -\frac{2}{3})$ and is perpendicular to the line whose equation is $\frac{3x}{7} = \frac{-y}{2}$.

$l_1$

$\frac{3x}{7} = \frac{-y}{2}$

$\frac{y}{2} = -\frac{3x}{7}$

$\frac{1}{2}y = -\frac{3}{7}x$

$\frac{2}{1}\left(\frac{1}{2}y\right) = \frac{2}{1}\left(-\frac{3}{7}x\right)$

$y = -\frac{6}{7}x$

$m = -\frac{6}{7}$

$m_{l_1} \perp m_{l_2}$

$l_2$ $m = +\frac{7}{6}$ $(\frac{5}{6}, -\frac{2}{3})$

$y = mx + b$

$-\frac{2}{3} = \left(\frac{7}{6}\right)\left(\frac{5}{6}\right) + b$

$-\frac{2}{3} = \frac{35}{36} + b$

$-\frac{2}{3} - \frac{35}{36} = b$

$-\frac{2}{3} \cdot \frac{12}{12} - \frac{35}{36} = b$

$-\frac{24}{36} - \frac{35}{36} = b$

$-\frac{59}{36} = b$

Equation:

$y = \frac{7}{6}x - \frac{59}{36}$

5. Find the equation of the line passing through point $(-\frac{1}{5}, -\frac{3}{5})$ given that this line is parallel to the line whose equation is $x - \frac{y}{5} = -1$.

$l_1$ $\parallel$ $l_2$ $\quad m = 5 \quad (-\frac{1}{5}, -\frac{3}{5})$

$m_1 = m_2$

$x - \frac{y}{5} = -1$

$-\frac{1}{5}y = -x - 1$

$-\frac{5}{1}\left(-\frac{1}{5}y\right) = -5(-x-1)$

$y = 5x + 5$

$m = 5$

$y = mx + b$

$-\frac{3}{5} = \left(\frac{5}{1}\right)\left(-\frac{1}{5}\right) + b$

$-\frac{3}{5} = \frac{-5}{5} + b$

$-\frac{3}{5} + \frac{5}{5} = b$

$\frac{2}{5} = b$

Eqn

$y = 5x + \frac{2}{5}$

6. Find the equation of the line passing through point ( -2, -1.5 ) given that this line is perpendicular to the line whose equation is $2x - 7y + 3 = 0$.

$l_1$ $\perp$ $l_2$ $\quad m = \frac{-7}{2} \quad (-2, -1.5$

$2x - 7y + 3 = 0$

$-7y = -2x - 3$

$\frac{-7y}{-7} = \frac{-2x}{-7} - \frac{3}{-7}$

$y = \frac{2}{7}x + \frac{3}{7}$

$m = \frac{2}{7}$

$y = mx + b$

$-1.5 = \left(-\frac{7}{2}\right)\left(\frac{-2}{1}\right) + b$

$-1.5 = 7 + b$

$-1.5 - 7 = b$

$-8.5 = b$

$-8\frac{1}{2}$ OR

$m_{l_1} \perp m_{l_2}$

Eqn: $y = -\frac{7}{2}x - 8.5$ OR $y = -\frac{7}{2}x - 8\frac{1}{2}$

7. Find the equation of the line which passes through $(\frac{4}{3}, 7.5)$ and is perpendicular to the line whose equation is $\frac{2}{3}x - \frac{3}{4}y + 3 = 0$.

$l_1$

$\frac{2}{3}x - \frac{3}{4}y + 3 = 0$

$-\frac{3}{4}y = -\frac{2}{3}x - 3$

$-\frac{4}{3}\left(-\frac{3}{4}y\right) = -\frac{4}{3}\left(-\frac{2}{3}x - 3\right)$

$y = \frac{8}{9}x + 4$

$m = \frac{8}{9}$

$m_{l_1} \perp m_{l_2}$

$l_2$ $m = -\frac{9}{8}$ $(\frac{4}{3}, 7.5)$

$y = mx + b$

$7.5 = \left(-\frac{9}{8}\right)\left(\frac{4}{3}\right) + b$

$7.5 = \frac{-36}{24} + b$

$7.5 = \frac{-36 \div 12}{24 \div 12} + b$

$7.5 = \frac{-3}{2} + b$

$7.5 + \frac{3}{2} = b$

$9 = b$

Equation

$y = -\frac{9}{8}x + 9$

8. Find the equation of the line which passes through $(-1, \frac{3}{5})$ and is parallel to the line whose equation is $\frac{2x}{5} = \frac{-y}{4}$.

$l_1$

$\frac{2x}{5} = \frac{-y}{4}$

$\frac{y}{4} = -\frac{2}{5}x$

$\frac{4}{1}\left(\frac{1}{4}y\right) = \frac{4}{1}\left(-\frac{2}{5}x\right)$

$y = -\frac{8}{5}x$

$m = -\frac{8}{5}$

$m_{l_1} \parallel m_{l_2}$

$l_2$ $m = -\frac{8}{5}$ $(-1, \frac{3}{5})$

$y = mx + b$

$\frac{3}{5} = \left(-\frac{8}{5}\right)\left(\frac{-1}{1}\right) + b$

$\frac{3}{5} = \frac{8}{5} + b$

$\frac{3}{5} - \frac{8}{5} = b$

$-\frac{5}{5} = b$

$-1 = b$

Equation

$y = -\frac{8}{5}x - 1$

9. Find the equation of the line which passes through $(2, \frac{1}{3})$ and is parallel to the line whose equation is $-\frac{x}{2} - \frac{y}{3} + 6 = 0$.

$l_1$

$-\frac{1}{2}x - \frac{1}{3}y + 6 = 0$

$-\frac{1}{3}y = \frac{1}{2}x - 6$

$-\frac{3}{1}\left(-\frac{1}{3}y\right) = \frac{-3}{1}\left(\frac{1}{2}x - 6\right)$

$y = -\frac{3}{2}x + 18$

$m = -\frac{3}{2}$

$l_2$ $m = -\frac{3}{2}$ $(2, \frac{1}{3})$

$y = mx + b$

$\frac{1}{3} = \left(-\frac{3}{2}\right)\left(\frac{2}{1}\right) + b$

$\frac{1}{3} = -\frac{6}{2} + b$

$\frac{1}{3} = -3 + b$

$\frac{1}{3} + 3 = b$

$3\frac{1}{3} = b$

OR $\frac{10}{3}$

Eqn

$y = -\frac{3}{2}x + 3\frac{1}{3}$

OR

$y = -\frac{3}{2}x + \frac{10}{3}$

10. Find the equation of the line which passes through $(\frac{1}{5}, -\frac{1}{7})$ and is perpendicular to the line whose equation is $\frac{3}{5}x - 3y + 1 = 0$.

$l_1$

$\frac{3}{5}x - 3y + 1 = 0$

$-3y = -\frac{3}{5}x - 1$

$\frac{-3y}{-3} = \frac{-\frac{3}{5}x}{-3} - \frac{1}{-3}$

$y = \frac{1}{5}x + \frac{1}{3}$

$m = \frac{1}{5}$

$-\frac{3}{5} \div \frac{-3}{1}$

$= -\frac{3}{5} \times \frac{-1}{3} = \frac{3}{15} = \frac{1}{5}$

$l_2$ $m = -5$ $(\frac{1}{5}, \frac{-1}{7})$

$y = mx + b$

$-\frac{1}{7} = \left(\frac{-5}{1}\right)\left(\frac{1}{5}\right) + b$

$-\frac{1}{7} = \frac{-5}{5} + b$

$-\frac{1}{7} = -1 + b$

$-\frac{1}{7} + 1 = b$

$-\frac{1}{7} + \frac{7}{7} = b$

$\frac{6}{7} = b$

Equation

$y = -5x + \frac{6}{7}$